

TAXATION, CORPORATE GROWTH AND FINANCIAL POLICY

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Abstract

This paper presents a corporate growth model in which investment and financial policies are simultaneously determined under the constraint that the corporate financial structure affects its borrowing rate of interest. Also, the effects of taxation and the degree of the bank-borrower (corporate) relation on investment are examined. As a result, it is shown that close bank-corporate relationship raises the corporate growth rate and its debt-capital ratio, and that the effect of tax rates on the growth rate become ambiguous due to the indirect effects through changes of financial policies.

1. Introduction

Over recent years a considerable number of theoretical and empirical studies have been made on the linkage between corporate investment

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and its financial structure and/or financial policy. The preferred theoretical description of investment is Q theory in which the firm faces strictly convex costs in adjusting its capital stock, and it derives the unique relation between investment and the so-called (marginal) Q. The theory of optimal financial structure and policy has been developed in the way of relaxing conditions for the well-known Modigliani and Miller [1958] irrelevance theorem in order to understand the real-world 'costs of capital'. As shown in Fazzare *et al* [1988], it has widely been recognized that "financing hierarchy" or "pecking order" theories have been useful to understand the above-mentioned linkage.

Since Stiglitz [1973], King [1974] and Auerbach [1979a, 1979b], financing hierarchy theories have correctly pointed out that the cost of new share issue (equity finance) is higher than that of retention earning (internal finance). Also, it is often assumed or concluded that the cost of (new) debt finance is larger than that of internal finance¹⁾. Because debt finance, particularly long-term debt, causes financial distress and agency problems, its cost increases with its level. If, however, there is a corporate income tax under which the interest payment is a deductive expense, having debt may reduce capital costs. At least in the steady (or stationary) state, the marginal benefit of the issue of debt may be equal to its marginal cost if the firm is rational.

Financing hierarchy theories also show that the firm never pay dividends when (new) debt and/or equity financing. In other words, it pays dividends only when the internal retention is larger than the total cost of investment. But in the real world the corporation pays dividends with debt and/or debt finance.

1) See, for instance, Chirinko [1987] and Hayashi [1985]

Little attention has been given to the point that the corporate financial structure affects the schedule of marginal efficiency of investment. Chirinko [1987] and Hayashi [1985] analyze this in the dynamic or quasi-dynamic framework. Usually, however, the schedule of marginal efficiency of investment is given independently from the corporate financial structure.

Aforementioned points are not shortcomings of these theories but show that there remain missing cases or regimes in financing hierarchy theories. In these cases the corporate does not face severe budget constraints so that it does not need the equity finance and even can use debt finance as the instrument to reduce the corporate tax payment. The main purpose of paper is, therefore, to throw a new light on these cases, and to explain the relation between the corporate growth and its financial policy. In the model discussed here the equity finance is not considered because we focus on the above cases.

As studies have been developed on the modification of theory of investment by taking account for the corporate capital structure, the importance of roles of financial intermediaries and of bank-borrower (corporate) relationships has been widely and deeply recognized.

Especially, a number of economists suggest that there are close bank-borrower (corporate) relationships in Japan. Nakatani [1984] stresses the existence of financial corporate groups and Horiuchi *et al* [1988] investigates the role of the well-known "*main bank*" relation. Some empirical studies, such as Hoshi and Kashyap [1990] and Hoshi *et al* [1991], investigate the relation between investment behavior of Japanese firms and their corporate groups. One of these paper's purposes is to examine effects of these relations on the corporate growth and its financial policy.

The organization of this paper is as follows. The next section presents the model discussed in this paper. Section-3 shows the conditions for optimality and checks the stability. Section-4 analyzes properties in the steady state. Section-5 investigates the transitional dynamic path. And final section provides some concluding remarks.

2. The Model

In this paper, a firm is assumed to act so as to maximize the market value of its existing equities $V(t)$.

The equilibrium (no-arbitrage) condition is:

$$(1-\theta_t) i_t P_t N_t = (1-c_t) \dot{P}_t N_t + (1-\theta_t) D_t, \quad (1)$$

where $\theta_t \equiv$ personal income tax rate, $i_t \equiv$ rate of return of riskless asset, $P_t \equiv$ equity price, $N_t \equiv$ number of existing equities, $c_t \equiv$ capital gains tax rate, $D_t \equiv$ dividend.

The instantaneous budget constraint of the firm is:

$$\Pi(K) + \dot{B} = rB + \Phi(I, K) + \tau [\Pi(K) - \Phi(I, K) - rB] + D, \quad (2)$$

where $\Pi(K) \equiv \max_L F(L, K) - WL$; value added function,

$L \equiv$ labor input, $I \equiv$ gross investment $K \equiv$ capital stock, $W \equiv$ real wage rate, $\Phi(\cdot) \equiv$ costs for investment (including adjustment cost), $B \equiv$ market value of debt issued, $r \equiv$ borrowing rate of interest, $\tau \equiv$ corporate tax rate, a dot ($\dot{\cdot}$) over the variable indicates its time derivative.

(Time arguments are suppressed when no ambiguity results.)

For the sake of simplicity, exogenous variables i , W and tax rates θ , c , τ and functions $F(\cdot)$ and $\Phi(\cdot)$ are assumed be constant over time.

The production structure of the firm is specified using the following simplifying assumptions.

(A.1) The production function is linearly homogeneous and has following properties,

$$F(L,K) = f(n)K, \quad n \equiv L/K, \quad f' > 0, \quad f'' < 0. \quad (3)$$

(A.2) The investment cost function is also linearly homogeneous and strictly convex,

$$\Phi(I,K) = \phi(g)K, \quad g \equiv I/K, \quad \phi' > 0, \quad \phi > 0. \quad (4)$$

From the above assumptions, the expression (2) is transformed as

$$d = (1-\tau) [\pi - \phi(g) - rb] + \beta b, \quad (5)$$

where $d \equiv D/K$, $\pi \equiv \max_n f(n) - Wn$, $b \equiv B/K$, $\beta \equiv \dot{B}/B$.

The firm's behavior must obey the following two dynamic constraints,

$$\dot{K} = (g - \delta) K, \quad K_t : \text{given}, \quad (6)$$

$$\dot{b} = (\beta - g + \delta) b, \quad b_t : \text{given}, \quad (7)$$

where $\delta \equiv$ physical depreciation rate of capital stock.

As in the real world, borrowing rates of interest may be differentiated according to firms' financial structures²⁾ and be affected by the increment of debt³⁾, we assume that the borrowing rate of interest is function of the debt-capital ratio b and the growth rate of debt β .

$$r=r(b,\beta), \quad r(0,0)=i, \quad r(b,0)>i \quad \text{for } b>0^4), \quad (8)$$

To simplify the analysis, we further assume that

$$\begin{aligned} r=r(b, \beta) &= \alpha(b; \gamma) + c(\beta; \eta), & (9) \\ \alpha_b &> 0, \quad \alpha_{bb} > 0, \quad \alpha_\gamma < 0, \\ c &> 0, \quad 0 < c_\beta < 1, \quad c_{\beta\beta} > 0, \quad c_\eta < 0, & \text{for } \beta > 0, \\ c &= 0, \quad c_\beta < 0, \quad c_\eta = 0, & \text{for } \beta \leq 0, \end{aligned}$$

where γ & $\eta \equiv$ shift parameters.

The close bank-borrower (corporate) relationships may economize on the information costs and reduce the agency costs. Especially, the main bank system and the financial corporate groups in Japan reduce

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- 2) The main reason for this is that the probability of bankruptcy depends on not the size of debt but the debt-capital ratio. This is pointed out by Hayashi (1985), too. Auerbach (1979b) suggests that r depends on the corporate leverage.
- 3) If the bond of a firm is differentiated from others, then its demand curve is downward-sloping. Though the main reason for this intuition is this demand curve does not shift instantaneously, it also implicitly includes bond issue and transactions costs.
- 4) In fact, $r(0,0)$ exists not actually but notionally. From the viewpoint of a lender, $i = E[r(0,0)]$, where $E[\cdot] \equiv$ expectation operator.

them largely through sharing risks between banks and borrowers. The large γ reflects these situations. The large η , on the other hand, implies the financing capacity of a firm itself in the bond market is large, and vice versa.

Dividend never be negative. This imposes the next constraint.

$$(1-\tau) \{ \pi - \Phi(g) - (\alpha + c) b \} + \beta b \geq 0^{5)}. \quad (10)$$

Now we define the market value of its existing equities is to be:

$$V \equiv PN. \quad (11)$$

From (1), (2), (5), (9) and (11), we obtaine the following differential equation.

$$\dot{V} = mV - m [(1-\tau) \{ \pi - \Phi(g) - (\alpha(b;\gamma) + c(\beta;\eta)) b \} + \beta b] K, \quad (12)$$

where $m \equiv (1-\theta)/(1-c)$.

Solving (12), the maximand $V(t)$ becomes:

$$V(t) = \int_t^{\infty} m [(1-\tau) \{ \pi(\alpha+c)b \} - \Phi(g) + \beta b] K e^{-m(s-t)} ds. \quad (13)$$

The optimization problem for the firm is to determine β and g so as to miximize $V(t)$ under the constraints of (6) and (7).

5) From now on, the gross profit rate π is assumed to be large enough to make this constraint ineffective.

3. Optimal Conditions and Stability

The dynamic system, which characterizes the optimal state and costate trajectories, is:

$$\dot{K} = (g - \delta) K, \quad K_t : \text{given}, \quad (14)$$

$$\dot{b} = (\beta - g + \delta) b, \quad b_t : \text{given}, \quad (15)$$

$$\dot{\mu} = (mi - \beta)\mu + m \{ (1 - \tau) (\alpha + c + \alpha_b b) - \beta \}, \quad (16)$$

$$\lim_{s \rightarrow \infty} b_s \mu_s \exp \left[- \int_t^s \{ \rho + \delta - g(s) \} ds \right], \quad (17)$$

where variables β and g must satisfy the following equations,

$$m(1 - \tau) c_\beta = m + \mu, \quad (18)$$

$$\phi'(g) = \frac{(1 - \tau) \{ \pi - (\alpha + c) b \} - \phi(g)}{mi + \delta - g} + \frac{(m + \mu) \beta b - mi \mu b}{m(mi + \delta - g)}. \quad (19)$$

Eq. (18) says that the marginal (tax-adjusted) cost for new debt issue is equal to the marginal (implicit) value of holding debt. Also, (19) implies that the marginal cost of investment is equal to the marginal present value of the firm. In other words, this equilibrium condition shows the investment-(marginal) Q relation.

We get the partial derivatives of β and g with respect to b , μ , and other parameters from (18) and (19), as shown in Table-1.

	β	g
b	0	(*)
μ	+	-
γ	0	+
η	0	+
π	0	+
i	0	-
τ	+	-
m	+	-

$$(*) \begin{matrix} + \\ 0 \\ - \end{matrix} \leq \dot{\mu} \leq \begin{matrix} + \\ 0 \\ - \end{matrix}$$

Table-1 Partial derivatives of β and g .

The sub-system, which is described by equations (15), (16) and (17) is closed itself. After this sub-system is determined, recursively the remaining part of the full-system (the optimal path of capital stock $K(t)$) is determined. From now on, we focus on only this sub-system.

We limit our dynamic analysis to a neighborhood around the steady state. To do so, we linearize (15) and (16) around the steady state and express them in matrix form.

$$\begin{pmatrix} \dot{b} \\ \dot{\mu} \end{pmatrix} = \begin{pmatrix} 0 & \beta\mu - g\mu \\ m(1-\tau)(2\alpha_b + \alpha_{bb}) & mi + \delta - g \end{pmatrix} \begin{pmatrix} b - b^* \\ \mu - \mu^* \end{pmatrix} \quad (20)$$

where b^* and μ^* denote their steady state values, respectively. The above system has two real roots, which are opposite signs. To ensure it, the determinant of Jacobian matrix (Δ_1) must be negative, which is obvious as:

$$\Delta_1 = -m(\beta\mu - g\mu)(1 - \tau)(2\alpha_b + \alpha_{bb}b) < 0.$$

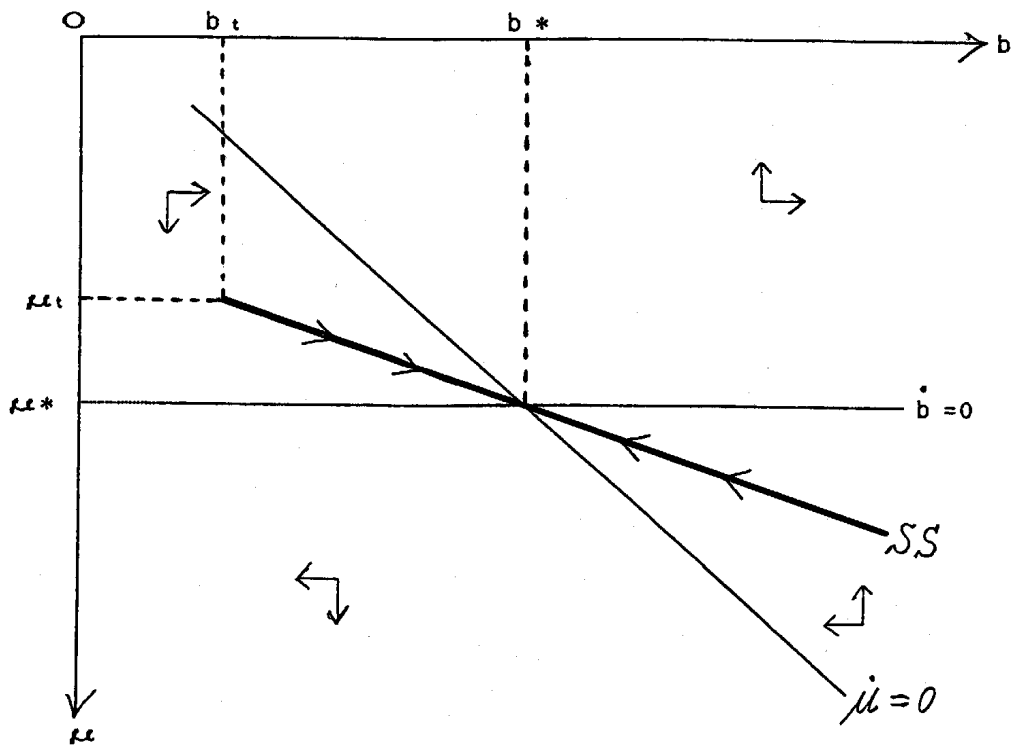


Figure-1 Phase diagram.

Figure-1 gives the phase diagram corresponding to (20). The $\dot{b} = 0$ locus is horizontal at $b = b^*$; the $\mu = 0$ locus is downward-sloping. The arrows indicate directions of motion. There is therefore a unique path converging to the steady state, the downward-sloping path SS .

4 . Steady State Properties

In the steady state $\dot{b} = 0$ and $\dot{\mu} = 0$ so that the optimal conditions are:

$$\beta = g - \delta, \tag{21}$$

$$mi = (1 - \tau) \{ \alpha + c + \alpha_b b + (mi + \delta - g) c_\beta \}, \tag{22}$$

$$\Phi'(g) = \frac{(1 - \tau) (\pi + \alpha_b b^2) - \Phi(g)}{mi + \delta - g} \tag{23}$$

Eq. (22) implies that the required rate of return of shareholders is equal to the marginal (tax-adjusted) cost of debt finance (MCB). The financial structure (the debt-capital ratio) and the growth rate of debt do not affect the borrowing rate of interest, then (22) becomes $mi = (1 - \tau)i$. In general, however, $m = (1 - \theta)/(1 - c) > (1 - \tau)$. Therefore, raising the debt level as possible is the best policy for the firm. In the absence of taxes there exist no optimal debt-capital ratios⁶⁾.

Similarly, if $\alpha_b = 0$, then (23) becomes:

$$\Phi'(g) = \frac{(1 - \tau) \pi - \phi(g)}{mi + \delta - g} \tag{24}$$

The above equation is the same as the famous Uzawa [1969]-Yoshikawa [1980]-Hayashi [1982] condition.

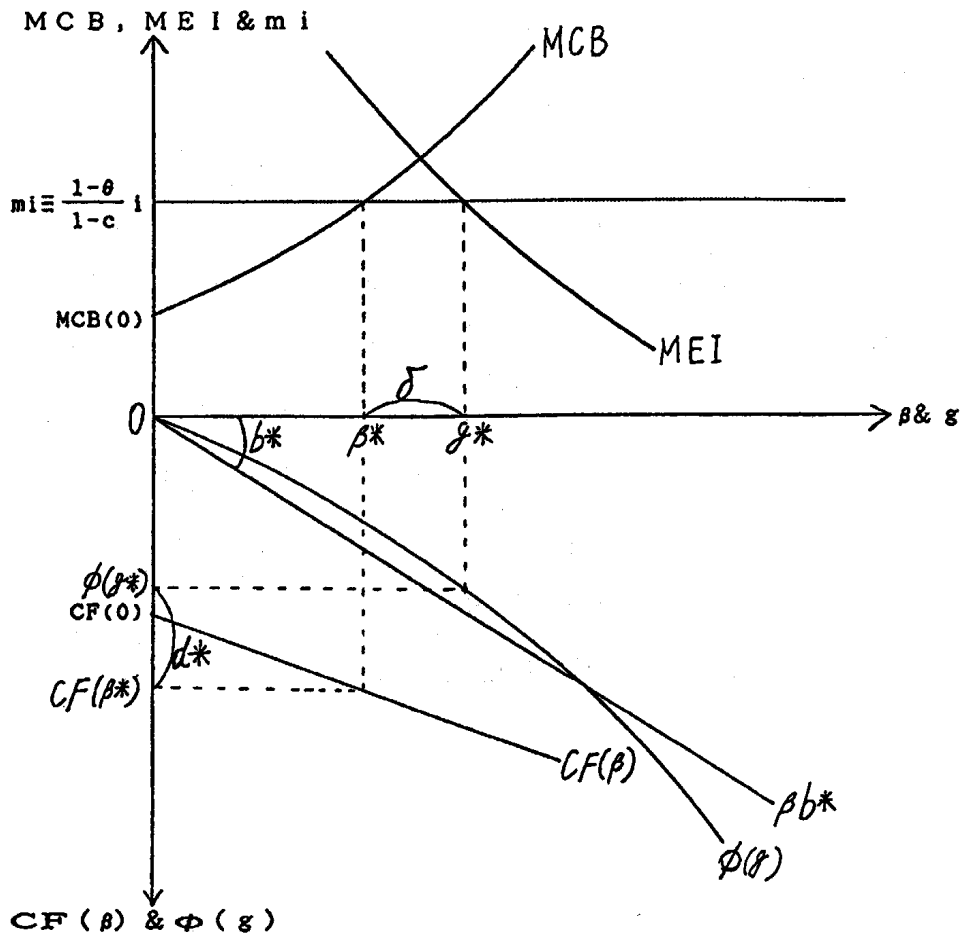
Eq.(23) is rewritten as:

6) This is consistent with the famous Modigliani-Miller theorem. See Modigliani and Miller [1958, 1963].

$$m_i = \frac{(1-\tau)(\pi + \alpha_b b^2) - \phi(g)}{\phi'(g)} - \delta + g. \quad (25)$$

The LHS of (25) is the cut-off rate of investment and the RHS is the marginal efficiency of investment (MEI).

Figure-2 illustrates the determination of corporate growth rate, debt growth rate, debt-capital ratio and the dividend rate.



where $CF \equiv (1-\tau)(\pi - (\alpha+c)b) + \beta b$.

Determination in the steady state.

Figure-2

If the non-negativity constraint on the dividend rate is crucial and effective, the intersection of MCB and MEI may give the optimal β and

g, as shown, for instance, in Hayashi [1985]. This situation may occur on the transitional path with large debt-capital ratio (b) in the model discussed in this paper. We, however, focus on properties on the steady growth path and around it. In this sense Figure-2 can be easily included in financing hierarchy theories.

Using (21), (22) and (23), comparative statics results in the steady state are obtained as shown in Table-2.

	b^*	$g^* (\beta^*)$
γ	+	+
η	+	+
π	-	+
i	+	?
τ	+	?
θ	-	?
c	+	?

Table-2 Comparative statics results in the steady state.

A rise in γ shifts MCB-curve downward and consequently it raises b^* (and β^*). The rise in b^* leads the upward shift of MEI-curve. As a result, g^* increases and becomes equal to $\beta^* + \delta$. The close bank-corporate relation facilitates the corporate growth and raises its debt-capital ratio.

The effects of a change in i , τ , θ and c on g^* become ambiguous. This is as follows by taking the case of rise in i as an example. Rise in i first raises the cut-off rate mi to raises b^* . But the rise in b^* leads the downward shift of MEI-curve. Thus the second effect through the change in b^* is different from the first effect.

5. Analysis on Transitial Path

Before doing comparative analysis, we must note that:

$$\frac{d\beta_t}{dx} = \frac{\partial\beta_t}{\partial x} + \frac{\partial\beta_t}{\partial\mu_t} \frac{d\mu_t}{dx}, \quad (26)$$

$$\frac{dg_t}{dx} = \frac{\partial g_t}{\partial x} + \frac{\partial g_t}{\partial\mu_t} \frac{d\mu_t}{dx}, \quad (27)$$

where x represents exogenous variable.

Let us call the first term in RHS of (26) and (27) the direct effect, the second term the indirect effect and sum of them the total effect, from now on.

5 - 1. Effect of a change in the initial debt-capital ratio b_t .

From Figure-1, $d\mu_t/db_t$ is negative. The direct effect of b_t on β_t ($\partial\beta_t/\partial b_t$) is zero and the indirect ($(\partial\beta_t/\partial\mu_t) \cdot (d\mu_t/db_t)$) is negative. The total effect is therefore negative. This means that the higher the initial debt-capital ratio, the lower the growth rate of debt. When the debt-capital ratio of a firm is already high, the firm decrease it so that the growth rate of debt is low.

In the neighborhood around the steady state, the direct effect of b_t on g_t ($\partial g_t/\partial b_t$) is small enough to be ignored and the indirect effect ($(\partial g_t/\partial\mu_t) \cdot (d\mu_t/db_t)$) is positive. As a result the total effect becomes positive.

This states that the higher the initial debt-capital ratio, the higher the growth rate of capital. This is because the firm must reduce debt-capital ratio in order to save the interest payment.

This corresponds the situation that the upward-shift of both MEI-curve and MEB-curve in Figure-2. The gap between g and β is therefore larger than δ and b will decline over time.

5 - 2 . Effects of a rise in the shift parameter γ .⁷⁾

Suppose that the shift parameter γ rises from γ_0 to γ_1 . From (15) and (16),

$$\left. \frac{d\mu}{d\gamma} \right|_{\dot{b}=0} = \frac{g\gamma}{\beta_{\mu} - g_{\mu}} > 0,$$

$$\left. \frac{d\mu}{d\gamma} \right|_{\dot{\mu}=0} = \frac{m(1-t)\alpha\gamma}{mi - \beta} > 0.$$

In this case, as Figure-3 shows, the optimal path moves upwards from SS to SS'.

7) Here we consider an unanticipated permanent change in each parameter.

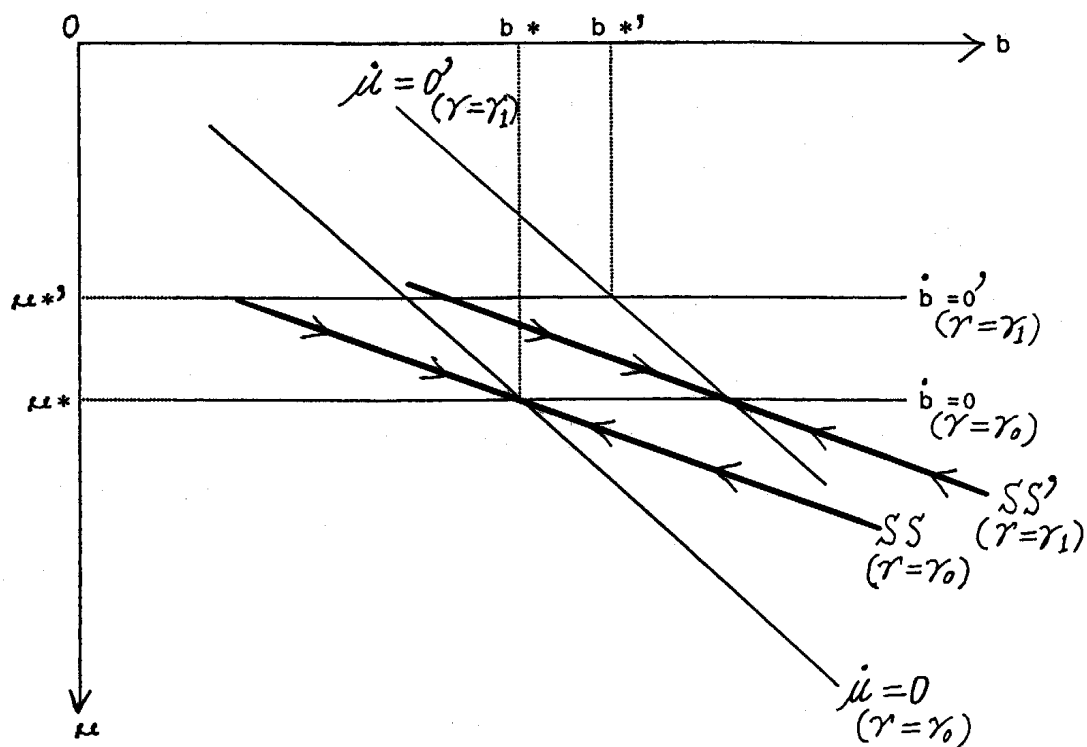


Figure-3 The effect of a rise in γ .

Using the above procedure, results in Table-3 are obtained⁸⁾.

8) See Appendix-D for the directions of the SS path.

	β_t			g_t		
	dir.	ind.	tot.	dir.	ind.	tot.
b_t	0	-	-	0	+	+
γ	0	+	+	+	-	?
η	0	+	+	+	-	?
π	0	+	+	+	-	?
i	0	-	-	-	+	?
τ	+	?	?	-	?	?
θ	-	?	?	+	?	?
c	+	?	?	-	?	?

where

dir. \equiv direct effect,

ind. \equiv indirect effect,

tot. \equiv total effect.

Table-3 Comparative dynamics results on the transitional path.

The usual results on the effects of a change in i, τ, θ and c on g_t are consistent with the direct effects in Table-3⁹⁾. In other words, so far little attention has been given to the indirect effects. We must note that the direct and indirect effects often have opposite signs.

When parameters change, on the transitional path there exists the

9) For usual results, see, for example, Hall and Jorgenson [1967], Abel [1982] and Auerbach [1984].

possibility that the direction of a change in β_t is different from that in g_t . If we assume that, as Poterba and Summers [1983], b_t is constant through time, this dynamic property may be missed.

6. Concluding Remarks

If holding debt or issuing new bonds has a negative effect on the value of a firm or its profit stream, it never have debt at least in a steady state. But in the real world a firm has debt. In the model discussed in this paper, a firm can receive the tax-allowance by holding debt. At the same time, however, the debt-capital ratio affects the borrowing rate of interest. These two factors yields the optimal financial structure, and the transitional adjustment path converges to it. In other words, a firm has the desirable level of the debt-capital ratio, and adjusts an actual debt-capital ratio to it by using both the bond issue and investment.

For a firm, the tax-allowance due to the holding debt is regarded as the additional source of revenue. In our formulation, corporate taxes have effects on investment through this tax allowance as well as through the direct reduction of the accounting profits. This makes the effect on investment ambiguous¹⁰⁾.

It is often cited that the degree of leverage and the growth rate of Japanese corporations are relatively higher than those in other industrial countries. At the same time, it is pointed out that there exist the close bank-corporate relationships in Japan. This paper also presents one theoretical explanation for the relation of these facts.

APPENDIX-A

Solving (6), we get

10) Stiglitz [1973] also shows that there is the possibility that a rise in the corporate tax rate increases investment.

$$K_s = K_t \exp \int_t^s (g_v - \delta) dv. \quad (\text{A-1})$$

Using (A-1), the original problem is rewritten as the following problem.

$$\text{Max}_{\beta, g} V(t) = \int_t^\infty m [1 - \tau] \{ \pi - (\alpha + c)b \} - \phi(g) + \beta b] K(0) e^{-x} ds. \quad (\text{A-2})$$

$$\text{s. t. } db = (\beta - g - \delta) b ds, \quad b_t ; \text{ given}, \quad (\text{A-3})$$

$$dx = (mi + \delta - g) ds, \quad X_t = 0, \quad (\text{A-4})$$

where

$$X \equiv \int_t^s \{ mi + \delta - g_v \} dv. \quad (\text{A-5})$$

From (A-4),

$$\frac{dX}{ds} = \frac{1}{mi + \delta - g} \quad \text{or} \quad ds = \frac{dX}{mi + \delta - g}. \quad (\text{A-6})$$

From (A-3) and (A-4),

$$\frac{db}{dX} = \frac{(\beta - g + \delta)b}{mi + \delta - g}. \quad (\text{A-7})$$

From the fact that the optimization problem is free from K_t , we can

set, without loss of generality, $K_t = 1$. Using (A-7), the above problem is further simplified as the following problem.

$$\text{Max}_{\beta, g} V(0) = \int_0^\infty \frac{m [(1-\tau) \{ \pi - (\alpha + c)b \} - \phi(g) + \beta b]}{mi + \delta - g} e^{-x} dX,$$

s. t., (A-7) and b_0 ; given.

To solve the problem, we define the Hamiltonian H as

$$H(g, \beta, b, \lambda) \equiv \frac{[(1-\tau) \{ \pi - (\alpha + c)b \} - \phi(g) + \beta b] + \mu(\beta - g + \delta)b}{mi + \delta - g}, \tag{A-8}$$

where $\mu \equiv$ auxiliary variable.

The first order conditions for this problem are:

$$\begin{aligned} \langle 1 \rangle \quad \frac{d(\mu e^{-x})}{dx} &= - \frac{\partial(H e^{-x})}{\partial b} \quad \text{or} \\ \frac{d\mu}{dx} &= \frac{(mi - g)\mu + m[1 - \tau](\alpha + c + \alpha_b b) - \beta}{mi + \delta - g}, \end{aligned} \tag{A-9}$$

$$\langle 2 \rangle \quad \text{Max}_{\beta, g} H.$$

Also, Arrow type transversality condition must be satisfied :

$$\lim_{x \rightarrow \infty} b_x \mu_x e^{-x} = 0 \quad \text{or} \quad \lim_{s \rightarrow \infty} b_s \mu_s \exp \left[- \int_t^s \{ \rho + \delta - g(s) \} ds \right] = 0. \tag{A-10}$$

From (A-6) and (A-9), the derivative of μ with respect to time is

$$\frac{d\mu}{ds} = \frac{d\mu}{dx} \cdot \frac{dx}{ds} = (mi-g)\mu + m[(1-\tau)(\alpha+c+\alpha_b b) - \beta] \quad (A-11)$$

Assuming an interior solution, the necessary conditions for

<2> are :

$$-m(1-\tau)c_{\beta} + m + \mu = 0. \quad (A-12)$$

$$\begin{aligned} & -m(1-\tau)\phi'(g)(mi+\delta-g) \\ & + m[(1-\tau)\{\pi-(\alpha+c)b\} - \phi(g) + \beta b] - (mi-\beta)\mu b = 0, \end{aligned} \quad (A-13)$$

APPENDIX-B

From (18) and (19) following results are obtained.

$$\beta_{\mu} \equiv \frac{\partial \beta}{\partial \mu} = \frac{1}{m(1-\tau)c_{\beta\beta}} > 0,$$

$$\beta_{\tau} \equiv \frac{\partial \beta}{\partial \tau} = \frac{c_{\beta}}{(1-\tau)c_{\beta\beta}} > 0.$$

$$\beta_m \equiv \frac{\partial \beta}{\partial m} = \frac{1-(1-\tau)c_{\beta}}{(1-\tau)c_{\beta\beta}} > 0.$$

$$g_b \equiv \frac{\partial g}{\partial b} = \frac{(mi-\beta)\mu + m\{(1-\tau)(\alpha+c+\alpha_b b) - \beta\}}{m(mi+\delta-g)\phi''} \begin{matrix} \geq 0 \\ < 0 \end{matrix} \Leftrightarrow \dot{\mu} \begin{matrix} \leq 0 \\ \geq 0 \end{matrix},$$

$$g_{\mu} \equiv \frac{\partial g}{\partial \mu} = -\frac{(mi-\beta)b}{m(mi+\delta-g)\phi''} < 0,$$

$$g_{\gamma} \equiv \frac{\partial g}{\partial \gamma} = -\frac{(1-\tau)\alpha_{\gamma} b}{(mi+\delta-g)\phi''} > 0,$$

$$g_{\eta} \equiv \frac{\partial g}{\partial \eta} = -\frac{(1-\tau)c_{\eta} b}{(mi+\delta-g)\phi''} > 0,$$

$$g_{\pi} \equiv \frac{\partial g}{\partial \pi} = -\frac{1-\tau}{(mi+\delta-g)\phi''} > 0,$$

$$g_i \equiv \frac{\partial g}{\partial i} = \frac{m[b\{1-(1-\tau)c_{\beta}\} - \phi']}{(mi+\delta-g)\phi''} > 0,$$

$$g_{\tau} \equiv \frac{\partial g}{\partial \tau} = -\frac{\pi - (\alpha + c)b}{(mi + \delta - g)\phi''} < 0,$$

$$g_m \equiv \frac{\partial g}{\partial m} = \frac{\beta b \{1 - (1 - \tau)c_{\beta}\} - mi\phi'}{m(mi + \delta - g)\phi''} < 0.$$

APPENDIX-C

We totally differentiate (22) and (23) with consideration of (21) and express them in matrix form.

$$\begin{pmatrix} -(1-\tau)(2\alpha_b + \alpha_{bb}b) & -(1-\tau)(mi + \delta - g)c_{gg} \\ (1-\tau)(2\alpha_b + \alpha_{bb}b)b & -(mi + \delta - g)\phi'' \end{pmatrix} \begin{pmatrix} db \\ dg \end{pmatrix}$$

$$= \begin{pmatrix} (1-\tau)\alpha_{\gamma}d\gamma + (1-\tau)c_{\eta}d\eta + \{(1-\tau)c_g - 1\}mdi \\ -\{\alpha + c + \alpha_b b + (mi + \delta - g)c_g\}d\tau + \{(1-\tau)c_g - 1\}idm \\ -(1-\tau)d\pi + m\phi'di + (\pi + \alpha_b b^2)d\tau + i\phi'dm \end{pmatrix}.$$

In order to ensure the sufficient condition the determinant of the coefficients matrix (Δ_2) must be positive, which is obvious as : $\Delta_2 = (1-\tau)(2\alpha_b + \alpha_{bb}b)\{(mi - \delta - g)\phi'' + (1-\tau)(mi + \delta - g)c_{gg}b\} > 0$.

Using Cramer's rule, we obtaine that :

$$\frac{db}{d\gamma} = -\frac{(1-\tau)\alpha_{\gamma}(mi + \delta - g)\phi''}{\Delta_2} > 0,$$

$$\frac{dg}{d\gamma} = -\frac{(1-\tau)^2\alpha_{\gamma}(2\alpha_b + \alpha_{bb}b)bb}{\Delta_2} > 0,$$

$$\frac{db}{d\eta} = -\frac{(1-\tau)c_{\eta}(mi+\delta-g)\phi''}{\Delta_2}$$

$$\frac{dg}{d\eta} = -\frac{(1-\tau)^2c_{\eta}(2\alpha_b+\alpha_{bb})b}{\Delta_2} > 0,$$

$$\frac{db}{d\pi} = -\frac{(1-\tau)^2(mi+\delta-g)c_{gg}}{\Delta_2} < 0,$$

$$\frac{dg}{d\pi} = \frac{(1-\tau)^2(2\alpha_b+\alpha_{bb})}{\Delta_2} > 0,$$

$$\frac{db}{di} = \frac{m(mi+\delta-g)[\phi'(1-\tau)c_{gg}-\phi''\{(1-\tau)c_g-1\}]}{\Delta_2} > 0,$$

$$\frac{dg}{di} = -\frac{m(1-\tau)(2\alpha_b+\alpha_{bb})[b\{(1-\tau)c_g-1\}+\phi']}{\Delta_2} ?$$

$$\frac{db}{d\tau} =$$

$$-\frac{m(mi+\delta-g)[\{\alpha+c+\alpha_b b+(mi+\delta-g)\phi''+(1-\tau)(\pi+\alpha_b b^2)c_{gg}\}]}{\Delta_2} > 0,$$

$$\frac{dg}{d\tau} = -\frac{(1-\tau)(2\alpha_b+\alpha_{bb})[(\pi+\alpha_b b^2)-b\{\alpha+c\alpha_b b+(mi+\delta-g)c_g\}]}{\Delta_2} ? ,$$

$$\frac{db}{dm} = \frac{i(mi+\delta-g)[\phi'(1-\tau)c_{gg}-\phi''\{(1-\tau)c_g-1\}]}{\Delta_2} > 0,$$

$$\frac{dg}{dm} = -\frac{i(1-\tau)(2\alpha_b+\alpha_{bb})[b\{(1-\tau)c_g-1\}+\phi']}{\Delta_2} ?$$

APPENDIX-D

5-3. Effects of a rise in the shifi parameter η .

$$\left. \frac{d\mu}{d\eta} \right|_{\dot{b}=0} = \frac{g_{\eta}}{\beta_{\mu}-g_{\mu}} > 0,$$

$$\left. \frac{d\mu}{d\eta} \right|_{\dot{\mu}=0} = -\frac{m(1-\tau)c_{\eta}}{mi-\beta} > 0.$$

The SS-curve shifts upward.

5-4. Effects of a rise in the profit rate π .

$$\left. \frac{d\mu}{d\pi} \right|_{\dot{b}=0} = \frac{g_x}{\beta_\mu - g_\mu} < 0,$$

$$\left. \frac{d\mu}{d\pi} \right|_{\dot{\mu}=0} = 0.$$

The SS-curve shifts upward.

5-5. Effects of a rise in the rate of return of riskless asset i .

$$\left. \frac{d\mu}{di} \right|_{\dot{b}=0} = \frac{g_i}{\beta_\mu - g_\mu} < 0,$$

$$\left. \frac{d\mu}{di} \right|_{\dot{\mu}=0} = -\frac{m\mu}{mi - \beta} < 0.$$

The SS-curve shifts downward.

5-6. Effects of change in the corporate tax rate τ .

$$\left. \frac{d\mu}{d\tau} \right|_{\dot{b}=0} = \frac{g_\tau - \beta_\tau}{\beta_\mu - g_\mu} < 0,$$

$$\left. \frac{d\mu}{d\tau} \right|_{\dot{\mu}=0} = \frac{m(a+c)}{mi - \beta} > 0.$$

The direction of the shift of SS-curve is ambiguous.

5-7. Effects of a change in the tax parameter m .

$$\left. \frac{d \text{vii}}{dm} \right|_{\dot{b}=0} = \frac{g_m - \beta_m}{\beta_\mu - g_\mu} < 0,$$

$$\left. \frac{d\mu}{dm} \right|_{\dot{\mu}=0} = \frac{i\mu + \{(1-\tau)(\alpha + c + \alpha_b) - \beta\}}{mi - \beta} > 0.$$

The direction of the shift of SS-curve is ambiguous.

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