Surface Waves along a Boundary of Single-Negative and Double-Positive Materials

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Abstract — **Existence of a surface wave along the boundary between the semi-infinite materials, one of which is a free-space and the other is a material with either negative permeability or negative permittivity, is presented theoretically in this paper. Surface waves exist in only limited combination of negative and positive sign of the material parameters. In addition, by analyzing the surface wave in a finite-thickness slab with negative permeability, the even- and odd-symmetry modal profiles have been obtained. From these results, the present paper predicts the possibility of a surface wave directional coupler based on a single slab transmission along its top and bottom surfaces.**

I. INTRODUCTION

Negative permeability and/or permittivity materials have received much attention from many researchers due to their extraordinary electromagnetic(EM) characteristics [1]-[5]. It has recently been reported that negative permittivity and permeability can be constructed by arranging split-ring resonators [2] and by arranging thin metal wires [3], [4] in the host media, respectively. It has already been known that the negative permittivity or permeability can be observed in a conducting material [6] or magnetized ferrite with a tensor permeability [7]. The surface plasmon, impedance surface wave [8], and magnetostatic surface wave (MSSW) are some examples propagating on the negative permittivity or permeability materials (single negative material) boundary. However, our study reorganizes those waves from the concept of scalar permittivity and permeability, and thus tries to cope with the application of metamaterials.

Any surface wave can not exist along the boundary of the ordinary semi-infinite materials with both positive material parameters. In this paper, as an application of the extraordinary EM characteristics of metamaterials with negative material parameter, we will theoretically demonstrate that the TE and TM surface wave along the boundary between two semi-infinite materials can propagate for only limited combination of the positive and negative sign of permeability and permittivity.

Furthermore, we will also show that surface waves propagate in a finite-thickness slab with negative permeability. From a numerical analysis, we will obtain the even- and odd-symmetric surface waves and its possibility for a directional coupler constructed with a single slab structure of negative permeability whose top and bottom surfaces work as the wave-guiding boundaries separately.

II. SURFACE WAVE ALONG SEMI-INFINITE MATERIAL BOUNDARY

As shown in Fig. 1, a semi-infinite material with its permeability μ or permittivity ε is negative in Region II, is faced at the x=0 surface with a semi-infinite free-space whose permittivity ε_0 and permeability μ_0 is both positive real in Region I. The coordinate system is also shown in the same figure, where the structure has no variation in the y- and z-directions. It is assumed that the time and z-dependence of the EM fields are $\exp\{j(\omega t - \beta z)\}$.

We have tried to find the TE and TM surface waves for all possible positive and negative material parameters combination of Region II by deriving the field components in each region from the Maxwell's equation and applying the appropriate boundary condition. Here, the combination of double-negative and double-negative materials is excluded in this paper, since our current interest is focused on the single-negative material. The results are shown in Table I, which indicates that the surface wave can exist only for a limited combination of material parameters.

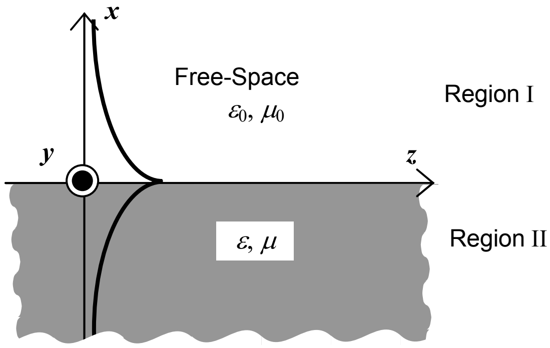

Fig.1 Semi-infinite single negative material and surface wave along it.

TABLE I
Existence of surface wave for material parameter combinations.

	$\varepsilon>0, \mu<0$	$\varepsilon<0, \mu>0$
TE wave (E_z=0)	○	×
TM wave (H_z=0)	×	○

III. Surface Wave in Finite-Thickness Slab with Negative Permeability

A. Field Expression and Eigenvalue Equation

Because the semi-infinite material can not be realized for the practical manufacturing, we analyze the TE surface wave in the finite-thickness slab with negative permeability where slab thickness is T as shown in Fig.2. The coordinate system is also shown in the same figure. Then, the slab region, that is $-T\le x\le 0$, is constructed with the positive permittivity $\varepsilon=\varepsilon_1$ and negative permeability $\mu=-\mu_1$ material, and the upper and lower regions of the slab, that is $x\ge 0$ and $x\le -T$, are the free-space with the material parameter ε_0 and μ_0. The EM field components of this surface wave can be expressed as follows,

$$\begin{cases} H_x = -\dfrac{j\beta}{\delta_0} e^{-\delta_0 x} \\ H_z = e^{-\delta_0 x} \\ E_y = \dfrac{j\omega\mu_0}{\delta_0} e^{-\delta_0 x} \end{cases} \quad \text{for } x\ge 0,$$

$$\begin{cases} H_x = \dfrac{j\beta}{\delta_1}\left(-A_1 e^{-\delta_1 x} + B_1 e^{\delta_1 x}\right) \\ H_z = A_1 e^{-\delta_1 x} + B_1 e^{\delta_1 x} \\ E_y = \dfrac{j\omega\mu_1}{\delta_1}\left(-A_1 e^{-\delta_1 x} + B_1 e^{\delta_1 x}\right) \end{cases} \quad \text{for } -T\le x\le 0, \qquad (1)$$

$$\begin{cases} H_x = \dfrac{j\beta A_2}{\delta_0} e^{\delta_0 x} \\ H_z = A_2 e^{\delta_0 x} \\ E_y = -\dfrac{j\omega\mu_0 A_2}{\delta_0} e^{\delta_0 x} \end{cases} \quad \text{for } -T\ge x,$$

where $\delta_0 = \sqrt{\beta^2 - \omega^2\varepsilon_0\mu_0}$ and $\delta_1 = \sqrt{\beta^2 + \omega^2\varepsilon_1\mu_1}$, and A_1, B_1, and A_2 are arbitrary coefficients. By applying the boundary condition at x=0 and x=$-T$ surface, and eliminating the coefficients A_1, B_1, and A_2, the following eigenvalue equation for the surface wave can be obtained

$$\left(\mu_1\delta_0 - \mu_0\delta_1\right)^2 = \left(\mu_1\delta_0 + \mu_0\delta_1\right)^2 e^{-2\delta_1 T}. \qquad (2)$$

B. Surface Waves along the Negative Permeability Slab

Solving Eq.(2) numerically, we can obtain the eigenvalues for the surface waves. The negative permeability could be realized by stacking the cubic cell with printed double split-ring resonators on its walls [2]. In this structure, the permeability becomes negative at some frequency. According to Eqs.(15) and (43) in the reference [2] with the resistivity σ=10.0Ω, at the fixed operating frequency 13.6GHz, the permittivity and the permeability of the material is estimated as $\varepsilon/\varepsilon_0$=1.144 and μ/μ_0=-6.071, respectively. In the following simulations, these values are used for the analysis.

Fig.3 presents the numerical results for the normalized propagation constant β/k_0 as a function of normalized slab thickness T/λ_0, where k_0 and λ_0 are the wave-number and the wavelength in free-space, respectively. From this result, two surface waves with different propagation constants are observed at any slab thickness. The normalized propagation constant for the semi-infinite structure with the same numerical parameters is also drawn in the same figure with the dotted dash line. As the slab thickness becomes larger, the propagation constant of the two surface waves approach to that of the semi-infinite structure.

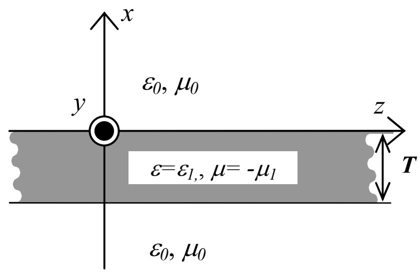

Fig.2 Finite-thickness slab with negative permeability.

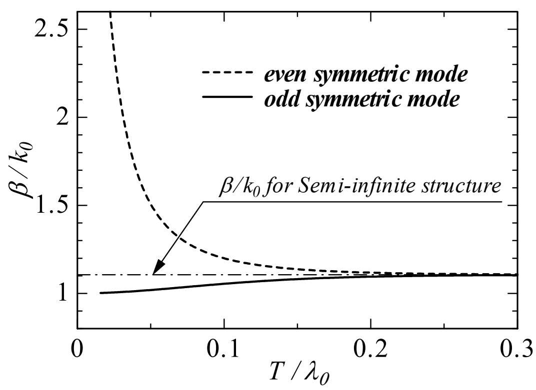

Fig.3 The normalized propagation constant as a function of normalized slab thickness, where $\varepsilon/\varepsilon_0$= 1.144 and μ/μ_0 = -6.071, respectively, at the fixed operating frequency 13.6GHz.

Next, we have estimated the field profiles of each surface wave. The result shown in Fig.4 tells that each wave has the even or odd symmetric profile with respect to the center of the slab, reflecting the symmetry of the structure, where T / λ_0=0.2. It should also be noted that there is no thickness mode as is the case for a normal dielectric slab of positive permittivity. For the negative permittivity slab, on the other hand, due to the duality of the electric and magnetic fields, it is easily demonstrated that a TM surface mode propagates with proper interchange of the electric and magnetic field components.

Changing the subject to applications, a surface wave directional coupler consisting of two YIG films separated by a dielectric sheet has been proposed [9]. Their directional coupler with double YIG films has very similar characteristics of propagation constant and field profiles with those for our single slab structure of negative permeability. Compare their Fig.2 in the reference [9] with our Figs.3 and 4. In our structure, one boundary surface confines the EM field, and thus the surface wave directional coupler can be constructed where its top and bottom surfaces of the slab exchanges energy as the wave-guiding regions. To confirm this phenomenon, we estimate the field distribution when the top surfaces is excited at z=0. Taking the propagation constants of the even- and odd-symmetric mode into consideration, the field distributions along the slab with negative permeability material $\Psi(x, z)$ can generally be expressed by the linear combination of the eigen-functions with the arbitrary constants C_1 and C_2 as

$$\Psi(x,z) = C_1 \Psi_e(x) e^{-j\beta_e z} + C_2 \Psi_o(x) e^{-j\beta_o z}, \tag{3}$$

where $\Psi_e(x)$ and $\Psi_o(x)$ are the modal profiles shown in Fig.4, and β_e and β_o are the propagation constants for even- and odd-symmetric mode, respectively. Considering the boundary condition at z=0 given above, Fig.5 shows the field distribution of $|H_z|$ for the slab thickness T / λ_0=0.1, where the other parameters are the same as Fig.3. It is confirmed that the field is bounded on the top and bottom surface of the slab and the maximum field intensity is periodically exchanged between two surfaces. From this result, the present slab structure was shown to work as a directional coupler based on two wave-guiding surfaces.

IV. Conclusions

It is theoretically studied that a surface wave propagates along the boundary between the semi-infinite materials, one of which is a free-space and the other is an either negative permeability or negative permittivity material. TE and TM type surface waves exist in only limited combination of negative and positive sign of the material parameters. In addition, by analyzing the surface wave in the finite-thickness slab with negative permeability, its mode profile has been obtained for two different types of symmetry. The possibility of a surface wave directional coupler based on a single slab transmission along its top and bottom surfaces is presented.

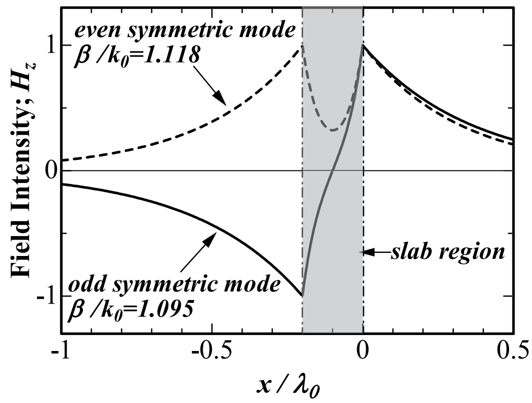

Fig.4 The mode profiles of the longitudinal field component, H_z, for each surface-wave at the normalized slab thickness T/λ_0=0.2. The other parameters are the same as those in Fig.3.

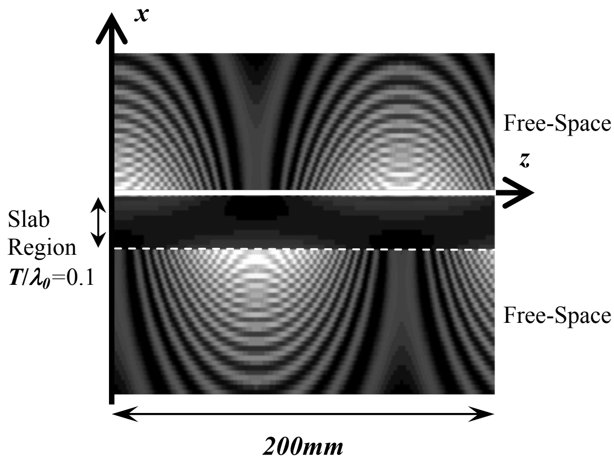

Fig.5 The field distribution, $|H_z|$, along the slab with negative permeability material for the thickness T/λ_0=0.1. The other parameters are the same as those in Fig.3.

Acknowledgments

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