A Model of Competing Rumours

TERAJI, Shinji

Abstract

This paper studies the effect of information transmission on the adoption of alternative products. We analyze the time evolution of a process with two competing rumours. We show that the decision rules whether or not to believe the rumour, based on optimizing behaviour, will result in a variety of equilibria. We then show that an inefficient product may survive in the market. The quality of the rumour for adopters has serious welfare consequences in equilibrium.

Keywords: competing rumours, information externalities, societal adoption of products.

1. Introduction

Learning from others is a central feature of most cognitive and choice activities, both in individuals and organizations (Levitt and March (1988)). In fact, which product you decide to buy will depend on what you learn from other purchasers, who already went through the same choice process in which you are currently engaged. And after you bought your chosen product, other people will ask you what product you had, so your experience will help inform their choices and hence their experiences as well.

In the societal adoption of products, information externalities arise that drive towards the emergence of some patterns of influences among individuals. Now suppose that there is a population of potential adopters, each with a chance of hearing that someone else has adopted a product. If an agent believes the information and passes it on, he or she becomes a source of information to others who may adopt it and so on. We call such a class of information processes *rumours*. The probability that someone hears the rumour depends on how many people already have it.

The diffusion of information is at the core of the pattern of structural change.[1] In Arthur and Lane (1993) and Lane and Vescovini (1996), a model of information contagion, in which the agents are Bayesian optimizers, is presented. Agents convert the information they receive from other agents into a choice between the two competing products. They show that the information feedback suffices to drive the market to domination by one or the other of the competing products. However, the setting of their model is based on a sequential choice of product where individuals enter the market one by one, observe the predecessors and take an irreversible action.[2] This assumption is unrealistic and restrictive. In fact, at the point one durable good wears out, it is hard to rule out that the consumer may make a new choice. Furthermore, since our goal is to study the long run equilibrium, it rules out consideration of migration of the population within the adoption process. It is more natural that we study the modelling of market situations in which agents are interacting simultaneously and modifying their actions at each period of time. Of course, these comments should not be viewed as a criticism of their studies. The present analysis immensely benefits from theirs.

Questions involving the diffusion of information have traditionally been

1) This terminology may be a close parallel to what Leibenstein (1950) called the "bandwagon effect" and "snob effect" in his classic study on the static market demand curve. By the bandwagon (snob) effect he referred to the extent to which the demand for a commodity is increased (decreased) because others are consuming the same commodity. See also Becker (1974) who incorporates a general treatment of social interactions into the modern theory of consumer demand.

2) See also Banerjee (1992) and Bikhchandai, Hirshleifer, and Welch (1992) for models in which agents observe the actions of every preceding adopter.

studied by anthropologists, sociologists and psychologists (see, for example, Coleman (1964), Funkhouser and McCombs (1972), Gersho and Mitra (1975) and Hamblin, Jacobsen and Miller (1973)). They have proposed diffusion as a significant driving force in the evolution of social systems. The basic approach adopted in most models is to decompose the total population into subgroups and to describe the diffusion process as one in which individuals move from one subgroup to another as the information spreads through the total population.[3] However, in these models the exchange of information is purely mechanical. The probability of making a choice whether to believe it and pass it on is assumed to be exogenous. For a notable exception, Banerjee (1993) characterizes the decision based on optimizing behaviour in a rumour diffusion process. However, Banerjee studies only one rumour and pays no attention to the market configurations that can arise in the information transmission processes.

This paper studies the effect of information diffusion on the societal adoption of alternative products, in which consumers do not have full information in the marketplace. There are two competing rumours. The rumour takes the form of information that someone has adopted product A or product B. At each date, every agent is present and decisions are reversible. A previous purchaser may lose interest in his or her product. Potential adopters assimilate the rumour that they hear via Bayesian updating, and they decide which product to adopt by maximizing expected utility. The adoption decision of any agent affects other agents in the market. The social interaction represents the structure of contacts. We examines how the parameters of the model affect the long run market structure through information transmission processes.

3) Social learning may be characterized by pre-existing social networks. See An and Kiefer (1995), in which agents base their adoptions on information obtained from their neighbors on d - dimensional lattice.

This paper focuses on a way in which agent characteristics and connectivity structure affect the market shares of the competing products. In particular, we describe some properties that seem desirable at the individual level, but which turn out to have undesirable effects at the aggregate level: what is good for each is, in a certain sense, bad for all.

The rest of the paper is arranged as follows. The model is presented in section 2. The case of competition between two products through rumour diffusion processes is presented in section 3. Section 4 presents some concluding remarks.

2. The model

2.1. Framework

In this section, we formulate a model for the diffusion of information through a population with time. We assume that the population can be partitioned into subgroups whose membership changes as time passes. Each group consists of individuals who behave similarly with regard to the transmission of information.

Suppose that two alternative products or technologies, A and B, are available in the market. We examine a model in which two competing rumours are spreading continuously in time.

Consider an economy composed of N agents. Let $X(t)$, $Y(t)$ denote the number of adopters that is using A, B, at time t, respectively. Then, the remaining $N-X(t)-Y(t)$ members do not, at this time, belong to either of the two groups; they are potential adopters who are deciding which brand to use in some time interval $[t, t+dt]$. Let $x(t)$, $y(t)$ denote the respective fraction of the total population in the adopters that is using *A*, *B*, at *t*. At $t = 0$, $x(0)$ and $y(0)$ are given as initial conditions on diffusion.

Let *UA*, *UB* be the payoff for an individual who adopts *A*, *B* respectively. In this paper, we suppose that $UA - UB = \Delta$ has two possible values such that $\Delta^{+} > 0 > \Delta^{-}$. The key assumption is that potential adopters do not know the true value of Δ when they choose between two products.[4] All the potential adopters assign common prior probability $q > 1/2$ to the event that $\Delta = \Delta^{+}$. We just assume that the value q is given. Furthermore, in this model, we suppose that $q\Delta^{+} + (1-q)\Delta^{-} > 0$, i.e. *ex ante* *A* is better than *B*. This is equivalent to the prior odds ratio, $q/(1-q)$, being strictly greater than $-\Delta^{-}/\Delta^{+}$, which we will denote by k.

Potential adopters, who do not know the realization of Δ, must value other sources of information about it. In our model, they do not observe the market share or popularity of each choice.[5] The only source of information is the rumour. The rumour takes the form of information that someone is using product *A* or product *B*. Each potential adopter can benefit from the information contained in it. The rumour need not, of course, reflect the true realization of Δ.

At any instant *t*, each potential adopter will have some probability of hearing the rumour between t and $t + dt$. It is assumed that dt is so short that at most one of these events happens in the time interval dt. Specifically, we will consider the case where the probability is proportional to the number of people who are using product *A* or *B* at any time. In general terms, we assume

4) The prices of the two products do not enter explicitly into the present model. If we suppose that these prices are known and fixed throughout the market—share allocation process, then we can suppose that the performance characteristics are measured in a way that adjusts for price differences.

5) In the setting in Vives (1993), the market participants do not see the previous actions of the participants directly, but they can act conditionally on past and current prices.

that potential adopters hear the rumour as a result of certain pairwise contact. Only meetings of two individuals among potential adopters and previous purchasers of A or B have the potential to information transmission. Let ρ denote the *contact rate* between individuals from potential adopters and previous purchasers of A or B.

Thus, if there is a contact between two members from potential adopters and previous purchasers of A (B) in some time interval $[t, t+dt]$, then a potential adopter will hear the rumour that a previous adopter is using A (B).

2.2. Agents

We assume that potential agents are Bayesian optimizers. Since they cannot observe the realization of Δ, they only have access to the rumour. A potential adopter believes that the rumour reflects the realization of Δ with some probability.[6] We assume that existence of competing rumours as follows: the rumour is either {someone else is using product A} or {someone else is using product B}; and it is linked to the true value of Δ through the following conditional probabilities:

$$\text{Prob (someone else is using product } A \mid \Delta = \Delta^{+}) = \mathrm{P}(A \mid \Delta^{+}) = 1 - w,$$

$$\text{Prob (someone else is using product } B \mid \Delta = \Delta^{+}) = \mathrm{P}(B \mid \Delta^{+}) = w,$$

$$\text{Prob (someone else is using product } A \mid \Delta = \Delta^{-}) = \mathrm{P}(A \mid \Delta^{-}) = w,$$

6) Since we focus on purely informational effects, we consider no strategic manipulation of information during the course of the market process.

Prob (someone else is using product B $|\Delta = \Delta^{-}$)

$= P(B \mid \Delta^{-}) = 1-w.$

For example, $P(B \mid \Delta^{-})$ is the probability that a person is using product B when the realization is Δ^{-}. The probability $1-w$ is assumed to be greater than $1/2$. It reflects the "quality" of the rumour for potential adopters. The value w is assumed to be same for all of them. The quality of the rumour is assumed bounded, i.e. $1-w < 1$. Indeed, by making the probability w small, we can make the rumour as precise as we like.

Once a potential adopter hears the either rumour, he or she updates his or her prior beliefs using Bayes' rule: the probability of event Δ^{+} given hearing the rumour that a person is using product A is:

$$P(\Delta^{+} \mid A) = \frac{P(A^{+} \mid \Delta)\ q}{P(A \mid \Delta^{+})\ q + P(A \mid \Delta^{-})\ (1-q)}$$

$$= \frac{(1-w)\ q}{(1-w)\ q + w\ (1-q)}\ .$$

Similarly, it follows that:

$$P(\Delta^{-} \mid A) = \frac{w\ (1-q)}{(1-w)\ q + w\ (1-q)}$$

$$P(\Delta^{+} \mid B) = \frac{wq}{wq + (1-w)\ (1-q)}$$

$$P(\Delta^{-} \mid B) = \frac{(1-w)\ (1-q)}{wq + (1-w)\ (1-q)}\ .$$

Potential adopters, who hear the rumour that a person is using A, will choose A when $P(\Delta^{+}|A)\Delta^{+} + P(\Delta^{-}|A)\Delta^{-} > 0$. This is equivalent to the posterior odds ratio, $P(\Delta^{+}|A)/P(\Delta^{-}|A)$, being strictly greater than $-\Delta^{-}/\Delta^{+}= k$, that is, $(1-w)q/w(1-q)$ exceeds k. Note that A is optimal under the prior beliefs such that the prior odds ratio $q/(1-q)$ exceeds k and that $(1-w)/w >1$ $(\because 1-w >1/2)$. Then potential adopters will choose A surely.

Note that $w/(1-w) < 1$. Potential adopters, who hear the rumour that an agent is using B, will choose A when $P(\Delta^{+}|B)\Delta^{+} + P(\Delta^{-}|B)\Delta^{-} > 0$, that is, the posterior odds ratio, $P(\Delta^{+}|B)/P(\Delta^{-}|B) = wq/(1-w)(1-q)$, is strictly greater than k. Thus, we can say that if the quality of the rumour is low for each potential adopter, he or she should follow his or her own prior belief. On the other hand, he or she will choose B when the posterior odds ratio is strictly less than k. Thus, if the quality of the rumour is high for each potential adopter, he or she should always follow the rumour. When the odds ratio is exactly k, an adopter is indifferent.

Let δ (λ) be the probability that each potential adopter chooses product A when he or she hears the rumour that an agent is using product A (B) between t and $t+dt$. Then the above conditions can be written as:

$$\delta = \{1\}, \tag{1}$$

$$\lambda = \begin{cases} \{1\} & \text{if } wq/(1-w)(1-q) > k, \\ [0,1] & \text{if } wq/(1-w)(1-q) = k, \\ \{0\} & \text{if } wq/(1-w)(1-q) < k. \end{cases} \tag{2}$$

2.3. Dynamics

If there is a pairwise contact between individuals from potential adopters and previous purchasers of A (at which a potential adopter hears the rumour that a person is using A), then a potential adopter chooses A with probability one. If there is a pairwise contact between individuals from potential adopters and previous purchasers of B (at which a potential adopter hears the rumour that a person is using B), a potential adopter chooses A with probability λ and chooses B with probability $(1-\lambda)$.

If the population size is N, there are ${}_N C_2 = (N-2)!\,2!/N!$ (where $N! = N(N-1)\cdots 1$) possible contacts. Consequently, at any instant t, the probability that a potential adopter will hear the rumour that a person is using A and B between t and $t+dt$ is given by:

$$(\rho X(t)\ (N-X(t)-Y(t))/\ {}_N C_2)\ dt,$$

and

$$(\rho Y(t)\ (N-X(t)-Y(t))/\ {}_N C_2)\ dt,$$

respectively. It depends on the number of people who are the rumour spreaders. Thus, we have the transition probabilities of the process as follows:

$$\begin{aligned} &\mathrm{Prob}[(X, Y) \rightarrow (X+1, Y) \text{ between } t \text{ and } t+dt] \\ &= (\rho X(t)\ (N-X(t)-Y(t))/\ {}_N C_2)\ dt \\ &\quad +\lambda(\rho Y(t)\ (N-X(t)-Y(t))/\ {}_N C_2)\ dt + o(dt), \end{aligned} \tag{3}$$

and

$$\begin{aligned} &\mathrm{Prob}[(X, Y) \rightarrow (X, Y+1) \text{ between } t \text{ and } t+dt] \\ &= (1-\lambda)(\rho Y(t)\ (N-X(t)-Y(t))/\ {}_N C_2)\ dt + o(dt). \end{aligned} \tag{4}$$

For simplicity, we suppose that the contact rate between previous purchasers (or rumour spreaders) is assumed to be zero. Then there is no contact between two members from previous purchasers of product A and previous purchasers of product B. Thus, each previous purchaser has no probability of getting information. This specific assumption may be justifiable in a situation where the options for potential adopters are important factors.

However, a previous purchaser in the process may be liable to lose interest in his or her product. Each may cease to be a rumour spreader and revert to the category of neutrals, i.e. potential adopters. Of course, he or she may, once again, join the ranks of the rumour spreaders. This leads to the transitions:

$$\begin{aligned}&\text{Prob}[(X, Y) \rightarrow (X-1, Y) \text{ between } t \text{ and } t+dt]\\&= \beta_1 X(t)\, dt + o(dt),\end{aligned} \tag{5}$$

and

$$\begin{aligned}&\text{Prob}[(X, Y) \rightarrow (X, Y-1) \text{ between } t \text{ and } t+dt]\\&= \beta_2 Y(t)\, dt + o(dt).\end{aligned} \tag{6}$$

where β_1 and β_2 are taken to be small positive constants.

For any t and small dt, we have from (3) and (5)

$$\begin{aligned}X(t+dt) = X(t) &+ (\rho X(t)\,(N-X(t)-Y(t))/{}_N C_2)dt\\&+\lambda(\rho Y(t)\,(N-X(t)-Y(t))/{}_N C_2)dt\\&-\beta_1 X(t) + o(dt),\end{aligned}$$

from which we obtain (suppressing the arguments through)

$$\begin{aligned}dX/dt = &(2\rho/N(N-1))X(N-X-Y)\\&+\lambda(2\rho/N(N-1))\,Y(N-X-Y)-\beta_1 X.\end{aligned}$$

In a similar manner we obtain from (4) and (6)

$$dY/dt = (1-\lambda)\ (2\rho/N\ (N-1))\ Y\ (N-X-Y)-\beta_2 Y.$$

The equations can be further simplified by shifting to population fractions. Also it is convenient to introduce $a = 2\rho/(N-1)$ as a parameter. Throughout this paper we will restrict the parameter values to satisfy the assumption below:

Assumption. $a > \beta_1$, $a > \beta_2$ and $\beta_1 \neq \beta_2$.

With this simplification the dynamical behaviour of our system can be described by:

$$dx/dt = ax\ (1-x-y) + \lambda ay\ (1-x-y) - \beta_1 x, \tag{7}$$

$$dy/dt = (1-\lambda)\ ay\ (1-x-y) - \beta_2 y, \tag{8}$$

and initial conditions, $x(0)$, $y(0)$, $x(0)+y(0) \leq 1$. They describe the time diffusion of information through a population. Specific predictions will vary according to the values taken by the parameters.

3. Analysis

In this section we investigate the behaviour of the differential equations system which consists of (7) and (8).

First, we examine the long run equilibria of the economy when $\lambda = 0$. Then we have:

$$dx/dt = ax\ (1-x-y) - \beta_1 x, \tag{7'}$$

$$dy/dt = ay\,(1-x-y) - \beta_2\, y, \tag{8)'}$$

in the state space $\{(x, y) \in R^2{}_+ \mid x + y \leq 1\}$. The first (second) one "dies" at rate β_1 (β_2) and "gives birth" at rate a. As a matter of fact, there are, besides the trivial state $(0, 0)$, two steady states from the Assumption:

$$(x^*, y^*) = ((a-\beta_1)/a,\ 0)$$

and

$$(x^{**}, y^{**}) = (0,\ (a-\beta_2)/a).$$

In order to check the local stability of (x^*, y^*), we express the Jacobian matrix of the dynamical system:

$$J^* = \begin{pmatrix} -2ax^*-ay^*+a-\beta_1 & -ax^* \\ -ay^* & -ax^*-2ay^* + a-\beta_2 \end{pmatrix}$$

$$= \begin{pmatrix} -(a-\beta_1) & -(a-\beta_1) \\ 0 & \beta_1-\beta_2 \end{pmatrix}$$

where

$$\text{trace } J^* = -(a-\beta_1) + (\beta_1-\beta_2),$$
$$\det J^* = -(a-\beta_1)(\beta_1-\beta_2).$$

In this case, if $\beta_1 < \beta_2$, it follows that trace $J^* < 0$ and det $J^* > 0$.

Then the steady state (x^*, y^*) is stable, and product B tends to vanish in the long run. If $\beta_1 > \beta_2$, (x^*, y^*) is a saddle point.

Likewise, at (x^{**}, y^{**}) the Jacobian matrix of the dynamical system is given by:

$$J^{**} = \begin{pmatrix} \beta_2 - \beta_1 & 0 \\ -(a - \beta_2) & -(a - \beta_2) \end{pmatrix}$$

where

$$\text{trace } J^{**} = -(a - \beta_2) + (\beta_2 - \beta_1),$$
$$\det J^{**} = -(a - \beta_2)(\beta_2 - \beta_1).$$

In this case, if $\beta_2 < \beta_1$, we have trace $J^{**} < 0$ and det $J^{**} > 0$. Then the steady state (x^{**}, y^{**}) is stable, and product A tends to vanish in the long run. If $\beta_2 > \beta_1$, (x^{**}, y^{**}) is a saddle point.

Thus, we can conclude as follows: suppose that $wq/(1-w)(1-q) < k$. Then, for each potential adopter, the quality of the rumour is high. If $\beta_1 < \beta_2$, the steady state (x^*, y^*) is stable. Then product B will be driven out of the market completely; and if $\beta_1 > \beta_2$, the steady state (x^{**}, y^{**}) is stable. Then product A dies out in the market. In the case where $\beta_1 > \beta_2$, each potential adopter will follow the rumour that a person is using B even though ex ante A is better than B. Then the resulting equilibrium (x^{**}, y^{**}) may be inefficient in the ex ante welfare sense. All potential adopters will choose B, even when they are not really sure that the rumour is completely right. Thus, an inefficient product can survive in the market.

When $\lambda = 1$, we have:

$$dx/dt = ax(1-x-y) + ay(1-x-y) - \beta_1 x, \tag{7}''$$

$$dy/dt = -\beta_2 y. \tag{8}''$$

As a matter of fact, there is, besides the trivial state (0, 0), one steady state: $(x^*, y^*) = ((a-\beta_1)/a, 0)$. Using the similar method, we can see that the steady state (x^*, y^*) is stable in the differential equations system which consists of (7)" and (8)". Thus, if $wq/(1-w)(1-q) > k$, product B will be driven out of the market completely. Then, for each potential adopter, the quality of the rumour is low. Thus, in an ex ante welfare sense, we can say when it is possible to end up with one efficient product dominating and the other inefficient product being driven out of the market completely.

Our emphasis on the information externalities derives striking results. Indeed, we have found that the quality of the rumour may be so severe in equilibrium. The welfare question is motivated by the information externalities. In terms of ex ante welfare, the economy may be better off if potential adopters are not allowed to hear the precise rumour.

4. Conclusion

In this paper we have documented different aspects of the adoption of products. We have improved our understanding of consumer choice under incomplete information. We have taken a methodology that is different from those used in the previous literature. A potential adopter does not find out the proportion of each product in the market. He or she gets to hear the rumour, which product someone else is using. The rumour processes take place in such a way that a potential adopter does not quite know whether or not to believe the information, and that the probability that he or she receives the information depends on how many people already have it. Furthermore, the adoption is revocable. It is natural that the agent may make a new choice. Our model allows for migration of the population within the adoption process.

We characterize the long run invariant distribution of the societal choice, and find out when it is possible to end up with one of the products dominating and the other being driven out of the market through the rumour processes. The inefficient product may survive in the market. Potential adopters make their choice according to information they receive, and the trajectory of product diffusion is determined through the information transmission processes. The existence of equilibria that present incorrect herding presents a possible challenge for policy.

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