Anomalous Effect of Weak Collisions on the Nonlinear Interaction in a Small-Cold-Beam, Warm-Plasma System

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(Received 17 March 1977)

It is found that the nonlinear wave-particle interaction based on the single-wave model can be changed strongly by introducing effects of weak collisions into plasma electrons. These collisions are too weak to change the linear stage. However, they alter the phase relation between the single wave and the trapped electrons in the nonlinear stage. As a result, the wave amplitude of the first minimum decreases anomalously and the persistent trapped-particle oscillations in the collisionless case are destroyed.

A nonlinear wave-particle interaction in a system of a small cold beam plus a plasma has been investigated both theoretically[1] and experimentally.[2] Theoretical predictions[1] based on the single-wave model agree with experimental observations[2] through the initial trapping of beam electrons and up to the first amplitude oscillation. Beyond this point, however, experiments[2] exhibit a rapid decay of the saturated wave rather than persistent trapped-particle oscillations predicted by numerical results[1] using the single-wave model. Recently, Dimonte and Malmberg[3] observed destruction of trapped-particle oscillations when the background plasma was simulated by a traveling wave tube.

The spatial evolution of a beam-plasma instability in a small-cold-beam, warm-plasma system has been studied using a particle simulation by Naitou and Abe.[4] The behavior of an unstable monochromatic wave is in agreement with the numerical results[1] up to the point where it saturates due to beam trapping. However, their simulation results are different from the numerical results[1] after this point. The wave damps strongly to the first minimum and fails to regrow to the amplitude of the first maximum. The ratio of the amplitude of the first maximum to that of the first minimum is about three times as large as the expected value from the numerical results.[1]

In order to examine some differences between numerical results,[1] laboratory experiments, and particle simulations, we have extended the single-wave model[1] to a more rigorous one including the collisional effect of the plasma electrons. In Ref. 1, the collisional effects and higher-order temperature effects have been neglected. In the warm background plasma with collisions, however, the behavior of the wave amplitude and beam electrons in the nonlinear stage can be changed strongly from that expected by the collisionless trapping model, even if collisions are too weak to change the linear stage. As an effective collision frequency, we may adopt the reciprocal of the slowing down time associated with Coulomb collisions in a dense plasma. Even when collisions are negligible, however, some weakly nonlinear processes (such as parametric instabilities) may result in an effective damping of the saturated wave.[3,5] Since the beam velocity may be much greater than the plasma thermal velocity, collisional effects of beam electrons with plasma electrons are neglected. When plasma is generated by the injected beam, however, electron-neutral collisions of beam electrons may be important in the nonlinear stage. These collisional effects may also have other effects which are different from that considered here.

We consider the spatial evolution of a single wave of frequency ω. Following Ref. 1, we can treat the plasma as a linear dielectric medium. The plasma dielectric function $\epsilon(\omega,k)$ is given as follows in standard notation:

$$\epsilon(\omega,k)=1-\frac{\omega_p{}^2}{\omega(\omega+i\nu)}\left(1+3\,\frac{k^2v_t{}^2}{\omega^2}\right). \tag{1}$$

Here ν is the effective collision frequency between plasma electrons. We obtain the following system of equations in terms of a natural extension of the model by O'Neil and co-workers[1] and Jungwirth and Krlin[6]:

$$\left(A+iB+i\frac{d}{d\eta}+C\,\frac{d^2}{d\eta^2}\right)E(\eta)=\frac{i}{N}\sum_{j=1}^{N}\exp(i\xi_j), \tag{2}$$

$$\frac{d^2\xi_j}{d\eta^2}=\left(1+\kappa\,\frac{d\xi_j}{d\eta}\right)^3E(\eta)\exp(-i\xi_j)+\text{c.c.}, \tag{3}$$

where

$$A = \kappa^2 \frac{\omega_{pt}^2}{\omega_b^2}\left[Z\frac{\delta\omega}{\omega_{pt}} + \left(\frac{\delta\omega}{\omega_{pt}}\right)^2\right],$$

$$B = \kappa^2 \frac{\nu}{\omega_{pt}}\frac{\omega_{pt}^2}{\omega_b^2}\left(1+\frac{\delta\omega}{\omega_{pt}}\right)^{-1},$$

$$C = \frac{\kappa}{Z}, \quad \kappa = \left(\frac{1}{6}\frac{n_b}{n_p}\frac{v_b^2}{v_t^2}\right)^{1/3},$$

and

$$\omega_{pt}^2 = \omega_p^2(1+3v_t^2/v_b^2).$$

Equation (2) is obtained algebraically from a Taylor expansion of the dielectric function up to the second order of $\delta k = k - k_0$ where $k_0 = \omega/v_b$. The real part of the dielectric function, $\epsilon_r(\omega, k_0)$, is not necessarily equal to zero. In these equations, $\delta\omega$ denotes a detuning,[5] i.e., the difference between the frequency of the most unstable mode (ω_{pt}) and ω, and κ is the spatial scaling factor, where $\frac{1}{2}\sqrt{3}\,\kappa k_0$ is the spatial growth rate in the collisionless warm plasma. The normalized electric field $E(\eta)$ of the wave and the spatial coordinate η are defined in terms of x and κ by $E(\eta) = eE(x)/mv_b\omega\kappa^2$ and $\eta = \kappa x(\omega/v_b)$, respectively. The phase-space coordinate ξ_j of the jth beam electron is defined as $\xi_j = \omega[t_j(x) - x/v_b]$. The function $t_j(x)$ is the time when the jth beam electron passes the point x. The velocity $\dot{x}_j$ in the laboratory frame is obtained from $\dot{\xi}_j = d\xi_j/d\eta$ by using the relation $\dot{x}_j = v_b/(1+\kappa\dot{\xi}_j)$.

Figure 1(a) shows the typical results of the effect of collisions on the wave. For numerical calculation, plasma parameters are chosen as follows: $n_b/n_p = 5\times10^{-4}$, $v_b/v_t = 9.9$, and $\delta\omega/\omega_{pt} = 0$. These parameters correspond to ones for the typical case of particle simulations.[4] The solid line is $E^2(\eta)$ when $\nu/\omega_{pt} = 2\times10^{-3}$. The dashed line is $E^2(\eta)$ in the collisionless case. The corresponding beam energy losses $\Delta W/W_0$ are shown in Fig. 1(b), where

$$\Delta W = -\left(\sum_{j=1}^{N} mv_j^2/2 - W_0\right)$$

and $W_0 = Nmv_b^2/2$. Figure 2 shows the real and imaginary parts of the wave number, which may be interpreted as the instantaneous phase shift and growth rate, respectively.

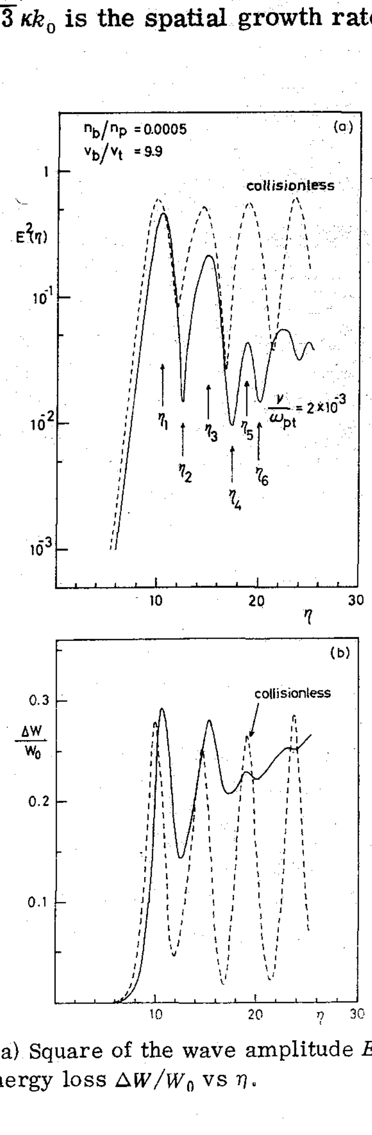

FIG. 1. (a) Square of the wave amplitude $E^2(\eta)$ vs η; (b) beam energy loss $\Delta W/W_0$ vs η.

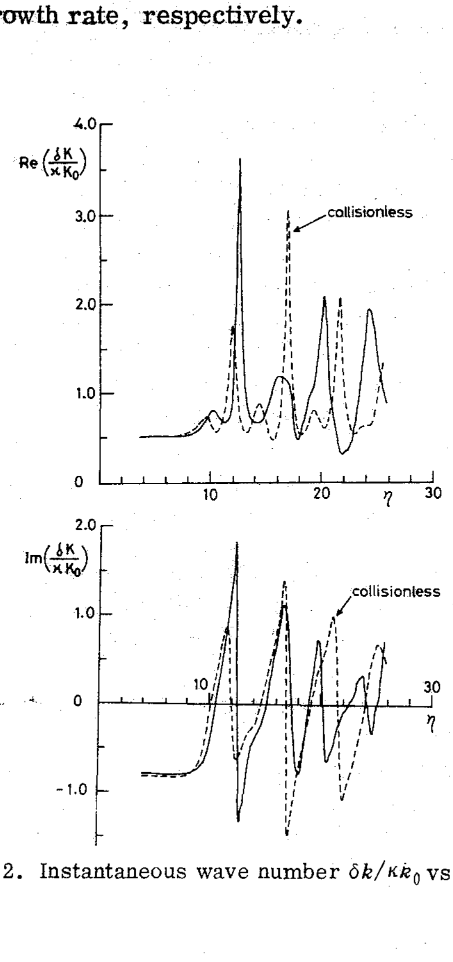

FIG. 2. Instantaneous wave number $\delta k/\kappa k_0$ vs η.

Before the first maximum, collisional effects do not significantly alter the spatial evolution of the wave except that the wave amplitude saturates at a slightly lower level than in the collisionless case. After this point, however, the two curves begin to depart from each other. The wave amplitude decreases to a value much smaller than that of the collisionless case and fails to regrow to the initial saturation level.

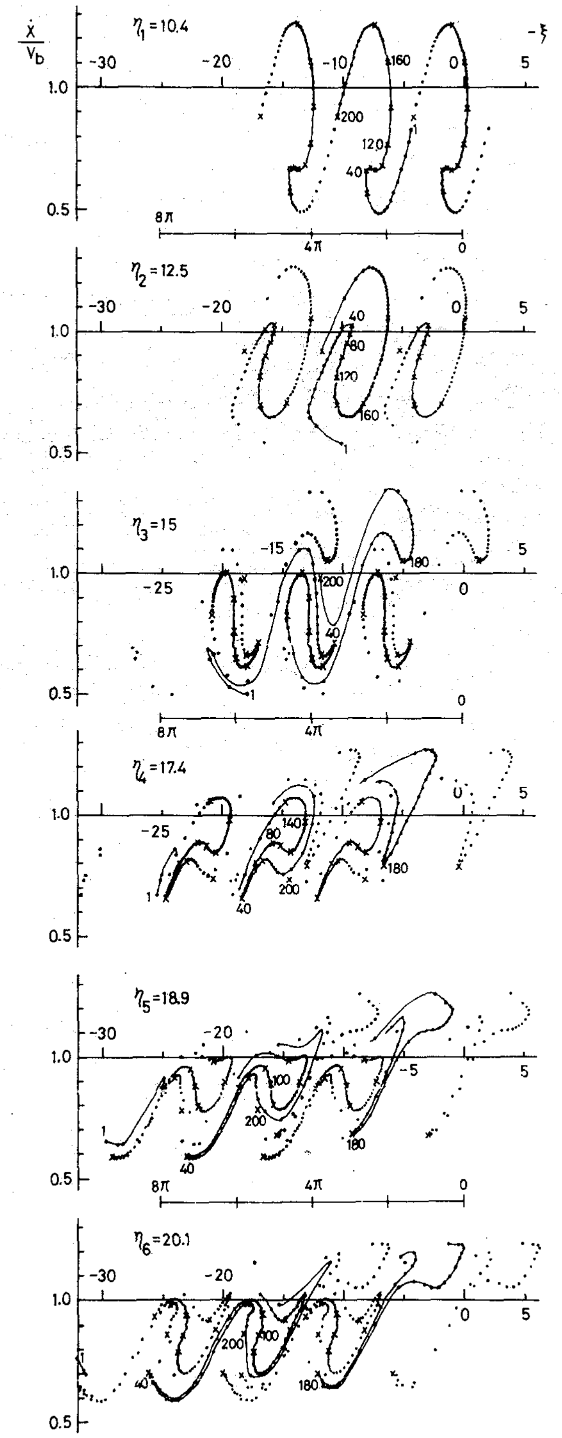

Figure 3 shows the phase-space evolution of the beam in the collisional case. Each locus is composed of the phase points for the beam electrons at the particular position denoted in Fig. 1(a). In accordance with the amplitude oscillation, the sloshing back and forth of the trapped beam electrons in the wave trough appears. Though the motion of the beam electrons in phase space is still a reversible process, after saturation their behavior changes gradually from that of the collisionless case. Near η_3, the locus splits into two parts. The beam electrons of the two parts begin to be smeared out and spill into adjacent wave troughs in an irreversible manner after η_4. On the other hand, the motion of the beam electrons in the collisionless case is still a reversible process after the second amplitude oscillation. Figure 4 shows the phase-space loci in the collisionless case at two positions, viz., (a) the third maximum and (b) the third minimum of $E^2(\eta)$. Although some of the beam electrons spill into an adjacent wave trough, most of them do not spread irreversibly but continue their phase rotation in a reversible manner.

The simulation results[4] agree qualitatively with numerical results in the collisional case for spatial evolution of the unstable wave and the behavior of the beam electrons in the phase space. It is clear that collisions play an important role in the nonlinear stage of the beam-plasma instability.

Plasma electrons support the unstable single wave in a linear dielectric medium. By analogy to an electric circuit, collisions between plasma electrons act as a phase shifter. According to the spatial evolution of $\mathrm{Re}(\delta k/\kappa k_0)$, the wave undergoes a rapid phase shift as well as a strong

FIG. 3. Phase-space loci for the beam electrons in the collisional case plotted for six positions denoted in Fig. 1(a). Each point gives the velocity $\dot{x}/v_b$ in the laboratory frame and the normalized spatial coordinate ξ for one of the beam electrons. Since Eqs. (2) and (3) are solved for $N = 200$ beam electrons which satisfy the initial conditions $\xi_j(0) = -2\pi(j/N)$ and $\dot{\xi}_j(0) = 0$, phase space is periodic in ξ with period 2π. In this figure, three groups of 200 electrons [the initial conditions $\xi_j(0)$ are $\xi_{-199 \le j \le 0} = -2\pi(j/N)$, $\xi_{1 \le j \le 200} = -2\pi(j/N)$, and $\xi_{201 \le j \le 400} = -4\pi(j-200)/N$] are displayed and every twentieth electron is marked. For reference, beam electrons with $\xi_{1 \le j \le 200} = -2\pi(j/N)$ are joined by a solid line.

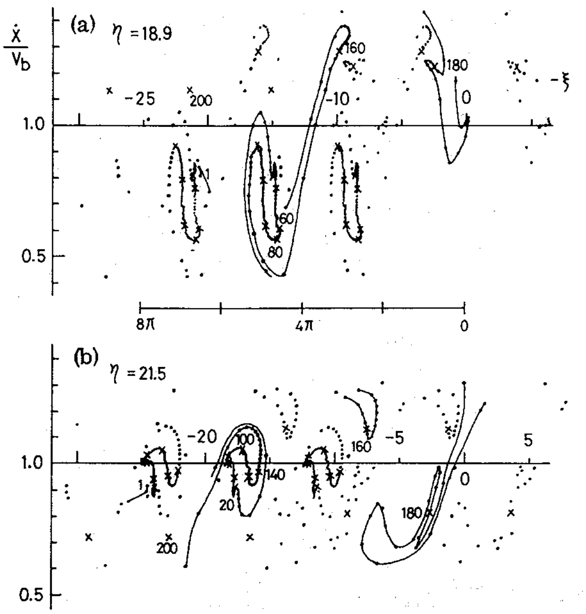

FIG. 4. Phase-space loci in the collisionless case plotted for two positions, viz., (a) the third maximum and (b) the third minimum of $E^2(\eta)$, respectively. Other conditions are the same as in Fig. 3.

damping of the amplitude near η_2. As is shown in Fig. 3, after η_2, the beam electrons spill into adjacent wave troughs and spread in phase space. This tendency becomes even greater beyond η_2. Furthermore, when we compared phase-space trajectories of some test particles in the $\dot{x}$-η plane in the collisional case with ones in the collisionless case, we found that they clearly departed from each other between η_3 and η_4. Motions of test particles in the collisional case become more irregular and their bounce periods also become longer because of the reduced amplitude of the wave. The energy exchange between the wave and the beam electrons ceases after η_3 as shown in Fig. 1.

Oscillations could be destroyed[3] as a result of particle phase mixing by either (a) wave damping or (b) modulation of the main wave by unstable sidebands. In the former case, a catastrophic effect on the oscillation is expected, similar to the results presented here.

We wish to thank Professor H. Momota and H. Naitou for their useful discussions. This work was carried out under the Collaborating Research Program at the Institute of Plasma Physics, Nagoya University.

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