Budget Deficits and the Control of Inflation.

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Introduction

Sargent and Wallace [9] recently presented a model that shows the difficulty of controlling inflation under persistent government budget deficits. Keeping Friedman's presidential address [3] in mind, they argue that the inflation control (even under monetarist conditions) tend to be included in the list of inpotency of the monetary policy.

When the fiscal authority independently sets its budgets and deficits (assumed to exist) which must be financed by bond sales and seignorage of money creation, the monetary authority has to finance with seignorage any discrepancy between the necessary amount of revenue requested by the fiscal authority and the amount of bond sales. Then, they showed two results (weak and "spectacular") that the restrictive monetary policy eventually leads to higher inflation due to the ceiling of per capita bond holding, and the policy even immediately raise inflation rate due to the demand for real balances depending negatively on the expected inflation rate.

Liviatan [5] and Drazen [2] intended to put SW analysis under more general conditions of the monetary growth model of Sidrauski type, for the SW framework is based on rather special assumptions of an overlapping generations model. They tried to show SW results using the monetary growth model with the perfect foresight. Their models are, however, characterized by unstable roots, and thus do not fit into the idea of the ceiling of per capita bond holding.

The purpose of this study is to see how the SW mechanism works in the framework of the monetary growth model with the stability. We will show that the SW weak result is not necessarily obtained under the adaptive expectations scheme. When the perfect foresight is introduced, it is shown that the model explodes. Finally, the result of a simple modification of the model to include foreign trade is presented. When the export drive works, the result that the long-run inflation rate is lower follows.

In the first chapter, SW, Liviatan and Drazen's models are briefly reviewed. In the second chapter, we modify Sidrauski's monetary growth model to include the issuance of government bonds and the persistent deficits, and try to look into the mechanism of the SW weak results. Summary of the analysis comes in the end.

I. Review of the previous models.

1. Sargent— Wallace (SW) model.

In SW [9], an example that the monetary authority's ability to control inflation is constrained under certain conditions even in an economy with monetarist conditions was presented. Suppose there exist persistent government deficits which must be financed by issuing bonds or seignorage (revenue from money creation), or by both, then the public's demand for interest bearing government bonds constrains the government by setting a ceiling on per capita real bonds holding (based on the size of the economy). When the fiscal authority dominates the monetary authority and independently sets budgets, and therefore determine necessary amount of revenue which should be raised from bond sales and seignorage, the monetary authority must finance any difference between the required revenue (suppose the deficits persist) and bond sales by seignorage (and thus by money creation).

Under these conditions, the monetary authority reduces the growth of high powered money to fight inflation while allowing the increase in the real stock of government bonds (by assuming that the real rate of interest on bonds exceeds economic growth rate and that government bonds are all one period debts, they introduce a key relation that real stock of bonds grows faster than the size of the economy). As soon as per capita bond holding hits the ceiling, the monetary authority is forced to raise the growth rate of money supply to finace the deficits (including the principal and interest due on the outstanding bonds). This causes "higher" inflation. We call this case the SW weak result. They even showed a numerical example of "spectacular case" that tight money today causes higher inflation immediately. This rather "extreme" example is shown based upon the expectation dominated demand for money equation. The expectation of price hike today causes the real cash holding to decrease immediately and leads to higher inflation.

To show the SW weak result, let $D(t)$, $H(t)$, $P(t)$, $R(t)$, $B(t-1)$ and $N(t)$, be the bugdet deficits defined as real expenditures (net of interest payment on bonds) minus tax revenue, the stock of high powered money, the price level, the real interest rate on one period bonds between time t and $t+1$, the government debt from the private sector between t and $t+1$, measured in terms of time t goods and the population (t denotes time). Then, the consolidated (the Treasury and the Federal Reserve System) budget constraint is written as

$$D(t)=\{[H(t)-H(t-1)]/P(t)\}+\{B(t)-B(t-1)[1+R(t-1)]\},$$
$$t=1,2,.... \tag{1}$$

implying that the deficit is financed both by money creation and issuing bonds (perfect foresight is assumed). The second term shows that one period bonds are redeemed with the interest paid and the difference between the current bond issuance and the amount needed to redeem bonds can be used for financing the deficits in current period. The population is assumed to grow at the constant rate n;

$$N(t+1)=(1+n)N(t), \quad t=0,1,2..... \tag{2}$$

with $N(0)>0$ and $n>-1$.

By (1) and (2), per capita expression of the budget constraint is;

$$B(t)/N(t)=\{[1+R(t-1)]/(1+n)\}[B(t-1)/N(t-1)]$$
$$+[D(t)/N(t)]-\{[H(t)-H(t-1)]/[N(t)P(t)]\}. \tag{3}$$

Now monetary policy is represented by alternative constant growth rate θ of $H(t)$ for $t=2,3,.....,T$ where $T>2$, and the per capita real stock of government bonds hits the ceiling at $t=T$ and stays at the same level($b_\theta(T)$) for $t>T$. The time path of $H(t)$ for $t>T$ is constrained by this condition on the government debts. Given $H(1)$, the time path of $H(t)$ is subject to

$$H(t)=(1+\theta)H(t-1), \quad t=2,3,\ldots,T, \tag{4}$$

where a smaller θ represents tighter monetary policy.

To characterize the monetarist economy, the price level is assumed to be proportional to per capita money supply,

$$P(t)=(1/h)[H(t)/N(t)] \tag{5}$$

where h is a positive constant. Equation (5) implies that $P(t)/P(t-1)=(1+\theta)/(1+n)$, thus the inflation rate is positively related to the monetary expanssion rate for $t=2,3,\ldots,T$.

Now they show the dependence of the inflation rate for $t>T$ on the one for $t\leq T$ through equation (3). Substituting the relation $B(t)/N(t)=B(t-1)/N(t-1)=b_\theta(T)$ for $t>T$ and $H(t)=hN(t)P(t)$ into (3),

$$1-[1/(n+1)][P(t-1)/P(t)]$$
$$=([D(t)/N(t)]+\{[R(t-1)-n]/(1+n)\}b_\theta(T))/h \tag{6}$$

is obtained. The right side of equation (6) must be less than one under assumption of $R(t-1)>n$, and this fact itself imposes an upper limit on $b_\theta(T)$. From (6), the higher $b_\theta(T)$ the higher the inflation rate. The next step is to show that the smaller θ makes $b_\theta(T)$ higher. Making use of (4) and (5), (3) can be written as

$$b(t)=\{[1+R(t-1)]/(1+n)\}b(t-1)+[D(t)/N(t)]$$
$$-[h\theta/(1+\theta)], \quad t=2,3,\ldots,T \tag{7}$$

where $b(t)=B(t)/N(t)$. By iteration on (7),

$$b_\theta(t)=\phi(t,1)b(1)+\sum_{s=2}^{s=t}\phi(t,s)[D(s)/N(s)]$$
$$-([h\theta/(1+\theta)]\sum_{s=2}^{s=t}\phi(t,s)), \quad t=2,3,\ldots,T. \tag{8}$$

is obtained where $\phi(t, t)=1$ and

$$\phi(t, s)=(\Pi[1+R(j)])/(1+n)^{t-s}$$

for $t>s$ Equation (8) shows that the smaller θ is the larger $b_\theta(T)$ is. From equations (6) and (8), it turns out that tighter monetary policy now leads to higher inflation later.

Also when taking account of the dependence of the demand for money on the expected inflation, the demand for money equation can be written as (derived from an overlapping generations model)

$$P(t)=(2/\gamma_1)\sum_{j=0}^{j=\infty}(\gamma_2/\gamma_1)^j[H(t+j)/N(t+j)]$$

where γ_1 and γ_2 are some positive constants. This equation suggests that when per capita money supply $h(t+j)=H(t+j)/N(t+j)$ is expected to increase in the future, the current price starts rising now.

2 Liviatan model.

He tries to recover the SW spectacular result in the framework of the monetary growth model of Sidrauski type. A representative family maximizes discounted utility over infinite time horizon, based upon per capita variables:

$$\int_{t=0}^{t=\infty} e^{-\sigma t}u[c, m]dt \qquad \text{s.t.} \tag{9}$$

$$\dot{a}(t)=(1-\tau)r(t)b(t)-nb(t)+(1-\tau)[y(t)+s(t)] \tag{10}$$
$$-[n+\Pi(t)]m(t)-c(t),$$

$$a(t)=m(t)+b(t), \quad \text{and} \tag{11}$$

$a(0)=a_0$: a given constant,

where u, c, m, b, y, n, r, τ and σ are standard utility function, consumption, real balances ($m = M/PL$, M is nominal balances, P is the price level, and L is the population), indexed government bonds,[1] income, the constant rate of population growth, gross real interest rate on government bonds, proportional income tax and the constant subjective discount rate. $\dot{x}$ denotes time derivative of x. $\Pi = \dot{p}/p$ is the inflation rate (There is no difference between expected and actual inflation rate). s denotes real transfer to the public (Time subscripts have been deleted).

Assuming that the utility function is separable

$$u(c, m) = \beta_1 \log c + \beta_2 \log m, \tag{12}$$

the Euler conditions are

$$\frac{\beta_1}{c} - \lambda = 0 \tag{13}$$

$$\frac{\beta_2}{c} - \lambda(n + \Pi) - q = 0 \tag{14}$$

$$\lambda[(1-\tau)r - n] - q = 0 \tag{15}$$

and the multiplier equation is

$$\dot{\lambda} = \delta\lambda - q. \tag{16}$$

National income identity on the macro level is

$$y = c + g \tag{17}$$

where g is per capita real government expenditure. The government deficit is assumed to persist at the positive level in the real term, and is expressed as

$$D = (1-\tau)s + (g - \tau y) + (1-\tau)rb = \dot{m} + (n + \pi)m + \dot{b} + mb. \tag{18}$$

The right side is the sum of $\dot{M}/PL$ and $\dot{B}/L$ where B is the aggregate amount of indexed bonds.

While defining $\mu = \frac{\dot{M}}{M}$, the relation $\frac{\dot{m}}{m} = (\mu - n - \Pi)$ holds. Therefore, along the steady state equilibrim path characterized by $\dot{c} = \dot{m} = \dot{b} = 0$, the effect of a reduction in μ on Π is shown as $\Delta\pi = \Delta\mu$, thus both of the SW paradoxical results does not show up. In the following analysis, he focuses on transition process from an equilibrium to another equilibrium. In doing that the definition of deficits is changed to the "net" government deficit to conform with the SW treatment as

$$D_N = g - \tau y + (1-\tau)s = D - (1-\tau)rb \tag{18'}$$

where g, s, τ are assumed to be constant.

Now as a policy experiment, μ is lowered throughout the period $[0, T]$ while the real stock of government bonds is raised and b stays at $b(T)$ for $t \geq T$, and the level of μ is determined endogenously. Thus, the concept of tight money here is to lower μ as in SW [9].

Combining equation (10) and (11), the equation

$$\dot{b} = \rho b - [(n+\Pi)m + \dot{m}] + (y+s)(1-\tau) - c \tag{19}$$

is derived where $\rho = (1-\tau)r - n$, and the relation $\dot{m} = (\mu - \pi - n)m$ gives another expression of (19) as

$$\dot{b} = \rho b - \mu m + Q_1, \quad Q_1 = (y+s)(1-\tau) - c \tag{20}$$

where Q_1 is claimed to be constant.[2)]

To form a dynamic system with respect to b and m, let $i = (1-\tau)r + \pi = \rho + n + \pi$. From the first order conditions, $\left(\frac{\beta_2}{\beta_1}\right)\left(\frac{c}{m}\right) = i$ obtatins. Together with the relation $\dot{m} = (\mu - \pi - n)m$, the equation

$$\dot{m} = (\mu + \rho)m + Q_2, \quad Q_2 = -c\left(\frac{\beta_2}{\beta_1}\right) \tag{21}$$

follows where Q_2 is again assumed to be constant.

Starting from a steady state, he observes that when μ is reduced to μ' over $[0, T]$, $\dot{m}(0) < 0$ holds in (21) (starting time is denoted by $t = 0$), therefore, $m(t)$ is decreasing over that time span while by (20) $b(t)$ is decreasing. The equation (21) is explicitly solved as

$$m(t) = \overline{m}(\mu') + [m_0 - \overline{m}(\mu')]e^{(\mu'+\rho)t}, \quad \overline{m}(\mu') = \frac{-Q_2}{\mu'+\rho} \tag{22}$$

where $m_0 - \overline{m}(\mu') < 0$. Then, by the optimality condition

$$\left(\frac{\beta_2}{\beta_1}\right)(\mathrm{c/m}) = i = \rho + n + \pi, \tag{23}$$

as m goes down due to the reduction in μ, π must go up (c is assumed to stay at a constant level[3)]). Thus tight money now generate higher inflation immediately as in the SW spectacular result. During $[0, T]$, he claims the relation $\dot{m} + \dot{b} = 0$ to have been holding due to the fact that $a = m + b$ must be constant at a steady state. Thus, per capita bond holding was increasing while $\dot{m} < 0$ was taking place. $b(t)$ attains its ceiling level $b(T)$ at T by assumption. Since the relation $\dot{m} + \dot{b} = 0$ continues to hold for $t > T$, $\dot{m} = 0$ must hold for $t > T$. Then, by equation (21),

$$0 = (\mu_1 + \rho)m(T) + Q_2 \tag{21'}$$

holds where μ_1 is the value of μ which attains $\dot{m} = 0$ for $t > T$ (μ jumps up from μ' to μ_1 at T). For $m(0) > m(T)$, the relation $\mu_1 > \mu_0 > \mu'$ must hold. From (23), when $m(t)$ stops falling at $m(T)$, $\pi(t)$ stays at $\pi(T) > \pi(0)$ for $t > T$. This completes his explanation of the SW mechanism for the spectacular result.

He mentions that "we have been put by μ_1 in a steady—state solution". However, by assuming away the variability of c, his analysis is entirely based on the steady state equilibrium path. Therefore, the relation $\Delta\mu = \Delta\pi$ is supposed to have been holding throughout the story, and the SW result should not have been recovered.[4) The problem is rough treatment of the relative movements of variables in equation (20), (21) and (23). Especially, in (23) the change in m can not be directly related to the change in π outside a steady state because c is varying. Besides the system of differential equations (20) and (21) can be shown to have exploding characteristic roots; Two characteristic roots can be shown to be $\mu+\rho$ and ρ where ρ is positive by assumption. Thus, only if $\mu < 0$ and $\mu+\rho < 0$, the system has the saddle point equilibrium. However, the fact that the growth rate of money supply can take only negative value contradicts the basic and implicit assumption in SW [9]. Especially, in Liviatan's model, if $\mu+\rho < 0$ in equation (20), $\overline{m}(\mu) < 0$ holds. Obviously, this is unacceptable. Thus, his model does not guarantee that the system converges to the steady state. Along the steady state path, the SW result does not hold, and the system explodes outside the equilibrium. This means that SW set—up of the ceiling for b does not fit into the model here. We thus need to have a model with stable roots.

3. Drazen model.

His model is a generalization of Liviatan model in the sense that the utility function takes general form $u(c, m)$ with only separability assumption (unlike Liviatan's exponential utility function). Both SW weak and spectacular cases are tried to recover in the framework of the monetary growth model. Notations and the structure of the model are similar to Liviatan's;

$$\max \int_{t=0}^{t=\infty} u[c(t), m(t)]e^{-\delta t}dt \quad \text{s. t.} \tag{24}$$

$$c(t)+[\dot{b}(t)+nb(t)][\dot{m}(t)+\pi(t)+n)m(t)] \\ = (1-\tau)(rb(t)+y(t)) \tag{25}$$

and

$$a(t) = m(t)+b(t). \tag{26}$$

From the Euler conditions and the multiplier equation,

$$u_c - q = 0 \tag{27}$$

$$u_m - q(\pi+n+\delta) = -\dot{q} \tag{28}$$

$$q[(1-\tau)r - n - \delta] = \dot{q} \tag{29}$$

are derived. The aggregate national income identity is

$$y = c+g \tag{30}$$

where g is fixed as in Liviatan's. In forming a dynamic system of m and b, he regards c as constant ("which by (30) is fixed"). But from the first order conditions, c is constant only when the Lagrange multiplier q is constant which is the steady state property. Therefore, this assumption is supposed to cause a conceptual difficulty when analyzing the time path of the system outside the steady state. Due to the constancy of q over time, $(1-\tau)r = n+\delta$ holds from (29). r is, therefore, treated as exogenous.

Equztions (25) and (28) provide

$$\dot{m}+\dot{b} = \delta b+(1-\tau)y-(\pi+n)m-c \tag{31}$$

and

$$u_m = q(\pi + n + \delta), \quad u_{mm} < 0, \tag{32}$$

respectively. Here, $\pi + n + \delta = \pi + (q - \tau)r$ is defined as the net nominal interest rate i. The relation on the real balances

$$\dot{m} = (\mu - \pi - n)m, \quad \mu = \frac{\dot{M}}{M} \tag{33}$$

is substituted into (31) to get

$$\dot{b} = \delta b - \mu m + [(1 - \tau)y - c] \tag{34}$$

where the square bracket term is fixed and denoted by D. Substitution of (32) into (33) gives

$$\dot{m} = \left(\mu + \delta - \frac{u_m}{q}\right)m. \tag{35}$$

The system of equations (34) and (35) describes the time path of m and b given the value of policy parameter μ.

To analyze the SW policy experiment on this system, μ is reduced from μ_0 to μ' during a finite period until time t_1. b and m are fixed at their t_1 level, and μ is adjusted to necessary level to make $m(t_1)$ and $b(t_1)$ steady state values. No open market operation is made, and thus discretionary changes in m and b are ruled out. μ jumps up to μ_1 at t_1.

Fig.-1 below is from Drazen[2].

The equations of $\dot{b} = 0$ and $\dot{m} = 0$ lines are

$$b = \left(\frac{\mu}{\delta}\right)m - \frac{D}{\delta} \tag{36}$$

$$u_m[m]/q = \mu + \delta. \tag{37}$$

$\dot{b} > 0$ holds below $\dot{b} = 0$ locus and $\dot{b} < 0$ above it while $\dot{m} > 0$ holds

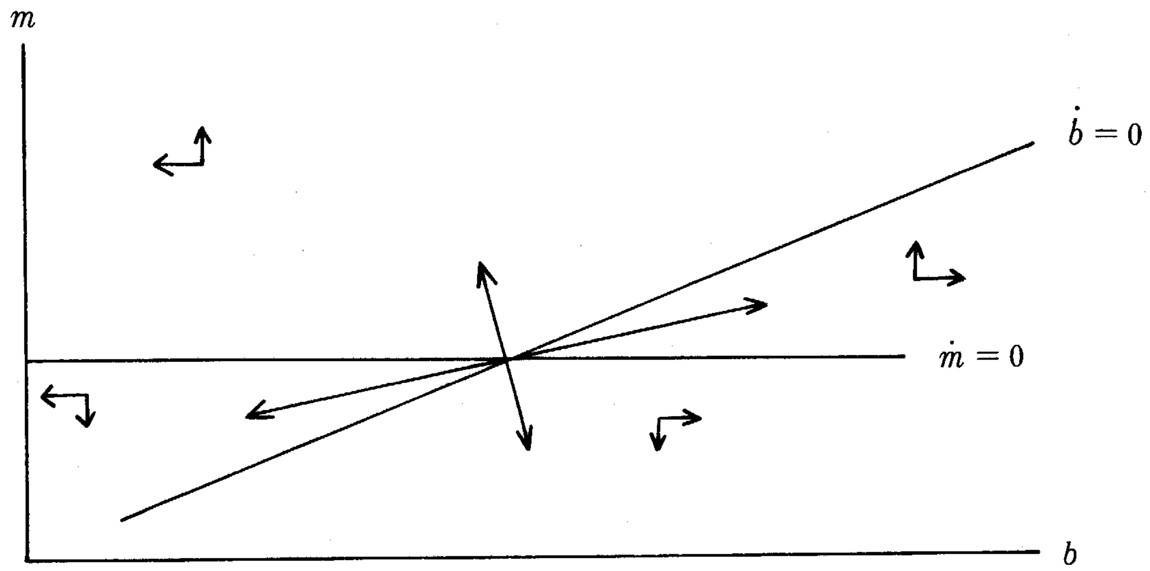

Fig. 1

above $\dot{m}=0$ locus and $\dot{m}<0$ below it.

As shown in fig.—1, Drazen's system is exploding like Liviatan's. Algebraically, two characteristic roots of the system are δ and $\mu+\delta-\frac{u_m}{q}$ where $\delta>0$ is given. So, $\mu+\delta-\frac{u_m}{q}<0$ must hold for the saddle point property of the equilibrium. But, from (32) it can be shown that $\mu+\delta-\frac{u_m}{q}=\mu+\delta-\pi-n-\delta=\mu-\pi-n=\frac{\dot{m}}{m}$. Therefore, $\dot{m}<0$ must be always assumed for the stability of the model. As in Liviatan's, this means that per capita real balances fall into negative values, but m must be frozen at t_1. This breaks the stability. As is shown in fig.—1, his model is of unstable nature. In fig.—1, the reduction in μ shifts the both loci up. The slope of the locus of the intersection between the lines depends on the relative shift of the lines. In fig.—2, the steady state locus(ss) is negatively sloped assuming $\dot{b}=0$ locus shifts up faster than $\dot{m}=0$ locus.

When moving along SS by open market operations, $\Delta\mu=\Delta\pi$ holds through (33), therefore the adjustment of b and m to the new steady state in his model was interpreted as being done through endogenous process.

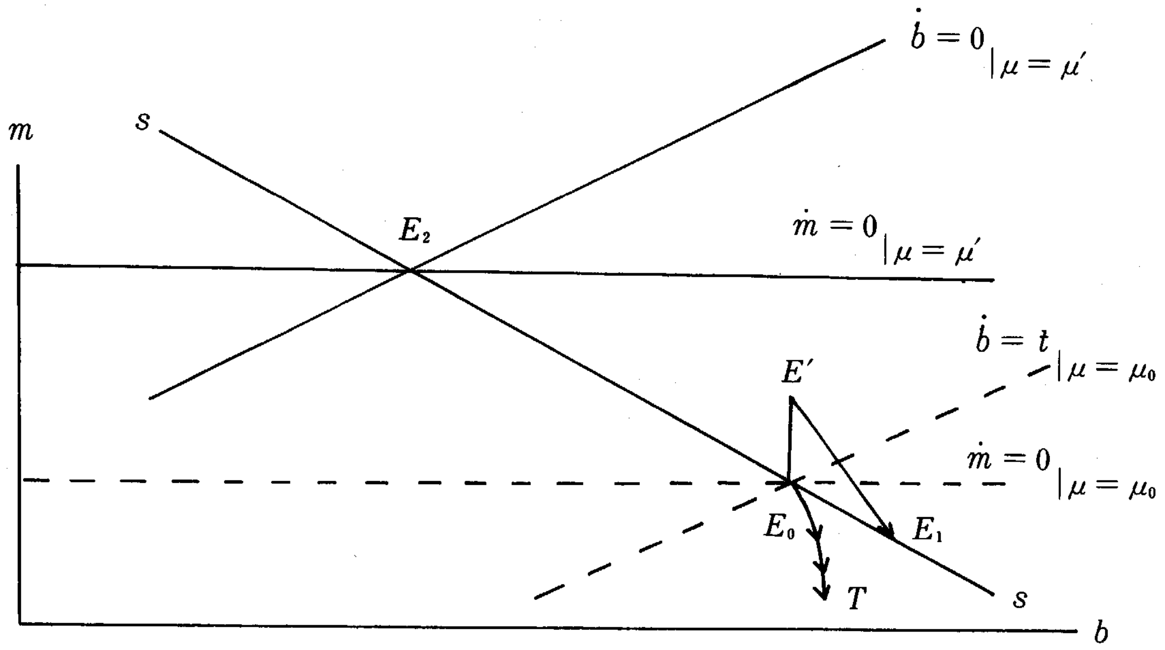

Fig. 2

From (36), if SS is vertical, the money demand elasticity with respect to μ is one $\left(\frac{dm}{d\mu}\frac{\mu}{m}=-1\right)$. If the elasticity is greater than one $\left(\frac{dm}{d\mu}\frac{\mu}{m}>-1\right)$, SS is positively sloped. When SS is positively sloped, the policy experiment is as follows: μ decreases at t_0, then from (36) m jumps up to E' in fig.—2 (b is momentarily fixed). The new steady state E_1 in this case will show higher m and b at t_1. Thus, from (32) π is lower than its initial value. The SW result does not hold (The reason why E_1 is to the right of E' is that "since it is known at t_0 that at t_1 μ_1 will be chosen so that $m(t_1)$ and $b(t_1)$ are steady state values, we jump at t_0 to a point which implies that we will just arrive to a point on SS at t_1", and b hits its ceiling at t_1 by assumption. However, what the monetary authority can do is to change the location of the steady state along SS line through the change in μ while the system will explode as time goes on. Therefore, it is not clear how the new steady state is achieved at E_1 on SS.

If SS is vertical, the system will stay at E' with higher m. So, π is lower than the initial value as well. Therefore, SS must be negatively sloping in order for SW results to hold. When SS is negatively sloping, he shows that whether SW results hold or not depends on the slope and the momentary change in m; Equation (31) together with (32) gives

$$\dot{a} = \delta(b+m) - m u_m / q + D \tag{38}$$

which is the equation of SS ($b = \frac{m u_m}{q\delta} - m - \frac{D}{\delta}$ by (36) and (37)) when $\dot{a} = 0$. Thus, total assets a is falling below SS and rising above it. This gives the slope of the paths from E' to E_1 and from E_0 to T in fig.—2; Depending whether the system is above, on, or below SS, the slope of the path is less than, equal to, or greater than one in absolute value(by the relation that if $\dot{a} > 0$, the change in $b(\dot{b} > 0)$ is greater than in $m(\dot{m} < 0)$, etc. Then, let the slope of SS be less than one (flat), and also let there be no jump at t_0, the path will fall out of SS (like E_0 T) because the momentary slope of the path is $-1(\dot{a} = 0)$, and below SS $\dot{a} < 0$ while implying the decrease in m is greater than the increase in b. In this case, the path will never hit SS. When m jumps down, the same will hold. Only when m jumps up, and by $\dot{a} > 0$ above SS the slope of the path is less than one but greater than SS, the steady state will be reached like E' E_1. Then, because m jumps up and eventually $m(t_1) < m(0)$, the SW weak result is derived through equation (32).

The spectacular result can be obtained if the slope of SS is greater than one in absolute value. Here, if m jumps up, the path will never hit SS. When m jumps down and the path has the slope less steeper than SS, it will hit SS, and through (32) the spectacular result is obtained.[5] Similarly, if SS has the slope equal to -1, and m shows no jump, the path will be identical with SS, and π immediately starts rising (as long as m is frozen at t_1 as he sets up, $\pi(t) = \pi(T) > \pi(0)$ for $t \geq T$).

The basic question on this analysis is how the monetary authority can manage the economy to come back to the steady state in the conformable direction with SW experimental conditions. While making use of (36), he could show the jump in m to E', then it is not clear how the path can hit SS while the same restrictive monetary policy is taken and how m can jump down (as required when SS has the slope greater than one in absolute value). If the system was stable, it would have been possible to derive the meaningful steady state relation among μ, m and π as in Sidrauski [10], so that the change in μ and π is supposed to have been shown explicitly. Obviously, the stability of the model is necessary here, too.

II Analysis of the SW weak result.

In this chapter, we try to construct a monetary growth model characterized by the stable equilibrim to make SW policy experiment. Only the weak result is analyzed. The spectacular case does not seem to arise under the adaptive expectations scheme, and if the perfect foresight is adopted as in the previous models, the system is shown to explode. A very brief treatment of the implication of the export drive will be shown in the current context while keeping the Japanese economy in mind where huge budget deficits coexist with a fairly low inflation rate.

1. modification of Sidrauski model and the policy experiment.

In this section, we slightly modify Sidrauski monetary growth model. Only change is made as to the inclusion of government bonds. Besides we assume a certain amount of deficits exists. We keep the basic structure of Sidrauski model in the spirit that no matter what level of deficits exists, economic agents decide their behavior in the context of intertemporal utility maximization over infinite time horizon.

The model consists of the followings:

$$\max \int_{t=0}^{t=\infty} [u(c, m)] e^{-\delta t} dt$$

Lagrangean;

$$L = u(c, m) + \lambda((1-\tau)[y(k)+v] + (1-\tau)rb - (n+\pi)m$$
$$-(u+n)k - (n+\pi)b - c) + q(a-k-m-b)$$

The Euler conditions;

$$u_c - \lambda = 0 \tag{40}$$

$$u_m - \lambda(n+\pi) - q = 0 \tag{41}$$

$$\lambda(1-\tau)y'(k) - \lambda(u+n) - q = 0 \tag{42}$$

The multiplier equation;

$$\dot{\lambda} = \delta\lambda - q \tag{43}$$

The transversality condition;

$$\lim a\lambda e^{-\delta t} = 0 \tag{44}$$

Constraints;

$$a = k + m + b \tag{45}$$

$$\dot{a} = (1-\tau)[y(k)+v] + (1-\tau)rb - (n+\pi)m$$
$$-(u+n)k - (n+\pi)b - c. \tag{46}$$

Notations; k per capita real stock of capital, v per capita transfer to the public, π expected inflation rate, u depreciation rate, b per capita real stock of bonds B/PL where B is total nominal bonds outstanding, P is the price level and L is population, λ and q multipliers. Other symbols are consistent with the above models.

From (42), $(1-\tau)y'(k)-(u+n)=\frac{\rho}{\lambda}=\rho>0$ (assumed to be positive as in Sidrauski [10]). From (41), thus $u_m=\lambda(n+\rho+\pi)$. Then, from equations (40)−(42), we have

$$c = c^0(\lambda, \rho, \pi) \tag{47}$$

$$m = m^0(\lambda, \rho, \pi) \tag{48}$$

$$k = k^0(\rho) \tag{49}$$

as in Sidrauski [10]. Equation (45) is written as

$$a = k^0(\rho) + m^0(\lambda, \rho, \pi) + b, \tag{45$'$}$$

so,

$$\rho = \rho(a, \lambda, \pi) \tag{50}$$

b was suppressed from (50) because of the assumption that agents regard B as exogenously given at each moment of time, and optimize their behavior. Price was represented by π

$$c = c'(a, \lambda, \pi) \tag{51}$$

$$m = m'(a, \lambda, \pi) \tag{52}$$

$$k = k'(a, \lambda, \pi) \tag{53}$$

Given π, the pair of equations (43) and (46) determine the time path of λ and a. While assuming that the effect of the change in bonds B does not alter (it might change the size of the partials, but not the direction) the signs of the partials $\frac{\partial k}{\partial a}$, $\frac{\partial m}{\partial a}$, $\frac{\partial c}{\partial a}$, $\frac{\partial k}{\partial \lambda}$, $\frac{\partial m}{\partial \lambda}$, and $\frac{\partial c}{\partial \lambda}$, and so the slopes of $\dot{a}=0$ and $\dot{\lambda}=0$ lines do not change, the stability of the model is maintained. This process is then exactly same as Sidrauski's, so under the condition of

$$(\pi+n)J_2+J_1<0$$

where

$$J_1 = u_{mm} - u_{cm}\frac{u_m}{u_c} < 0$$

and

$$J_2 = u_{cc}\frac{u_m}{u_c} - u_{cm} < 0,$$

the system of equation (43) and (46) has a saddle point equilibrium.

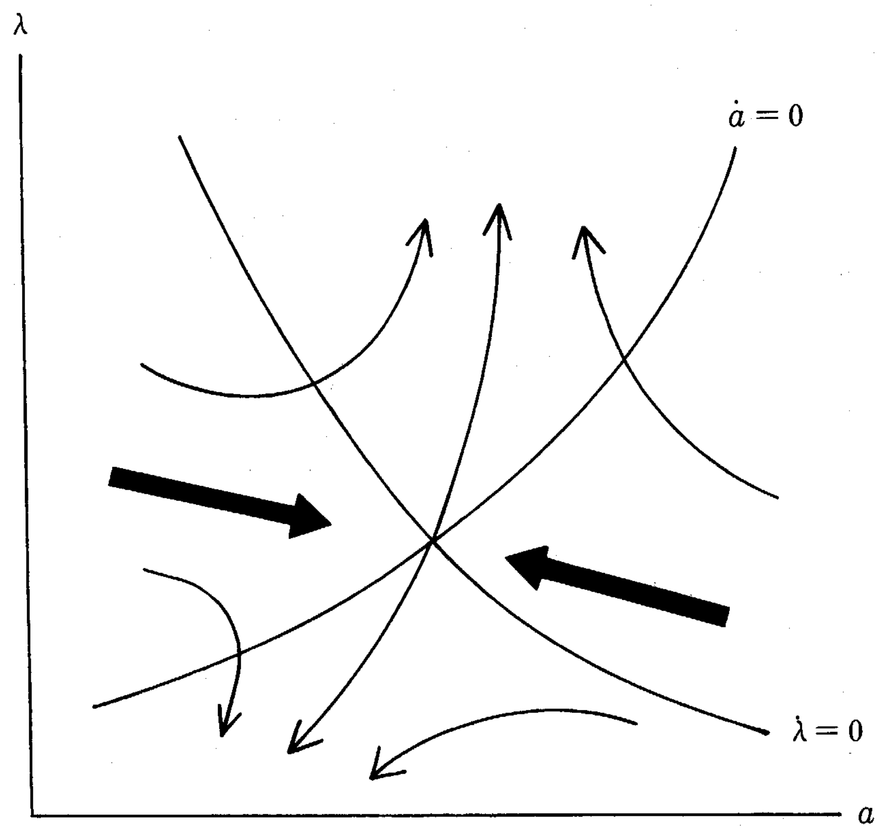

Fig. 1a

In the figure above, there is corresponding λ to each a on the heavy arrows. Optimal λ allocates the stock of wealth into capital and real cash balances through (45)′, (51), (52) and (53) (given B), and determines the optimal allocation of disposable income into consumption and saving. Obviously, the changes in π and v shift the optimal path of λ, so

$$\lambda = \lambda(a, \lambda, v) \tag{54}$$

By substituting (54) back into (51), (52) and (53), we have

$$c = c(\mathrm{a}, \pi, v) \tag{55}$$

$$m = m(\mathrm{a}, \pi, v) \tag{56}$$

$$k = k(\mathrm{a}, \pi, v) \tag{57}$$

Now expectation formation process is introduced by the adaptive scheme,

$$\dot{\pi} = \alpha(\dot{P}/P - \pi), \quad \alpha > 0. \tag{58}$$

In the money market,the damand and supply of real money balances are equal in equilibrium,

$$\frac{M}{PL} = m(a, \pi) \tag{59}$$

where

$$a = \frac{M}{PL} + k + b. \tag{60}$$

Equations (58)−(60) form a system of three equations in six unknowns M, L, k, P, a and π. In oder to have a complete system describing the behavior of the economy over time, three additional equations are required. We define the growth rate of money and population as

$$\dot{M}/M = \theta \tag{61}$$

$$\dot{L}/L = n. \tag{62}$$

Also, since all output that was not consumed necessarily becomes capital accumulation, the capital accumulation rate is

$$\dot{k} = y(k) - (u + n)k - c(a, \pi). \tag{63}$$

We now have a system of six equations in six unknowns. We dropped the transfer payment v out of (59) and (63) because unlike Sidrauski, this variable does not have a role of changing money supply. Instead of this money supply route, we assume that there exists a certain amount of government debts

$$D=(1-\tau)v+(g-\tau y)+(1-\tau)rb=\dot{m} \qquad (*)$$
$$+(n+\dot{P}/P)m+\dot{b}+(n+\dot{P}/P)b,$$

and monetary policy is such that $\dot{\theta}$m+ $\dot{\beta}$ $b=\dot{D}$.[6)]

For the individual economic agent, B, β and θ are given at each moment of time, and the signs of the partial derivatives influencing the slopes of $\dot{a}=0$ and $\dot{\lambda}=0$ lines are assumed not to be altered by the above deficit financing policy (It does not seem to be a strong assumption). Therefore, the saddle point property of the system consisting of equations (43) and (46) is invariant throughout the policy experiment.

Now from (57) and the substitution between bonds and real balances $(\frac{\partial m}{\partial \theta}>0$ through the relation $\frac{\dot{m}}{m}=(\theta-\mathrm{n}-\frac{\dot{P}}{P}))$, we can write

$$m=g_1(a,\ \pi,\ \theta). \tag{64}$$

From(60)

$$a=g_2(m,\ k,\ \pi) \tag{65}$$

where π is included because $b=B/PL$ depends on θ. Substituting (65) back into (64), we have $m=g_1(g_2(m,\ k,\ \pi),\ \pi,\ \theta)$. So,

$$m=m(k,\ \theta,\ \pi) \tag{66}$$

is obtained. Also we have $c=c(k,\ \theta,\ \pi)$ through (55) and (65). Using (66), money market equilibrium condition (59) can be written as

$$\frac{M}{PL}=m(k,\ \theta,\ \pi) \tag{67}$$

Where given M, L, k, θ, π, the price level is determined. Substituting (65) and (66) back into (63), the equation of the capital accumulation is written as

$$\dot{k}=y(k)-(u+n)k-c(k,\ \theta,\ \pi) \tag{68}$$

Where c is the desired per capita consumption for the price level P which equilibrates the money market at each moment of time.

The system of equations is now formed by (58), (61), (62), (67) and (68) for five unknowns M, L, k, π and P. Differentiating equation (67) with respect to time and using (58), (61), (62) and (68), we have the expression for the rate of change in π as

$$\pi=\frac{\alpha}{\left(1+\alpha\dfrac{\partial m}{\partial \pi}\dfrac{l}{m}\right)}\left(\theta-\pi-n-\frac{l}{m}\frac{\partial m}{\partial k}[y(k)-(u+n)k-c(k,\ \theta,\ \pi)]\right) \tag{69}$$

By letting $\dot{k}=\dot{\pi}=0$, equations (68) and (69) give

$$c^*=y(k^*)-(u+n)k^* \tag{70}$$

$$\pi^*=\theta-n \tag{71}$$

where n represents both the rate of population and economic growth along such a path.

This development is exactly the same as Sidrauski's except that money supply mechanism is now by the substitution between bonds and cash balances rather than through the transfer payment by the government. But from our assumption that the changes in B, β and θ are exogenously given at each moment of time, and agents optimize their behavior follwing the first order conditions, we have the same sign conditions of partials (derived from the first order conditions) as in Sidrauski [10] ;

$$\frac{\partial c^*}{\partial k^*} > 0, \quad \frac{\partial c^*}{\partial \theta} > 0, \quad \frac{\partial c^*}{\partial \pi^*} > 0. \tag{72}$$

$$\frac{\partial m^*}{\partial k^*} > 0, \quad \frac{\partial m^*}{\partial \theta} > 0, \quad \frac{\partial m^*}{\partial \pi^*} > 0. \tag{73}$$

We now look into the stability conditions of (k^*, π^*) of the system of (68) and (69).

Linear Taylor approximations to (68) and (69) are

$$\dot{k} = \left[y'(k) - (u+n) - \frac{\partial c}{\partial k} \right](k - k^*) - \frac{\partial c}{\partial \pi}(\pi - \pi^*) \tag{74}$$

$$\dot{\pi} = \left(\sigma_1 \left[\theta - \pi - n - \frac{1}{m}\frac{\partial m}{\partial k}\left(y(k) - (u+n)k - c\right) \right] \right. \tag{75}$$

$$+ \sigma_2 \left[\left(-\frac{\partial m}{\partial k}\frac{1}{m^2}\frac{\partial m}{\partial k} + \frac{1}{m}\frac{\partial^2 m}{\partial k^2} \right)(y(k) - (u+n)k - c) \right.$$

$$\left. \left. + \frac{1}{m}\frac{\partial m}{\partial k}(y'(k) - (u+n) - \frac{\partial c}{\partial k}) \right] \right)(k - k^*)$$

$$+ (\sigma_3 \left[\theta - \pi - n - \frac{1}{m}\frac{\partial m}{\partial k}(y(k) - (u+n)k - c \right] + \sigma_1$$

$$\times \left[-1 - \left(-\frac{\partial m}{\partial \pi}\frac{1}{m^2}\frac{\partial m}{\partial k} + \frac{1}{m}\frac{\partial^2 m}{\partial k \partial \pi} \right) \left(y(k) \right. \right.$$

$$\left. \left. \left. - (u+n)k - c \right) + \frac{1}{m}\frac{\partial m}{\partial k}\frac{\partial c}{\partial \pi} \right] \right)(\pi - \pi^*)$$

where

$$\sigma_1 = -\alpha^2 \left[\frac{\partial^2 m}{\partial m \partial k}\frac{1}{m} + \frac{\partial m}{\partial \pi}\left(-\frac{\partial m}{\partial k} \right)\frac{1}{m^2} \right] \frac{1}{\left[1 + \alpha \frac{\partial m}{\partial \pi}\frac{1}{m} \right]^2}$$

$$\sigma_2 = \frac{\alpha}{\left[1 + \alpha \frac{\partial m}{\partial \pi}\frac{1}{m} \right]}$$

$$\sigma_3 = -\sigma^2\left[\frac{\partial^2 m}{\partial \pi^2}\frac{1}{m} + \frac{\partial m}{\partial \pi}\left(-\frac{\partial m}{\partial \pi}\right)\frac{1}{m^2}\right]\frac{1}{\left[1+\alpha\frac{\partial m}{\partial \pi}\frac{1}{m}\right]^2}.$$

By making use of (70) and (71), (74) and (75) become

$$\dot{k} = \left[y'(k)-(u+n)-\frac{\partial c}{\partial k}\right](k-k^*)-\frac{\partial c}{\partial \pi}(\pi-\pi^*) \tag{76}$$

$$\dot{\pi} \mp \sigma_1\frac{1}{m}\frac{\partial m}{\partial k}\left[y'(k)-(u+n)-\frac{\partial c}{\partial k}\right](k-k^*)+\sigma_1\left[-1+\frac{1}{m}\frac{\partial m \partial c}{\partial k \partial \pi}\right] \times(\pi-\pi^*). \tag{77}$$

These equations are denoted as

$$\dot{k} = b_1(k-k^*)+c_1(\pi-\pi^*) \tag{78}$$

$$\dot{\pi} = b_2(k-k^*)+c_2(\pi-\pi^*) \tag{79}$$

where

$$b_1 = [y'(k)-(u+n)-\frac{\partial c}{\partial k}]$$

$$c_1 = -\frac{\partial c}{\partial \pi}$$

$$b_2 = \left(\sigma_1\frac{1}{m}\frac{\partial m}{\partial k}\left[y'(k)-(u+n)-\frac{\partial c}{\partial \pi}\right]\right)$$

$$c_2 = \left(\sigma_1\left[-1+\frac{1}{m}\frac{\partial m \partial c}{\partial k \partial \pi}\right]\right).$$

Routh-Hurwitz conditions for the stability of the equilibrium $b_1+c_2<0$ and $b_1c_2-b_2c_1>0$ are satisfied when

$$y'(k)-(u+n)-\frac{\partial c^*}{\partial k^*}<0 \tag{80}$$

$$\left(1+\alpha\frac{\partial m^*}{\partial \pi^*}\frac{1}{m^*}\right)>0 \tag{81}$$

together with the conditions (72) and (73) hold.

The equilibrium (k^*, π^*) of the pair of differential equations (68) and (69) is a stable node[8] as shown in fig.—2.) Thus,when monetary expansion θ is constant, the economy will monotonically approach the equilibrium growth path. To see how the SW weak result arises, suppose θ is reduced from θ_0 to $\theta_1(\theta_0>\theta_1)$ to cope with inflation at E_0 (intersection of k_0 and π_0) while government bond issuance is increasing (temporarily we keep this statement. We will see later that the change in β depends on whether it is necessary to just finance the deficits). The first impact of this policy is the reduction in consumption ((72)) which raises capital accumulation ((68)) as

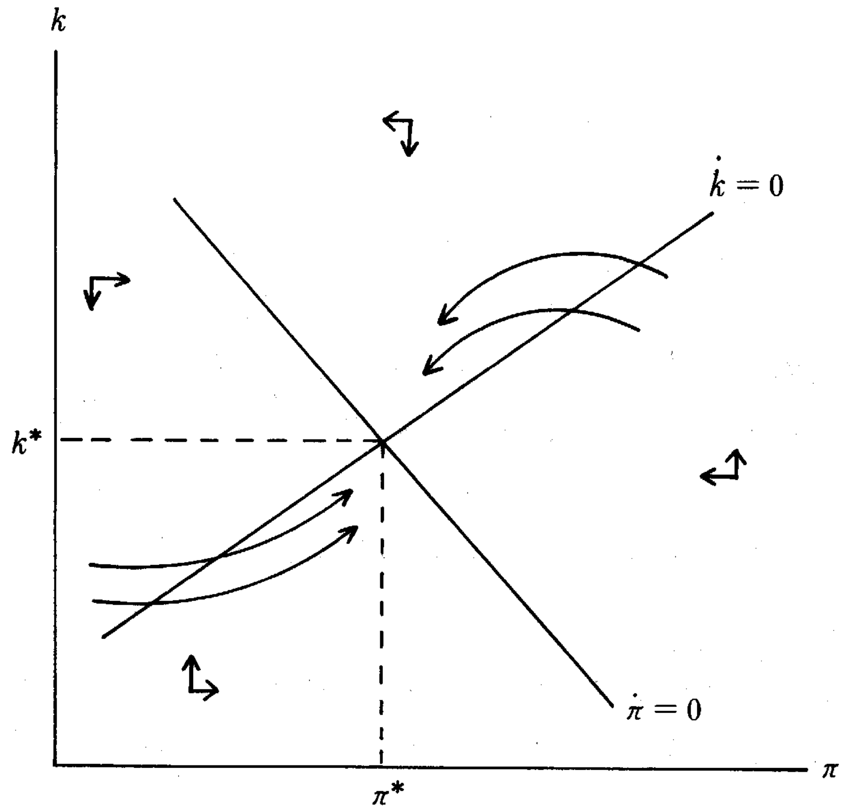

Fig. 2

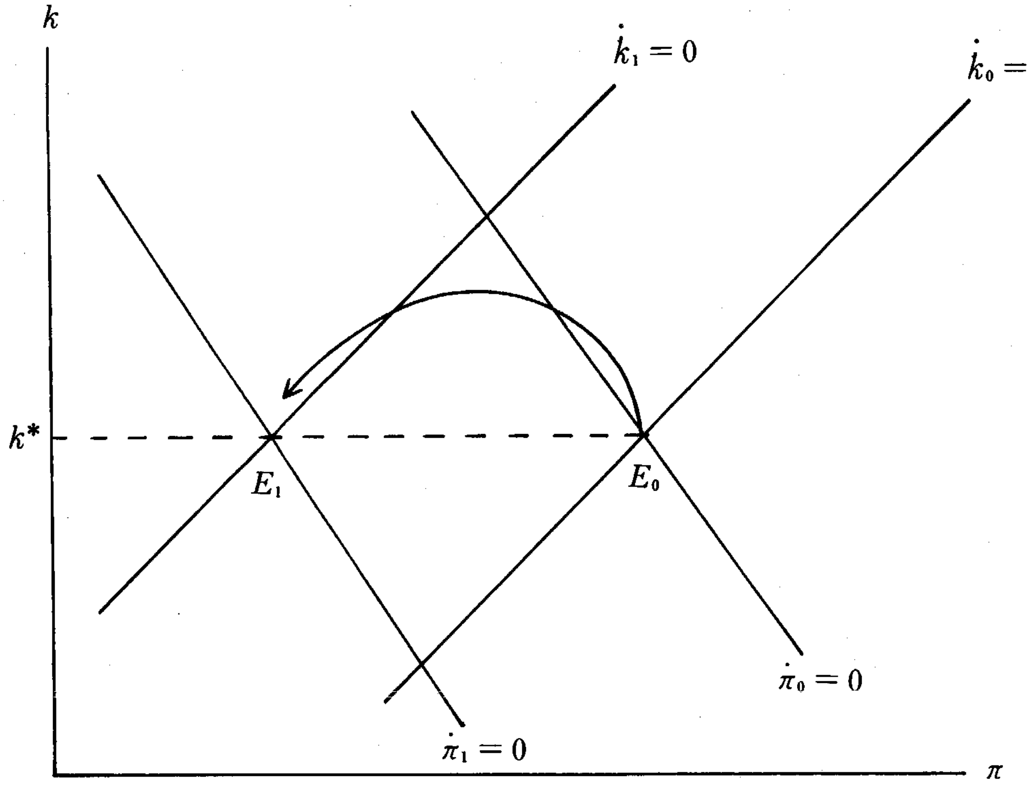

Fig. 3

well as the decrease in π ((69)). Both the $\dot{k}=0$ and $\dot{\pi}=0$ lines shift to the left in fig.—3. Since the long-run capital stock depends only on the rate of depreciation, the population growth rate, the subjective discount rate and the tax rate (this can be shown by setting $\dot{\lambda}=0$ in (43) and using (42)), and these factors are independent of θ, $\dot{k}_1=0$ and $\dot{\pi}_1=0$ lines will have the intersection at the same level of capital stock $k^*=k^*_0$ and at the lower π^*_1 where $\pi^*_1=\theta_1-n<\pi^*_0$($k^*_0$ are corresponding to E_0). The equilibrium moves from E_0 to E_1, and the restrictive monetary policy succeeds in reducing inflation rate.

According to SW, this process can not persist because there exists a ceiling for the per capita bond holding, and as soon as this ceiling is hit as a result of monetary contraction and the accumulation of goverment bonds, monetary expansion rate must be raised to finance a certain amount of goverment deficits.

To continue the SW policy experiment, we define the "net" deficits as

$$D_N = D - (1 - \tau) rb = \theta m + [\beta - (1 - \tau) r] b \qquad (**)$$

where $\beta - (1 - \tau) r > 0$ must be holding to raise fund by issuance of bonds. The relation

$$\dot{b} = (\beta - \dot{P}/P - n) b \qquad (82)$$

shows that $(\beta - \dot{P}/P - n) \geq 0$ must hold in order for b to rise and hit its ceiling (this condition shoud be thought of as one of the necessary conditions for the SW result). When budget surplus shows up, β might take negative value, however, we focus on the events during the period of the budget deficits here to analyze the mechanism of the SW phenomenon. As a standard case, we suppose $(\beta - \dot{P}/P - n) = 0$ initialy holds. The restrictive monetary policy reduces θ. Then, from (40) and (41) the value of m (m_1) will be higher than initial value (m_0) $(\frac{u_{mm}u_c - u_{cm}u_m}{u_c^2} < 0$ by $J_1 < 0$). But, θm can increase, decrease, or constant across the time in the deficits financing formula $D_N = \theta_m + [\beta - (1 - \tau) r] b$.

Depending on the change in θm and D_N (SW said only that the time path of D_N is given), the value of β is such that the funds raised in the right side of the relation is just enough to finance the deficits D_N. This implies that there are nine cases in total to be analyzed in the policy experiment. Depending on the situation, b might not necessarily hit the ceiling (for example, if D_N is decreasing and θm is increasing, β must decrease, and the reduction in β might result in $\dot{b} < 0$ in the relation $\dot{b} = (\beta - \dot{P}/P - n) b$ where $\dot{P}/P$ will be decreasing from the adaptive scheme (58)). We will analyze the cases in which b hits the ceiling in the following. However, the condition to make b hit the ceiling must be thought of as one of tions for the SW result to arise.

Corresponding to three cases of the change in θm , we set three cases of the change in D_N; D_N is increasing, decreasing and constant across the time. Our concern is around the period [0, T] where b hits the ceiling b (T) at $t = T$. We assume nothing about the time path of D_N long after the time T. In fig.—4 below (the position of T is flexible depending on the situation), three types of time path of D_N are shown as (1), (2) and (3). Nine combinations are shown in fig.—5.

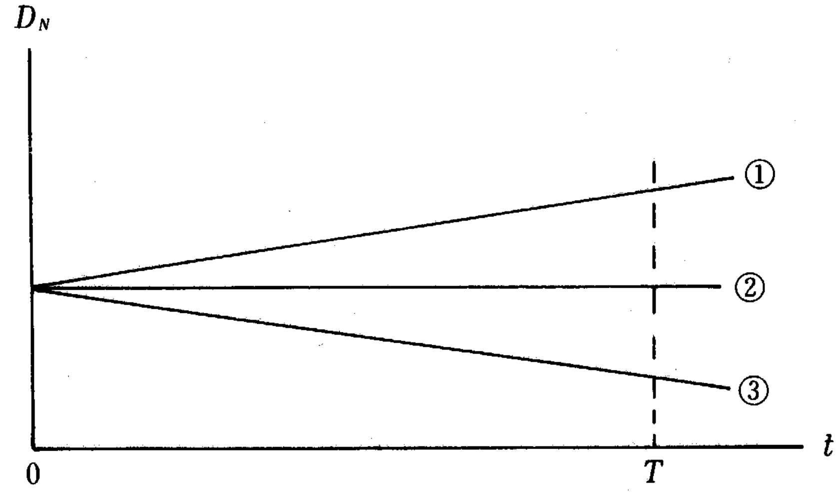

Fig. 4

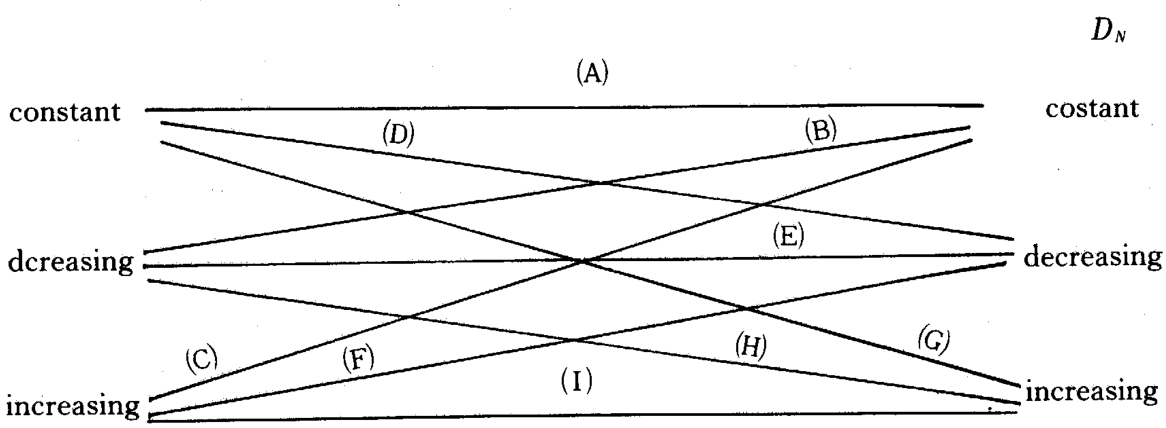

Fig. 5

Can the SW weak result hold in each case ? While using the relation $\dot{b} = (\beta - \dot{P}/P - n)\, b$ and $D_N = \theta m + [\beta - (1-\tau)r]\, b$, the answers to this question are:

(A) NO. As θ decreases, $\dot{P}/P$ decreases. By fixing β for a while, b will hit the ceiling. Then, β must decrease to the level such that $(\beta_1 - (\dot{P}/P)_1) = (\beta_0 - (\dot{P}/P)_0)$ to have $\dot{b} = 0$ where β_0, β_1 and $(\dot{P}/P)_0$, $(\dot{P}/P)_1$ are initial and ex post (after the ceiling being hit) values of each quantities. Then, there are three possibilities in the deficits financing equation; $D_N = (>, <)\theta_1 m_1 + [\beta_1 - (1-\tau)r]\, b(T)$. If the fund is less than D_N, the monetary authority can raise θ and so $\dot{P}/P$, but θ does not need to return to θ_0 because then $\dot{P}/P$, β and m return to the initial levels and the available fund will be exceeding D_N due to $b(T) b_0$.

So that the restrictive monetary policy can reduce $\dot{P}/P$ permanently. If D_N=fund, then β_1 can stay at this level and θ stays θ_1. $\dot{P}/P$ can be reduced permanently. If $D_N <$ fund, then θ can be reduced further, and by this reduction in θ_1, β and $\dot{P}/P$ can further decrease. $\dot{P}/P$ decreases permanently.

(B) NO. $\theta_1 m_1 < \theta_0 m_0$. There are three cases $D_N = (>, <)$ fund when $\beta = \beta_1$ (β_1 is such that $\dot{b} = 0$). If $D_N >$ fund, θ will be raised while keeping $(\beta_1 - (\dot{P}/P)_1) = (\beta_0 - (\dot{P}/P)_0)$, so that β is increasing as θ decreases, but $\dot{b} = 0$ is kept. θ does not need to return to θ_0 because then $\theta_0 m_0 + [\beta_0 - (1-\tau)r]b(T)$ is greater than D_N. Thus, $\dot{P}/P$ will be permanently lower than $(\dot{P}/P)_0$. If D_N=fund with β_1 as it is, $(\dot{P}/P)_1$ stays at that level. If $D_N <$ fund, θ can be further reduced so that β can be lower while keeping $\dot{b} = 0$. $\dot{P}/P$ will be even lower.

(C) NO. When β_1 is just enough to finance D_N, $\dot{P}/P$ stays at $(\dot{P}/P)_1$ permanently. No matter what the relation $D_N > (<)$ fund is, θ and β can not return to θ_0 and β_0 due to $b(T) > b_0$, so that the SW result does not arise.

(D) NO. The same reasonig as in (A) holds, but β_1 here will be lower than in (A) if $D_N =$ fund does not hold with β_1 (due to the reduction in D_N).

(E) NO. The same inference as in (B) holds, and β_1 here will be lower than in (B) if $D_N \neq$ fund with β_1 holds (for the same reason as in (D)).

(F) NO. By the same token as in (C). β_1 here will be less than in (C) due to the reduction in D_N.

(G) YES. But only when β_1 is not sufficient to finance D_N, θ might go up beyond the initial value θ_0 so that β can rise with $\dot{P}/P$ and more fund can be raised. There is no guarantee that β will go beyond β_0, however. If β_1 is just or more than enough to finance D_N, the SW result does not arise.

(H) YES. Only when β_1 is insufficient to finance D_N, this result can arise. Here, θ must go up from θ_1 to raise β, but then there are less possibility that the result holds because when θ increases, θm increases, so that β does not need to go up as much as in the case (G). In other cases of $D_N > (<)$ fund, the result does not hold.

(I) YES. When β_1 is insufficient to finance D_N, the result can arise. Because the increase in θm when θ decreased was not enough to compensate the reduction in β (to have $\dot{b} = 0$ while $\dot{P}/P$ has decreased), the increase in θ and β beyond θ_0 and β_0 will sufficiently finance D_N. Similarly, if β_1 is more than enough to finance D_N, the final value of θ and β might be above θ_0 and β_0. If β is just enough to finance D_N, the result does not hold.[9)]

As we have seen, the SW result has a limited applicability. Several conditions must have been set for it. If $(\beta - \dot{P}/P - n) < 0$, the monetary authority can engage in the counter inflation policy without hitting the ceiling of b. Then, by reducing β along with $\dot{P}/P$, the ceiling might not be hit permanently. Although their result could be explained in the cases (G), (H) and (I), some contingencies remained.[10)]

We investigated the SW mechanism in the absence of the perfect foresight. Previous models are based on the perfect foresight assumption. In the next section, we will see that the model loses the stability in the presence of this assumption. This result is consistent with the fact that Liviatan and Drazen models are of unstable nature.

2. Perfect foresight.

When the perfect foresight assumption (as rational expectations hypothesis in the absense of stochastic term) is introduced instead of the adaptive scheme, the equilibrium of the system ((68) and (69)) turns out to be unstable.

When the expectation is perfect, $\pi = \dot{P}/P$ holds. While deleting (58) out of our macro system, the characteristic equations are expressed as

$$k^2 - (b_1 + c_2)k + (b_1 c_2 - b_2 c_1) = 0$$

where

$$b_1 = \left[y'(k) - (u+n) - \frac{\partial c}{\partial k} \right]$$

$$c_1 = -\frac{\partial c}{\partial \pi}$$

$$b_2=\frac{\frac{\partial m}{\partial k}}{\frac{\partial m}{\partial \pi}}\left[y'(k)-(u+n)-\frac{\partial c}{\partial k}\right]$$

$$c_2=\frac{1}{\frac{1}{m}\frac{\partial m}{\partial \pi}}\left[-1+\frac{1}{m}\frac{\partial m \partial c}{\partial k \partial \pi}\right]$$

Routh-Hurwitz necessary and sufficient conditions are $b_1+c_2<0$ and $b_1c_2-b_2c_1>0$. But the sign conditions (72) and (73) tell that $b_1<0$, $c_2>0$, $b_2>0$ and $c_1>0$. The second condition for the stability is observed not to hold. Thus, in the presence of the perfect foresight, the SW set-up (the existence of the ceiling) for the policy experiment does not fit into the system.

3. The export drive

In Japan, it has been historically argued that firms prominently tend to sell more products abroad in the recession periods than in good periods in which they tend to be domestic market oriented. The export drive in the recession periods has helped improving the balance of payment. Since the middle of 1970s, the economic growth rate of Japan significantly decreased. The basic structure of the trade frictions between Japan and many parts of the world seems to be in the slow down of the Japanese economic growth and the continuous working of the export drive.

We try to see the implication of the export drive under the budget deficits context[11] in a very simple specification. The per capita real net export is defined as

$$ne = ne\left(\frac{\pi}{\pi_f}\right), \quad ne' < 0 \tag{83}$$

where π is the expected inflation rate in Japan and π_f is the actual (average) inflation rate in the rest of the world that is assumed to be exogenously given. Net export (per capita) decreases as π increases because if domestic firms expect higher inflation in the country, they tend to reduce export while $\dot{P}/P > \pi$ holds by the adaptive scheme (58), so that import will increase. This makes ne smaller. On the other, if π is lower, firms will try to sell more products abroad while $\dot{P}/P < \pi$, and thus import will be relatively smaller, so ne increases.

When net export is included,[12] the capital accumulation equation becomes

$$\dot{k} = y(k) - (u+n)k - c - ne\left(\frac{\pi}{\pi_f}\right) \tag{84}$$

Linear Taylor expansion is now

$$\dot{k} = \left[y'(k) - (u+n) - \frac{\partial c}{\partial k}\right](k - k^*) - \left(\frac{\partial c}{\partial \pi} + \frac{\partial ne}{\partial \pi}\right)(\pi - \pi^*)$$

, and the equation of $\dot{k}=0$ line is

$$k = \frac{c_3}{b_1}\pi + \frac{1}{b}(b_1 k^* - c_3 \pi^*)$$

where $b_1 < 0$ has been assumed above and $c_3 < 0$ holds (therefore, the condition was not altered). However, c_3 is greater than c_1 in absolute value, thus $\dot{k}=0$ line here is steeper and the intercept is greater than above. On the other hand, in the linear Taylor approximation of equation (69), the coefficient c_2 (denoted as c_4 here) becomes

$$c_4 = \frac{\alpha}{\left[1 + \alpha \frac{\partial m}{\partial \pi} \frac{1}{m}\right]} \left[-1 + \frac{1}{m} \frac{\partial m}{\partial k} \left(\frac{\partial c}{\partial \pi} + \frac{\partial ne}{\partial \pi}\right)\right] \tag{86}$$

which is negative (sign unaltered), but the absolute value has increased.

Then, the slope and the intercept of $\dot{\pi}=0$ line is shown in

$$k=-\frac{c_4}{b_2}\pi+\frac{1}{b_2}(b_2k^*+c_4\pi^*) \tag{87}$$

where $b_2 < 0$ by the assumption. Thus, the absolute value of both the slope and the intercept increases.

We saw that both the slopes and the intercepts of $\dot{k}=0$ and $\dot{\pi}=0$ lines increase in the inclusion of net export. However, as mentioned above the long-run stock of capital does not change by this modification. Therefore, these lines will intersect at the same level of k^* as in the absence of the trade, but it can be said that the equilibrium inflation rate π^* will be smaller as shown in fig.— 6 where π^*_0 denotes the equilibrium expected inflation rate in the absence of trade.The interpretation of this result is that the supply of goods is added during inflation periods, and the export gets larger when it does not accelerate inflation. In the context of the policy problem

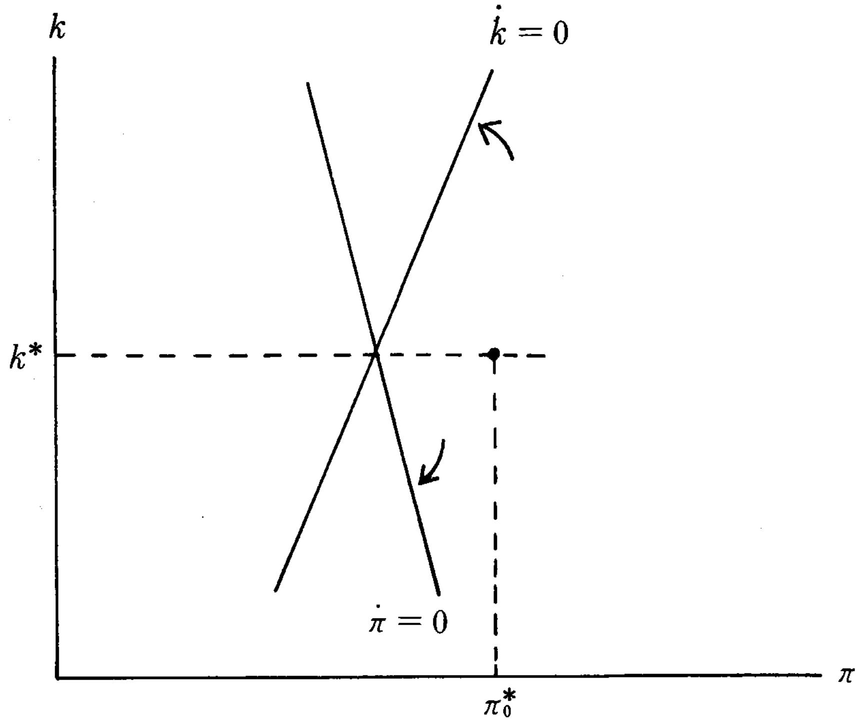

Fig. 6

under budget deficits, this result indicates that the difficulty of controlling inflation will be less in a country with such an export mechanism as shown above than otherwise.

Summary

We saw that the SW weak result can arise only when the deficit is increasing in the model with the stability in the absence of perfect foresight. when the perfect foresight was introduced, the model lost the stability. Also the possibility of the weak result arising is ambiguous. The more rapidly the deficit is growing, the higher the possibility is. When D_N is decreasing or constant across the time (over $[0, T]$), the restrictive monetary policy succeeds in repressing inflation despite the existence of budget deficits. In this case, per capita real balances increase and real consumption stays the same, therefore the welfare will be improved as a result of the policy. Even if D_N is increasing, the welfare level can still be improved depending on how rapidly the deficit is growing.

When the net export is included, equation (70)becomes

$$c^* = y(k^*) - (u+n)k^* - ne\left(\frac{\pi^*}{\pi_f}\right) \tag{88}$$

where π^* is lower than in the absence of foreign trade. c^* will be also lower while the real balances will be still higher. Therefore, the welfare comparison with the case that does not have the foreign trade turns out to be ambiguous.

Footnote

(1) The assumption of bonds being indexed is justified by perfect foresight arguments.

(2) In equation (17), g is exogenous, so if c is constant, y and Q_1 are also constant. However, c is constant only along the ateady state path where the relation $\Delta\mu = \Delta\pi$ holds. There seems to be a conceptual confusion here.

(3) This derivation can also be questioned because $\dot{c} = 0$ is being assumed while m and b are diverging from the steady state.

(4) When $\dot{c} = 0$, the relation $\dot{m} + \dot{b} = 0$ must hold with both of $\dot{m} = 0$ and $\dot{b} = 0$ holding.So, the substitution between m and b during $[O, T]$ is not shown.

(5) These path are not actually the path of the system. The motion of the system was supposed to be described by the equations (34) and (35). Besides μ must increase in order for the system to jump down from E_0, however, μ was supposed to be reduced to fight inflation in the beginning the policy experiment. This point can be indicated in the case of the slope of SS being -1 below.

In general, he did not show whether the combination of m and b at t_1 is consistent with the amount of deficits to be financed by them. The time path of deficits was assumed to be given after SW [9]. Thus, even if he expects that there exists a certain value of μ which makes $m(t_1)$ and $b(t_1)$ the steady state values, it must have been shown if they are the combination just financing the deficits. The stability of the model has an important meaning in this context, too.

(6) $\dot{m} + [n + \dot{P}/P]\, m = \theta_m$ and $\dot{b} + [n + \dot{P}/P] = \beta\, b$ where $\theta = \dot{M}/M$, $\beta = \dot{B}/B$, and B is total nominal bonds outstanding. So, $\theta m = \dot{M}/M$ and

$\beta\, b = \dot{B}/PL$ hold. At any given time t, the substitution between money creation and bonds issuance is expressed by the relation $-\dot{\theta}m = \dot{\beta}b$ that keeps the level of government deficits constant (The real rate of interest on bonds is assumed to be constant).

(7) The serial number of figures including the ones in the previous chapter are the same as in the original papers.

(8) By the exogenous change in B and, therefore, m, the intercepts and slopes of $\dot{k} = 0$ and $\dot{\pi} = 0$ lines will change. However, the long-run constancy of k implies that these lines will cross at the same level of k while maintaining the stability of the equilibrium through the sign conditions (72) and (73) (so that the steady state relations (70) and (71) are maintained).

(9) Throughout this policy experiment, we do not need to fix β until b hits the ceiling. However, after the ceiling was attained, the results will be as was shown above. So, we treated β as if it is fixed over $[O,\ T]$ for simplicity.

(10) The inflation could be controlled during the first half of the 1980s in the U.S. while the per capita government debt holding grew rapidly. Our analysis is consistent with this fact.

(11) Table- 3 shows that the per capita bond holding in Japanese economy grew faster than in the U.S. since 1970s while the inflation rate in Japan after 1978 is consistently lower than in the U.S. (table- 2). Japanese growth rates in table- 1 are vividly lower than in the previous decade.

(12) With this change, the asset accumulation equation (46)becomes

$$\dot{a} = (1-\tau)[y(k)+v] + (1-\tau)rb - (n+\pi)m - (u+n)k - (n+\pi)b - c - ne\left(\frac{\pi}{\pi_f}\right).$$

However, in linear Taylor approximation, π is fixed on the microlevel, so that the stability of the system of equations (43) and (46) is maintained.

Table 1
ecconomic growth rate (real) %

	Japan	U. S.
1975	3.6	−1.2
76	5.1	5.4
77	5.3	5.5
78	5.1	5.0
79	5.3	2.8
80	4.6	−0.3
81	3.5	2.6
82	3.3	−1.9

Table 2
imflation rate (CPI) %

	Japan	U. S.
1975	11.8	9.1
76	9.3	5.8
77	8.0	6.5
78	3.8	7.6
79	3.6	11.3
80	8.0	13.5
81	4.9	10.4
82	2.7	6.1

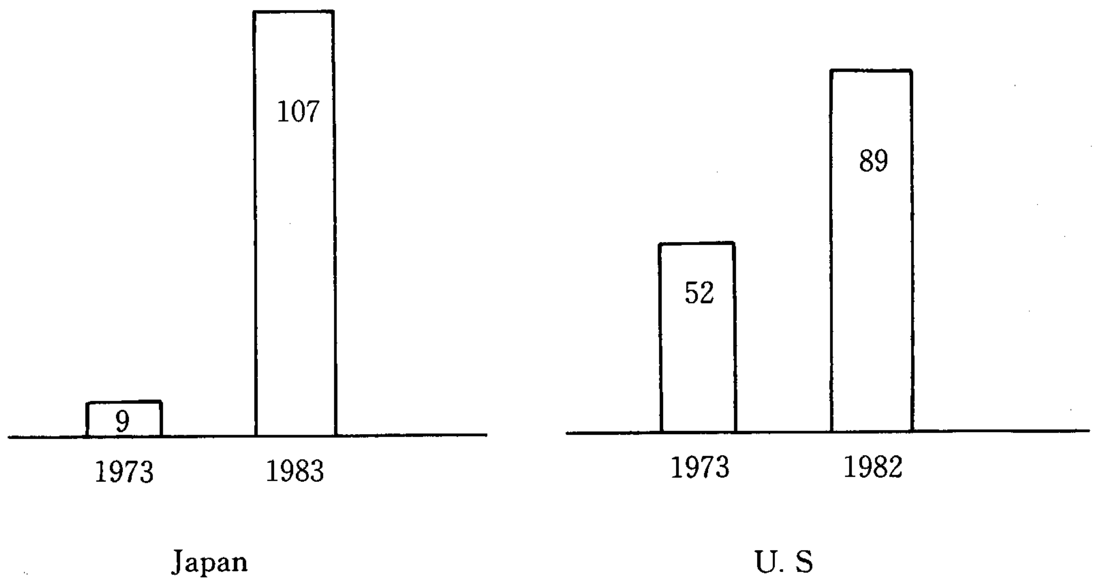

Table 3
Per capita long term goverment debt holding (10,000 yen).

Source : The data file 1984, PHP laboratory (in Japanese).

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