Distributive Justice and Bargaining Theory

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1. INTRODUCTION

The purpose of this paper is to consider the problem of fair allocation in economic environments based on *asymmetry*. A solution associates with each economic environment a set of feasible allocations, interpreted as desirable for the economy. While the concept of economic efficiency is well understood, there is still considerable debate concerning the appropriate definition of economic equity. It is since Foley [1967] that an ordinal concept of equity,the concept of an envy-free allocation, has been available for economic analysis. It selects the set of allocations at each of which no agent prefers the bundle of any other agent to his own. The no-envy notion is quite appealing, but the set of envy-free allocations may be quite large. The object is to formulate properties of solutions and to look for small subsolutions that satisfy these properties together.

Varian [1974] shows that at any efficient allocation there is someone that no one envies and someone that envies no one. Thus at any efficient allocation there is a natural way to say which agents are best off(the agents that envy no one) and which agents are worst-off(the agents that no one envies). This provides an appealing interpretation of Rawls's [1971] maximin principle. He wishes to maximize the bundle of *primary goods* going to the least well off. This is generally interpreted as requiring some sort of interpersonal comparisons of utility. However, if we consider the worst off agents to be the ones that no one envies, and we make them as well off as possible, we are led to the condition of an envy-free allocation.

The most prominent nineteenth-century theory of distributive justice was utilitarianism, the view that society should seek "the greatest good for the greatest number." A common position in social choice theory, at least until a decade ago, has been that a desirable allocation mechanism use information only on the possible outcomes. When the outcome is welfare, this position has been coined "welfarism" by Sen [1979] . Utilitarianism is an example of a welfarist allocation mechanism. Indeed, any allocation mechanism that can be represented as maximizing a function of the utilities of the agents is welfarist. Resource egalitarianism is one of several theories of justice that can be called "resourcist"as opposed to "welfarist." Resourcism is the position that economic structure(of resource availabilities and preferences)underlying the outcome matters. A resourcist wants to pay attention to information concerning the distribution of goods or resources to evaluate the justness of a state. Rawls [1971] proposes a theory of justice in which distribution is determined by the difference principle, which states that, first, one should look at *primary goods*, instead of utility, and second, that justice requires the adoption of that economic mechanism which maximizes the bundle of *primary goods* that the group that is worst off receives. Sen [1980] calls for distributing resources to equalize the basic capabilities of people, not their utilities. One should not, for example, equalize the amount of money(a primary good)that all receive, but rather the capabilities to function that money falilitates. In

1981, Ronald Dworkin, in two papers, offers a third proposal for what egalitarians should concern themselves with. For Dworkin, resources include not only objects in the world external to people, but also their internal resources, their talents and handicaps. Thus, a resource-egalitarian mechanism must take account of hidden, nontransferable resources that exist within the agents. It must distribute the endowment in a manner consistent with how it would distribute it if it were required explicitly to compensate agents in regard to the amounts of nontransferable resources that they possess.

The approach of studying the class of allocation rules on a large domain of possible worlds began in economics with axiomatic bargaining theory, as developed by Nash [1950].[1] Recently, Thomson and Lensberg [1989] have applied these techniques to the study of egalitarianism.[2] However, these bargaining theories are inadequate for the task of trying to salvage Dworkin's attempt, as they possess no language for representing resources, but only utility. These theories assume an axiom of welfarism,[3] namely, that the only information that is relevant about a possible world is the set of utility possibilities that it generates for its inhabitants.

Roemer [1986, 1988] attempts to model equality-of-resources by specifying axioms on resource allocation mechanisms. Many contemporary theories of distributive justice are resourcist, not welfarist, and require a language with which resources and preferences can be discussed, a language which bargaining theory lacks. His work is the

1) See Kalai and Smorodinsky [1975] and Kalai [1977, 1985] for the literature of bargaining theory.

2) See Thomson and Varian [1985] , Moulin [1987] and Alkan, Demange,and Gale [1991] for the literature of fairness theory.

3) See also Moulin [1988] for a unified and comprehensive study.

generalization of bargaining theory to get a thicker informational description of possible worlds. Its main mathematical idea is the CONRAD axiom, which says consistency of resource allocation across dimension.

However, his approach relies on interpersonal comparability of utility and treat agents identically. It is called Axiom Sy(Symmetry) and imposed on allocation mechanisms that if two agents have the same utility function, they are treated identically. In the following analysis, we consider the problem of distributive justice with the no-envy solution not to involve interpersonal comparisons of utility. And its concept of equity can treat agents as *asymmetry* and take account of hidden, nontransferable resources explicitly.

This article presents what constitutes a generalization of bargaining theory. The next section defines the model, lists the axioms required for the analysis and reformulate the no-envy solution. We obey the axioms except Axiom Sy in Roemer's analysis and look for a subsolution in which no-envy solutions satisfy these axioms. Section 3 proves the main theorem. Section 4 presents concluding comments of the article.

2. THE MODEL

In this section economic environments are defined and I show how resources can be allocated in a given economic environment. The world consists of some fixed bundle of resources, and of individuals who have ways of processing these resources into the utility. For the sake of simplicity in exposition, I assume that there are just two agents.

An *economic environment* is a vector

$\xi = \langle n\ ;\ \bar{x}\ ;\ \bar{u},\ \bar{v} \rangle$,

where $n \geqq 1$ is the dimension of the commodity space, $\bar{x} \in R^n_+$ is the aggregate resource endowment vector to be allocated between two agents with utility function$\bar{u}$ and $\bar{v}$, and $\bar{u}$ and $\bar{v}$ are arbitrary continuous, weakly monotone increasing, concave real-valued function of n variables, with $\bar{u}(0) = \bar{v}(0) = 0$. Let $U^{(n)}$ be the set of all these functions defined on R^n_+. Let the class of all admissible economic environments of dimension n be $\Sigma^{(n)}$. Let $\Sigma \equiv \cup_n \Sigma^{(n)}$.

The *utility possibility set* for ξ, denoted $A(\xi)$, is the set of attainable utility pairs achievable by various distributions of $\bar{x}$, that is,

$$A(\xi) = \{(\bar{u}, \bar{v}) \in R^2_+ \mid \exists \bar{x}^1, \bar{x}^2 \in R^n_+,\ \bar{x}^1 + \bar{x}^2 \leqq \bar{x},\ \bar{u} = \bar{u}(\bar{x}^1),\quad \bar{v} = \bar{v}(\bar{x}^2)\},$$

where $\bar{x}^i$ is the share of resource vector $\bar{x}$ to person i. Here, $A(\xi)$ is a closed, comprehensive, convex set in R^2_+ containing the origin. Convexity of $A(\xi)$ follows from the concavity of $\bar{u}$ and $\bar{v}$.

The analysis here is limited to economic environments without production. This is an assumption made for simplicity's sake, but much of the theory can be generalized to economic environments with convex production sets.

An *allocation mechanism* F is a function, defined onΣ, which assigns to each economic environment ξ an allocation of the resources in that environment, that is,

$$F(\xi) = (\bar{x}^1, \bar{x}^2) = (F^1(\xi), F^2(\xi)),$$

where $F^i(\xi) = \bar{x}^i$ is the share of resource vector $\bar{x}$ assigned by F to person i. Specifying an allocation mechanism amounts to naming a rule for distributing resources in any environment.

F induces a function in utility space and the induced utility mapping is as follows;

$$\mu_F(\xi) = (\bar{u}(F^1(\xi)), \bar{v}(F^2(\xi))).$$

I propose that any allocation mechanism F should satisfy the following axioms.

A 1: *Unrestricted domain.* The allocation mechanism F is defined on all economic environments $\xi = < n ; \bar{x} ; \bar{u}, \bar{v} >$.

A 2: *Weak Pareto Optimality.* $F(\xi)$ is a weakly Pareto optimal allocation for ξ, that is, $\nexists (\bar{u}, \bar{v}) \in A(\xi) ; (\bar{u}, \bar{v}) > \mu_F(\xi)$.

A 3: *Resource Monotonicity.* Let $\xi = < n ; \bar{x} ; \bar{u}, \bar{v} >$ and $\xi' = < n ; \bar{x}' ; \bar{u}, \bar{v} >$ be two economic environments and $\bar{x} \geqq \bar{x}'$. Then

$$\mu_F(\xi) \geqq \mu_F(\xi').$$

A 4: *Continuity.* For any dimension n, $\mu_F(\xi)$ is continuous in its arguments. The topology for the functional arguments is pointwise convergence.

A 5: *Consistency of Resource Allocation across Dimension.* Let $\Xi = < n + 2m ; (\bar{x}, y^1, y^2) ; u(x, y^1), v(x, y^2) >$ be an economic environment where there are n dimensions of the x goods, m dimensions of the y goods, and $u, v \in U^{(n+m)}$. Suppose that each y good is useful to, or liked by, at most one of the agents. Let $F(\Xi) = ((\bar{x}^1, y^1), (\bar{x}^2, y^2))$. Define u, v by

$$\bar{u}(\bar{x}^1) \equiv u(\bar{x}^1, y^1) \qquad \bar{v}(\bar{x}^2) \equiv v(\bar{x}^2, y^2).$$

If $\bar{u}, \bar{v} \in U^{(n)}$, consider the restricted environment, $\xi = < n ; \bar{x} ; \bar{u}, \bar{v} >$. If $A(\Xi) = A(\xi)$, then $F(\xi) = (\bar{x}^1, \bar{x}^2)$.

Axiom A1 says that the mechanism can be seen as a constitution which prescribes an allocation for any economic environment. Axiom A2 is clearly desirable and requires no motivation. Axiom A3 says that if two environments differ only in that the resource endowment in the first one dominates the endowment in the second one, then no agent should end up worse off in the first environment under$F$'s action. Axiom A4 is necessary for the "weak" part of Axiom A2. Axiom A5 is perhaps most controversial. Suppose that a mechanism F acts on a dimensionally large environment Ξ. Each y good is useful to, or liked by, only one agent. By A2 and A4, each agent gets all of the goods that only he or she uses or likes. Now define the restricted utility functions $\bar{u}(x)$ and $\bar{v}(x)$ which are derived from $u(x, y)$ and $v(x, y)$ by fixing the allocation of the y goods in the way that has just been determined. Then, how should the mechanism distribute $\bar{x}$ in the dimesionally smaller environmentξ? Axiom A5 says that $\bar{x}$ should be distributed the same way in Ξ as $\bar{x}$ is in ξ.[4)]

Actually, the y goods are hidden,nontransferable resources that exist within the agents.I suppose that $y^1 > y^2$. And I must write more properly

$$\bar{u}(x^1) = u(\bar{x}^1, y^1, y^2) = u(\bar{x}^1, \bar{y}^1)$$
$$\bar{v}(\bar{x}^2) = v(\bar{x}^2, y^1, y^2) = v(\bar{x}^2, y^2), \qquad y^1 > y^2.$$

This article presents solutions satisfying the fundamental notion of equity, that is no-envy, more explicitly.

Definition. (*No-Envy*) The no-envy solutions are the pairs($\bar{x}^1$, $\bar{x}^2$)that

4) For elaboration of this point, see Roemer [1986] .

satisfy $u(\bar{x}^1, y^1) \geqq u(\bar{x}^2, y^2)$ and $v(\bar{x}^2, y^2) \geqq v(\bar{x}^1, y^1)$.

3. NO-ENVY and BARGAINING THEORY GENERALIZED

A. Monotone Utility Path Solutions

Definition. F is a *monotone utility path* mechanism on a class of economic environments if and only if μ_F is the intersection in utility space of the Pareto frontier of environments with some monotone path emanating from the origin. Call the class of such mechanisms *MUP*.

A monotone path is a continuous curve from the origin(threat point), in utility space,which is strictly increasing except that it may contain a horizontal or vertical segment containing the origin. I prove the allocation mechanisms satisfying the five axioms are *MUP*.

Lemma 1. There exist *MUP* solutions if and only if the mechanisms satisfy the axioms A1～A5.

Proof. We can pick any two economic environments $\Xi = <n+2m\,;\,(\bar{x}, y^1, y^2)\,;\,u, v>$ *and* $\Xi' = <n+2m\,;\,(\bar{x}', y^1, y^2)\,;\,u, v>$ by A1. Their mechanisms are $F(\Xi)$ and $F(\Xi')$. By A5, the restricted environments $\xi = <n\,;\,\bar{x}\,;\,\bar{u}.\bar{v}>$ and $\xi' = <n\,;\,\bar{x}'\,;\,\bar{u}, \bar{v}>$ have the allocation mechanisms $F(\xi)$ and $F(\xi')$ repectively. They satisfy A2. Define

$$F_H = \sup\{F(\xi)\ ,\ F(\xi')\}.$$

To fix ideas take $F_H = F(\xi)$. By A3, we have $\mu_F(\xi) > \mu_F(\xi')$. Then we suppose $F_H = F(\xi)$ for all environments in $\Sigma^{(n)}$. Define $\Gamma^{(n)}$ as the subclass of environments whose Pareto frontiers contain only strongly

Pareto optimal points. And $\Gamma \equiv \cup_n \Gamma^{(n)}$. A4 enables us to prove the lemma immediately. Consider environments $\xi' \in \Sigma \setminus \Gamma$. We can choose a sequence of environments $\{\xi'_{(i)}\}_{i=1,\infty}$, in $\Sigma \setminus \Gamma$ that converge to ξ in Γ in the stated topology, as follows. Define $\xi'_{(i)} = < n ; \bar{x} - \varepsilon_{(i)} ; \bar{u}(\bar{x} - \varepsilon_{(i)}), \bar{v}(\bar{x} - \varepsilon_{(i)}) >$, where $\{\varepsilon_{(i)}\}$ is a sequence of positive numbers converging to zero. For any sequence $j < k$, by A3 and A4 we have $\mu_F(\xi'_{(j)}) < \mu_F(\xi'_{(k)})$. So these sequences are bounded from below by the origin and from above by $\mu_F(\xi)$. Thus there exist *MUP* solutions.

The other part is straightforward.

B. Transfer from Equal Division

We suppose that the mechanism assigns the allocation associated with an initial endowment in which each agent receives $\bar{x}/2$ in an environment Ξ. Since it is Pareto optimal, it must allocate all of y^1 to the agent with u and all of y^2 to the agent with v. By assigning each person equal division of $\bar{x}$, the agent with y^2 prefers the bundle of the agent with y^1 to his own. So we need to transfer some of $\bar{x}/2$ from the agent with y^1 to the agent with y^2. We denote such a transfer by $t(\Xi)$. Thus, the agent with y^1 would end up with $\bar{x}/2 - t(\Xi)$, and the agent with y^2 would end up with $\bar{x}/2 + t(\Xi)$.

We define the acceptable transfer level $t^*(\Xi)$ as the lowest level of them such that assigning the allocation associated with it yields a no-envy condition and suppose each agent accepts this rule. That is,

$$t^*(\Xi) \equiv \inf\{t > 0 \mid \exists((\bar{x}/2 - t, y^1), (\bar{x}/2 + t, y^2)) \in F(\Xi) : u(\bar{x}/2 - t, y^1) \geqq u(\bar{x}/2 + t, y^2) \text{ and } v(\bar{x}/2 + t, y^2) \geqq v(\bar{x}/2 - t, y^1)\}.$$

Assumption. There exists a unique transfer level $t^2(\Xi)$ in $(0, \bar{x}/2)$ such that $v(\bar{x}/2 + t^2(\Xi), y^2) = v(\bar{x}/2 - t^2(\Xi), y^1)$.

Lemma 2. The allocation mechanism satisfies a no-envy solution if and only if the transfer from equal division is $t^*(\Xi) = t^2(\Xi)$.

Proof. Suppose that we can find the allocation sequence as follows, under the sequence $\{t_{(i)}\}$ that goes more and more as i does. That is,

$(\bar{x}/2 - t_{(1)}, \bar{x}/2 + t_{(1)}), (\bar{x}/2 - t_{(2)}, \bar{x}/2 + t_{(2)}), \cdots\cdots.$

Thus, we have

$u(\bar{x}/2 - t_{(1)}, y^1) > u(\bar{x}/2 - t_{(2)}, y^1) > \cdots\cdots,$

and

$v(\bar{x}/2 + t_{(1)}, y^2) < v(\bar{x}/2 + t_{(2)}, y^2) < \cdots\cdots.$

Then, if $u(\bar{x}/2 - t_{(i)}, y^1) = u(\bar{x}/2 + t_{(i)}, y^2)$ holds in the sequence before $v(\bar{x}/2 + t_{(i)}, y^2) = v(\bar{x}/2 - t_{(i)}, y^1)$, the agent with y^1 doesn't transfer more than that and a no-envy solution doesn't exist. So that $t^*(\Xi) = t^2(\Xi)$ holds is necessary for no-envy solution. Furthermore, we define $t^2(\Xi) = t^2_{(l)}(\Xi)$. If the sequence goes to infinity as i does, the agent with y^1 wouldn't accept the transfer sequence more than $l+1$. Hence, the sequence must be bounded after all. We have $\lim_{i \to l} t_{(i)}(\Xi) = t^2(\Xi) = t^*(\Xi)$. We prove now that if and only if that, the allocation mechanism satisfies a no-envy solution.

C. The Theorem

In the previous assumption, $t^2(\Xi)$ also depends on the value $\bar{x}$, and increases as the value increases. Here, we denote it as $t^2(\Xi\,;\,\bar{x})$.

Finally I prove the following theorem.

Theorem. The allocation mechanism F satisfying the axioms A1 ~A5 chooses a unique allocation associated with the no-envy solution in economic environments. It is one of strongly Pareto optimal points.

Proof of Theorem. Lemma 2 shows that each economic environment $\Xi' = \langle n+2m\,;\,(\bar{x}', y^1, y^2)\,;\,u, v\rangle$ has a no-envy solution, $t^2(\Xi'\,;\,\bar{x}')$ respectively. By A5, in each restricted economic environment $\xi' = \langle n\,;\,\bar{x}'\,;\,\bar{u}.\ \bar{v}\rangle$ the allocation $(\bar{x}'/2 - t^2(\Xi'\,;\,\bar{x}'),\ \bar{x}'/2 + t^2(\Xi'\,;\,\bar{x}'))$ exists, and by A2 it it Pareto optimal. As the mechanism satisfy the five axioms, from Lemma 1, we have *MUP*. So $\xi'_{(i)}$ converges to ξ in Γ.In general, $F(\xi) = (\bar{x}/2 - t^2,\ \bar{x}/2 + t^2)$ chooses the no-envy solution, and it is strongly Pareto optimal. This proves Theorem.

4. CONCLUDING COMMENTS

Equality is a popular but mysterious ideal. It does not call for a definition of the word "equity" or an analysis of how that word is used in ordinary language.

Taking as a primitive notion of equity that of an envy-free allocation, my objective is to investigate the set of possible allocation mechanisms on a large domain of possible worlds. This article implemented this methodological approach, and characterized a resource allocation mechanism with a set of five axioms.

As Dworkin proposes a kind of comprehensive resource egalitarianism, we have to include internal talents that people possess into resources. We have to decide what degree of compensation with transferable resources is due a person with a low endowment of nontransferable ones.

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