Collisional and detuning effects on wave enhancement caused by a dc electric field

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The effect of a dc electric field on the spatial evolution of an external wave in a small cold beam–plasma system is investigated numerically. The wave enhancement is strongly dependent on the sign of detuning $\delta\omega(=\omega-\omega_0)$ (ω is the wave frequency and ω_0 is the frequency of the most unstable mode). Negative detuning, i.e., negative $\delta\omega$, leads to an anomalous increase in the wave power as compared with the wave enhancement for zero or positive detuning. Weak collisions also can play an important role in limiting the wave enhancement.

I. INTRODUCTION

It was shown both theoretically[1] and experimentally[2] that an electrostatic two-stream instability in a low-density fast electron beam–plasma system leads to the growth of a nearly monochromatic electron wave (single mode) with an amplitude that saturates by trapping the beam electrons within its potential troughs. Recently, it has been predicted that the application of an external dc electric field to particles trapped in the potential well of a wave can result in an increase in wave amplitude.[3] This effect has been observed experimentally in a traveling wave tube (TWT).[4]

On the practical side, saturation by beam trapping limits the maximum energy that can be extracted in a TWT. Therefore, control of the trapped beam offers the possibility of enhancing the efficiency of energy extraction out of beam devices. Moreover, the energy exchange process in a free-electron laser (FEL) is quite similar to that of a TWT; although in the case of the FEL the beam is relativistic, and the potential well is magnetic instead of electrostatic. In the FEL the application of a dc electric field to trapped beam electrons has been proposed as a possible wave power enhancement scheme by Lin.[5]

As the equations that describe the wave–particle interaction in a TWT are identical to those that describe the beam–plasma instability in the small cold beam limit, the modification of the dynamics of beam trapping caused by a dc field is of interest to basic plasma physics studies. Our purpose is to study, by means of a single wave model,[1] the collisional and detuning effects on wave enhancement caused by a dc electric field. Previously, we showed that collisional effects[6] play an important role in the nonlinear wave–particle interaction even if collisions are too weak to change the linear stage. In addition, in the case of the interaction of a premodulated beam with a plasma, waves can be excited with detuning.[7]

II. BASIC EQUATIONS

In the present simulation, we consider the spatial evolution of a single wave of frequency ω. According to our previous model,[6,8] the following system of equations is obtained in standard notation:

$$\left(A+iB+i\frac{d}{d\eta}+\frac{s}{2}\frac{d^2}{d\eta^2}\right)E(\eta)=\frac{i}{N}\sum_{j=1}^{N}\exp(i\xi_j), \tag{1}$$

$$\frac{d^2\xi_j}{d\eta^2}=\left(1+s\frac{d\xi_j}{d\eta}\right)^3[E(\eta)\exp(-i\xi_j)+\text{c.c.}]-\left(1+s\frac{d\xi_j}{d\eta}\right)^3E_{\text{dc}}, \tag{2}$$

where

$$A=s^2\frac{\omega_0^2}{\omega_b^2}\left[2\frac{\delta\omega}{\omega_0}+\left(\frac{\delta\omega}{\omega_0}\right)^2\right],$$

$$B=s^2\frac{\nu}{\omega_0}\frac{\omega_0^2}{\omega_b^2}\left(1+\frac{\delta\omega}{\omega_0}\right)^{-1},\quad s=\left(\frac{1}{6}\frac{n_b}{n_p}\frac{v_b^2}{v_t^2}\right)^{1/3},$$

and $\omega_0^2=\omega_p^2(1+3v_t^2/v_b^2)$. In these equations, $\delta\omega$ denotes the frequency difference between the given frequency ω and the frequency of the most unstable mode ω_0, i.e., detuning $\delta\omega=\omega-\omega_0$. The coefficient A is derived from a detuning effect. The coefficient B is derived from a collisional effect because ν is the effective collision frequency between plasma electrons.[6] The quantity s is the spatial scaling factor. The normalized electric field of the wave $E(\eta)$ and the spatial coordinate η are defined by $E(\eta)=eE(x)\exp(-ix\omega/v_b)/(mv_b\omega s^2)$ and $\eta=sx(\omega/v_b)$, respectively. Here E_{dc} is also the normalized dc field. The phase-space coordinate of the jth beam ξ_j is defined as $\xi_j=\omega[t_j(x)-x/v_b]$. The function $t_j(x)$ is the time when the jth beam passes the point x. The velocity $v_j=\dot{x}_j$ in the laboratory frame is obtained from $\dot{\xi}_j=d\xi_j/d\eta$ by using the relation $\dot{x}_j=v_b/(1+s\dot{\xi}_j)$.

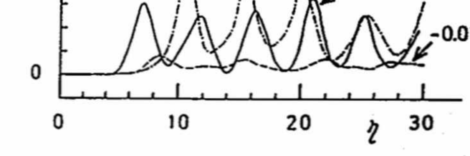

FIG. 1. Effect of detuning on spatial evolution of wave energy in the absence of E_{dc}. Plasma parameters are $n_b/n_p=5\times10^{-4}$, $v_b/v_t=9.9$, $\nu/\omega_0=0$, and $s=0.2$.

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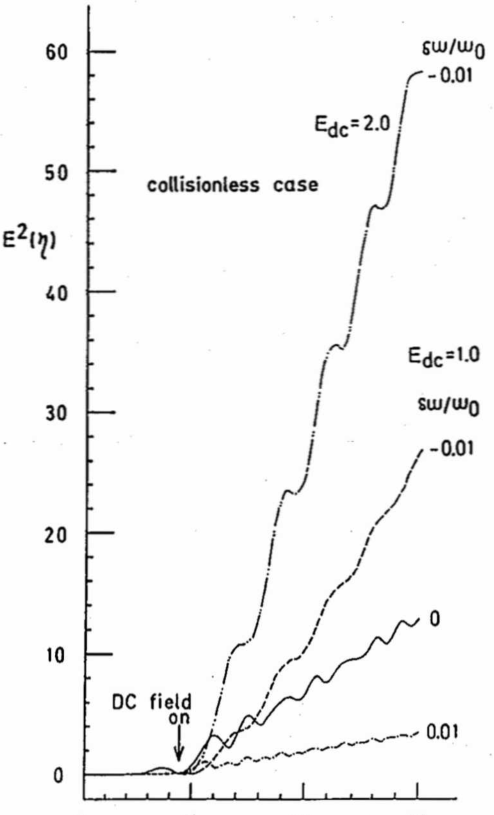

FIG. 2. Effect of detuning on spatial evolution of wave energy in the presence of E_{dc}. Plasma parameters are the same as those in Fig. 1, and E_{dc} is turned on at $\eta = 9.0$.

III. NUMERICAL RESULTS AND DISCUSSION

Figure 1 shows typical examples of the wave evolution with detuning in the absence of E_{dc}. The saturation level of the wave amplitude depends on the sign of $\delta\omega$.[7,8] Negative detuning, i.e., negative $\delta\omega$, leads to a decrease in the saturation amplitude comparable to the one without detuning. On the other hand, positive detuning leads to an increase in the saturation amplitude.

These results are well explained qualitatively by the characteristics of the linear dispersion relation. Generally, ω_0 is determined from the point where the branch of the electron plasma wave and that of the beam space charge wave intersect. Therefore, with the increase in wave amplitude, ω_0 is varied from the initial point to the higher side because of the slowing down of the injected beams, and the excitation condition of the wave with positive $\delta\omega$ becomes more resonant to the slowed beam–plasma system.[8]

When trapped electrons are accelerated by E_{dc}, however, the dependence of amplitude saturation of the wave on the sign of $\delta\omega$ may become quite different from the results in Fig. 1. Figure 2 exhibits the spatial evolution of the wave for the case $E_{dc} = 1.0$. In addition, the wave evolution with negative $\delta\omega$ is also plotted for $E_{dc} = 2.0$. Although the wave in every case is observed to grow secularly, the E_{dc} effect on wave enhancement depends strongly on the sign of $\delta\omega$. The wave with negative $\delta\omega$ grows anomalously to large amplitude.

Figure 3 shows the phase-space plots at $\eta = 26$, corresponding to the wave evolutions in Fig. 2. When $\delta\omega$ is zero or positive, the beam electrons split into two groups [see Figs. 3(b) and 3(c)]: The one with high velocity has runaway electrons and the other, with low velocity, has trapped electrons. The average velocity of runaway electrons increases secularly. The ratio of runaway electrons to total electrons is 44.5% in Fig. 3(b) and 80.5% in Fig. 3(c), respectively. As the wave–particle resonance is detuned by increasing runaway electrons, wave enhancement for positive $\delta\omega$ is not so remarkable (see Fig. 2). In the case of negative $\delta\omega$, however, runaway electrons are not produced, and the wave–particle resonance is continued; no runaway electrons are produced even if E_{dc} is 2.0.

The simple picture of the E_{dc} effect is as follows.[3,4] When the particles are trapped in the well of the wave, the response of the particles to E_{dc} cannot be a uniform acceleration because they are constrained to move at the phase velocity of the wave. Namely, the particle momentum cannot change in response to E_{dc}. Hence, the wave power can be increased. When E_{dc} is strong enough to detrap the particles, however, the particles accelerate and the wave enhancement

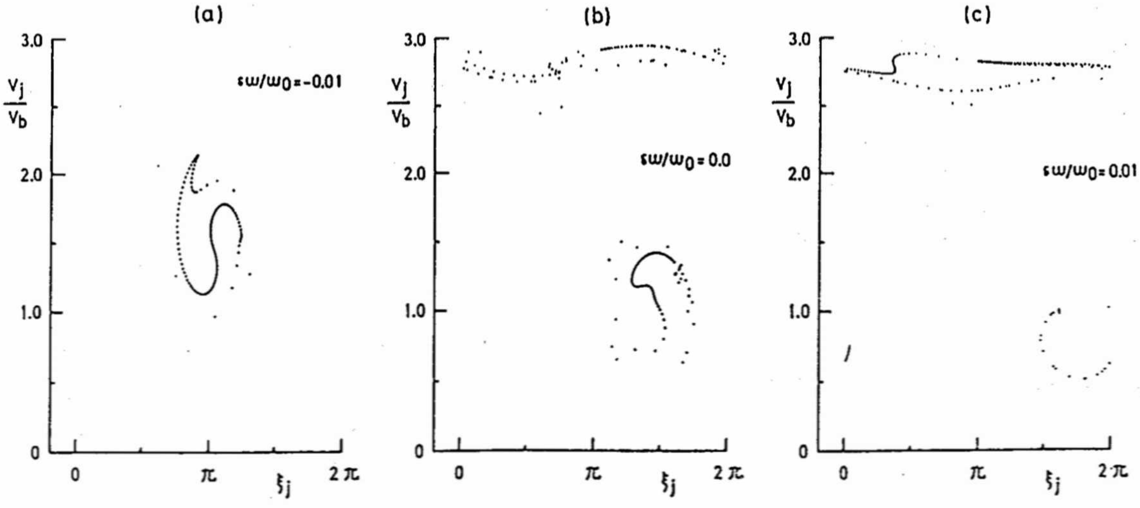

FIG. 3. Phase-space dynamics of beam electrons at $\eta = 26.0$. They are corresponding to the wave evolutions in Fig. 2, where $E_{dc} = 1.0$. Each point gives the velocity $\dot{x}/v_b$ and the coordinate ξ for one of the beam electrons. Equations (1) and (2) are solved for $N = 200$ beam electrons, which satisfies the initial conditions $\xi_j(0) = 2\pi(j/N)$ and $\dot{\xi}_j(0) = 0$. In this figure, $\xi_j(\eta)$ is transformed in the interval $0 \leqslant \xi_j(\eta) \leqslant 2\pi$.

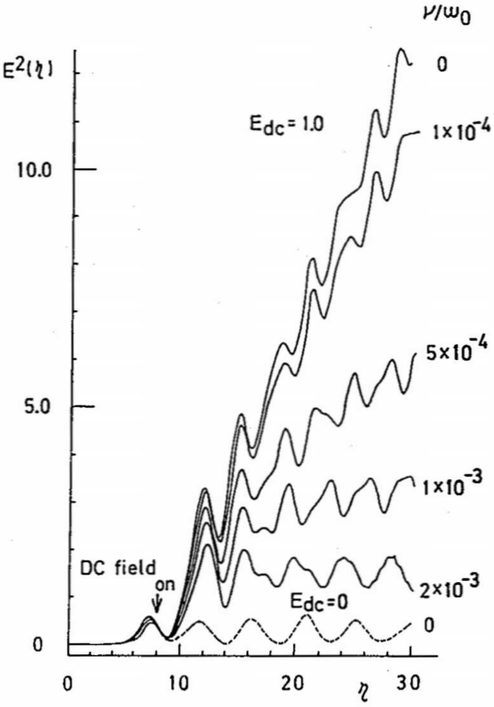

FIG. 4. Spatial evolution of wave energy in the collisional case, where E_{dc} is 1.0 and is turned on at $\eta = 8.0$. Plasma parameters are the same as those in Fig. 1.

does not occur. Qualitatively, the E_{dc} effect is classified into two cases, i.e., production of runaway beams and wave enhancement, based on a criterion $E_{dc} > E_s$ or vice versa. Here E_s is the saturation amplitude for $E_{dc} = 0$. Of course the system can be satisfied by having both states simultaneously, i.e., some of the beam electrons run away while others remain clamped, as shown in Fig. 3.

From a viewpoint of this picture, the system with detuning behaves quite anomalously. According to the results in Fig. 1, E_s for negative $\delta\omega$ is about 0.41 while E_s for zero $\delta\omega$ is 0.78. Some of the beam electrons run away for zero $\delta\omega$ when $E_{dc}/E_s = 1.28$, i.e., $E_{dc} = 1.0$. However, no beam electrons run away for negative $\delta\omega$ even if $E_{dc}/E_s = 4.9$, i.e., $E_{dc} = 2.0$.

Figure 4 illustrates the evolution of the wave in the presence of weak collisions. For reference, the evolution of the wave without E_{dc} in the collisionless case is also plotted. With increasing collisional effect, wave enhancement is strongly limited to a lower level. Furthermore, according to the phase-space analysis, the runaway electron ratio becomes high with increasing ν/ω_0, i.e., 37% for the collisionless case and 49% for $\nu/\omega_0 = 2\times10^{-3}$.

In summary, we have confirmed detuning and collisional effects on wave enhancement caused by E_{dc} in a small cold beam–plasma system. As detuning can play an important role in causing the beam to be detrapped, the wave enhancement has been found to be strongly dependent on the sign of $\delta\omega$. In particular, appreciable enhancement has been obtained for negative $\delta\omega$. Weak collisions can also play an important role in limiting the wave enhancement.

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