Intergenerational Mobility in Poverty State of the Chronically Poor in Rural Bangladesh: A Markov Chain Model Approach

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Abstract:

Based on a household survey in Bangladesh, the chronically poor group was identified. Intergenerational mobility among three poverty states is elucidated utilizing the Markov Chain Model.

Transition probability matrices for three generations reached stationary state only after 28th generation. Markov dependence was tested significant. The expected stay and mobility measures have been estimated. They show that the best state has the shortest staying period, implying that households who are in the most favorable state within the chronically poor in Bangladesh were most mobile and fell in deteriorating situation within a short period of time by being unable to stay in the same state.

Keywords:
Poverty state, intergenerational mobility, chronically poor, Markov-chain model, rural area, Bangladesh

I . Introduction

There are copious amount of concepts, definitions and terms that are used to portray poor and poverty in the literature. Different authors and researchers have characterized poor and poverty in diverse ways such as "absolute poor", "extreme poor", "hardcore poor", "ultra poor","primary poor","transitory poor" etc. Recently many authors use the terms "descending non-poor", "ascending poor" and "chronically poor". Each term is used to delineate separate characteristics of poverty of a household. The "headcount ratio", "poverty gap ratio" and "squared poverty gap ratio" are widely used as an aggregate static measure of poverty in any given area. But poverty is a very fluid condition since it has been observed that many households move in and out of poverty in the course of time. To measure such movement or transition we need panel data but it is quite arduous job to obtain such data. Until recent years, poverty had been measured by headcount ratio. But it suffers from much deficiency in sketching poverty since it does not adequately tell us about dynamics of poverty.

Like poverty, cognition of the concept of chronic poverty and its dynamics is very tricky and complex since it involves sets of underlying factors. The major concern of this research is to focus on poverty dynamics of chronically

poor[1] households - the changes in well-being and ill-being that households have been experiencing over one, or more than one generation. This work is, therefore, to a large extent, aimed on intergenerational mobility of chronically poor households. In addition to the increased allocation in successive development plans of Bangladesh for poverty alleviation, the government has implemented several target-group oriented programs and projects for the poor. Unfortunately, these efforts have failed to dent the poverty situation of chronically poor and the benefits of these efforts have bypassed them. The survey results indicate that more than 65 percent of chronically poor households inherited poverty from their parents and the rest experienced poverty since decomposition of their households from parent's households.

In order to measure mobility of poverty state of chronically poor households between two generations, transition probability matrix has been estimated. The present analysis is, therefore, confined to the changes of poverty state of chronically poor households.

Ⅱ. Source and Nature of Data

The current analysis is based on 510 chronically poor households spread over 32 villages in rural Bangladesh. A three-stage stratified random sampling design was followed for selecting final sampling units (chronically poor households). At the first stage 8 least developed districts were selected (2 districts from each old administrative division of Bangladesh). In order to select least developed districts, a composite index was computed on the basis of three simple indicators such as percentage of agriculture labor households, percentage of landless households and cropping intensity to capture at least of part of reality of development of 64 districts of the country. From each selected least developed district, 4 villages were selected at random with probability proportional to size (PPS) approach. In selected village complete list of households was prepared with certain indicators such as income, household size, landholding, and poverty status. These indicators along with opinion of household heads were used to classify households into four economic categories such as (i) non-poor, (ii) descending non-poor, (iii) ascending poor and (iv) chronically poor. From the list of chronically poor households, 16 households were selected at random from each selected village and the complete list of household in a village by category was treated as sampling frame. Thus a total of 510 chronically poor households were selected for the present analysis. During the field survey the chronically poor were asked to state the poverty status of their grandfather and father and him/herself. Their assessment regarding poverty status was verified with oldest member of the household and fathers of the respondents if they were alive during the survey. Thus,

1) Household's heads whose mean income or expenditure is always below the poverty line and sometime they inherit poverty from their parents are treated as chronically poor.

probing was an important technique in an attempt to improve the quality and reliability of data on poverty status, food security and history of the households.

Ⅲ. Methods of Analysis

The methodology applied in this study is designed to elucidate intergenerational mobility of poverty state from grand father to father and father to sons by Markov chain approach. The mobility of a household from one state of poverty to another state is somewhat erratic, fluctuating, multidirectional, and unpremeditated by nature. The future poverty status of a household in terms of poverty state cannot, therefore be prophesied with certainty but it has to be done only in a probabilistic framework. Due to this conundrum and because of scarce scope of getting reliable information on poverty status of three or more generations ago, we have in particular, used the Markov chain model, which assumes that current outcome depends only on the previous state and not on those of the further past. In order to test the null hypotheses that the order of a Markov chain, is of order zero such that

$$H_0 : P_{ij}^0 = P_j \text{ ; i, j = 1, 2, ..., m} \cdots\cdots\cdots\cdots (1)$$

against the alternative that the chain is order 1, the test statistic developed by Anderson and Goodman (1957) under H_0 is used and is given by

$$-\log\Lambda = 2\sum_{i=1}^{m}\sum_{j=1}^{m} n_{ij}\log\frac{(N)n_{ij}}{(n_i)(n_j)}$$

which has an asymptotic χ^2 distribution with $(m-1)^2$ degrees of freedom.

Let P_{ij}^t be the one step transition probability of a time-dependent process X(t). Symbolically it may be written as

$$P_{ij}^t = p[X(t+1)=j / X(t)=i]$$

Suppose we wish to test the null hypothesis relating to stationarity of the transition probability matrix such that

$$H_0 : P_{ij}^t = P_{ij}(t=1,2,.....,T)$$

Under the null hypothesis H_0, $-2 \ln \Lambda$ has a χ^2 distribution with (T-1)[m(m-1)] degrees of freedom and

$$-2 \ln \Lambda = 2 [L(\hat{P}^t) - L(P)]$$

$$= 2\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{m} n_{ij}^t \ln\frac{n_{ij}^t}{n_i^{t-1}p_{ij}} \cdots\cdots\cdots\cdots (2)$$

For the present context, the Markov chain $\{X_n\}$ is defined in terms of poverty state of chronically poor households under the assumption that the probability of poverty state of son depends on the poverty state of father. In other words, intergenerational transformation of poverty state from father to son constitutes a Markov chain. Let us consider a Markov chain with state space S{S=1,2,3} representing State 1 with the households that could provide adequate food for 3 meals and bear educational and medical expenses. Also, State 2 with the households that could provide adequate food

for 3 meals only, and State 3 with the households that could provide neither adequate food nor bear education and medical expenses.

As a next step, the limiting behavior of transition probabilities has been examined, as suggested by Feller (1965) using Chapman-Kolmigorov equation. Then by recursive relation $\|P^{(n)}\|=P^{n-1}.\ P=P^n$

If n is large, P^n is then equivalent to

$$\lim_{n\to\alpha} P^n=V \quad \cdots\cdots(3)$$

where $V=(v_1,v_2,v_3)$ with $0<v_i<1$ and $\sum_{j=1}^{s-1} v_j=1$.

The probability vector $V=(v_1,\ v_2,\ v_3)$ satisfies the relation $VP=V$, which gives the desired distribution of the process. It can also been shown that as $n\to\alpha$, $p^{(n)}$ tends to a limit v_j independent of the initial state i. This is called the stationary or equilibrium distribution.

The mobility of a continuing household in each state of poverty can be measured by the mean duration of stay in a particular state of poverty by the following methods.

Let m_i denote the number of generations required up to and including for moving from i-th state to another state. Again, let $m_i=n$, if and only if first (n-1) generations result in immobility and at the nth generation yield first mobility. Then m_i follows a geometric distribution and the mean of this distribution, which measures the mean time of stay in a state i may be estimated by

$$\lambda_i=E(m_i)=\frac{1}{1-P_{ii}} \quad \cdots\cdots(4)$$

where P_{ii} is the probability that a household will remain in state i from one generation to the next.

If v_i is compared with similar measure for an ideal situation of the poverty state, we can have a measure of mobility of poverty state. Prais (1954), however, considered perfectly mobile situation as one, whose transition probability matrix can be attained by the limiting distribution of the Markov chain. Then the standardized mean for the i-th state of poverty is

$$\lambda_i^s=\frac{(1-v_i)}{(1-P_{ii})};\ i=1,2,\ldots\ldots \quad \cdots\cdots(5)$$

where v_i is obtained for the Markov chain.

An appealing interpretation of λ_i^s is that in a mobile state it will be small and in an immobile state, it will be large [Bartholomew (1982)].

Ⅳ. Intergenerational Mobility Matrices of Poverty State

In the present study the term 'state space' (S) with 3 levels (grand father, father, son) has been used to depict the poverty status of the chronically poor households. The households who can provide adequate food for 3 meals a day and bear educational and medical expenses is termed as state-1, household which can provide adequate food only but cannot bear educational and medical expenses is termed as state-2, and household which cannot provide

neither adequate food not bear educational and medicate expenses is termed as state-3. Thus, the state space (S=State-1, State-2 and State-3) comprehends state-1, state-2 and state-3 for poverty status of chronically poor households.

A simple cross-tabulation of sample households according to poverty state of grandfather and father of the respondents is shown in matrix form in Table 1, while father to son/daughter (respondent) is shown in Table 2. These tables show the transition and the direction of alteration of poverty state of household between two generations.

From the juxtaposition of marginal totals of row and column of Tables 1 and 2, it reveals that there is a distinct downward change in poverty state though a few households experienced upward mobility. It appears from Table 1 that about 48 percent of the households showed upward mobility during father's period (1 household could provide only adequate food during grandfather's period but during father's period it could provide food and bear educational and medicare expenses, 6 households moved from state-3 to state-1 and 238 households from state-3 to state-2). Although 245 households accomplished to move up during father's period, majority (238 households) of them moved from state-3 to state-2 indicating very marginal change. On the other hand, about 31 percent of the total households showed downward mobility and 21 percent could not change their position and remained in the same state of poverty for two subsequent generations. It is worth noting that, poverty status has deteriorated when the respondents formed separate household. It is observed that 55 father's households could

Table 1: Transition Count Matrix of Poverty State for Grandfather and Father of Chronically Poor Respondents

Level of Poverty State		Father's Poverty State			Marginal Total
		State-1	State-2	State-3	
Grand Father's Poverty State	State-1	48	38	60	146
	State-2	1	34	60	95
	State-3	6	238	25	269
Marginal Total		55	310	145	510

Table 2: Transition Count Matrix of Poverty State for Chronically Poor Respondents and Their Father

Level of Poverty State		Respondent's Poverty State			Marginal Total
		State-1	State-2	State-3	
Father's Poverty State	State-1	1	37	17	55
	State-2	2	290	40	332
	State-3	2	47	74	123
Marginal Total		5	374	131	510

provide adequate food and bear educational and medical expenses, but only 5 respondent's households could currently maintain that status (Table 2). More than 71 percent of the chronically poor households live in the same state of poverty and they remain immobile from father's period to present period. Only 10 percent of the households showed upward mobility (2 households moved away from state-2 to state-1, 2 households from state-3 to state-1 and 47 households from state-3 to state-2), while 18 percent of the households showed downward mobility. It can also be proclaimed from Table 1 and Table 2 that majority of the chronically poor households inherited poverty from their parents and failed to ameliorate their level of living due to lack of productive resources and human capital. They process less productive assets (land), less social capital, less education, less skill, less employment opportunity etc. Moreover, majority of the chronically poor households are female-headed and large number of them are widowed (47%), divorced or separated (10%). These households had fewer numbers of adult male members which resulted in sparse earning opportunity for their livelihoods 〔Rahman, Matsui & Ikemoto (2005)〕.

Ⅴ. Transition Probabilities and Markov Chain Matrices

The first generational gradual changes expressed in terms of the conditional probabilities that the father will be in state-1, state-2 or state-3 given that grandfather was in state-1, state-2 or state-3 are evident in Table 3.

The transition between poverty states of the successive generations in a household may be regarded as transition of a Markov chain with the above transition probabilities. The transition probability matrix obtained from Table 3 may be denoted by $P=[P_{ij}]$, where P is a square Markov matrix with non-negative elements and

$$P=\begin{bmatrix} 0.32876712 & 0.2602740 & 0.4109589 \\ 0.01052632 & 0.3578947 & 0.6315789 \\ 0.02230483 & 0.8847584 & 0.0929368 \end{bmatrix}$$

The diagonal elements of P indicate the probability that a household will remain in the same state of poverty from one generation to the next. For instance, given that grandfather was in state-1, after one generation the probability that his son (father of respondent) will be in state-1 is 0.32876712.

Table 3: Conditional Probabilities between Grandfather and Father for 3 Poverty States

Level of Poverty State		Father's Poverty State		
		State-1	State-2	State-3
Grandfather's Poverty State	State-1	0.32876712	0.2602740	0.4109589
	State-2	0.01052632	0.3578947	0.6315789
	State-3	0.02230483	0.8847584	0.0929368

From matrix multiplication, we get

$$P^2 = P \cdot P = \begin{bmatrix} 0.11999392 & 0.5423195 & 0.3376865 \\ 0.02131528 & 0.6896231 & 0.2890616 \\ 0.01871928 & 0.4046823 & 0.5765984 \end{bmatrix}$$

From above matrix, we can interpret that, given that grandfather was in state-1, after two generation the probability that his grandson will be in state-1 is 0.11999392. The off diagonal elements show transitions from one state to another. For instance, given that the grandfather was in state-1, after one generation the probability that his son will be in state-2 is 0.2602774 and after two generations the probability that his grandson will be in state-2 is 0.5423195.

Ⅵ. Limiting Behavior of Transition Probability Matrix

The probability matrices are shown in Table 4 to have an idea about limiting behavior between grandfather and father. Table 4 divulges that the limiting $\lim_{n\to\alpha} P^n$ is equivalent to P^{28}, which implies that the Markov chain will occupy any state, which is independent of the initial state, and the poverty status will be stable after 28 generations. In other words, if a household starts initially with any state of poverty, then after 28 transitions the probability of getting that household in state-1, state-2 or state-3 is 0.02247594, 0.5702687, 0.4072554 respectively. For n=29 or more, no further change in transition probability will occur. It may also be noted that though the poverty status appears to be stable after 28 generations, the process of stability starts after 5th generation. Moreover, the transition matrix constructed from grandfather and father's poverty status interprets the stationarity of the poverty status at the succeeding 28th generation.

Table 4: Limiting Behavior of Transition Probability Matrix (Grandfather versus Father)

n	P^n		
1	0.32876712	0.2602740	0.4109589
	0.01052632	0.3578947	0.6315789
	0.02230483	0.8847584	0.0929368
2	0.11999392	0.5423195	0.3376865
	0.02131528	0.6896231	0.2890616
	0.01871928	0.4046823	0.5765984
3	0.05269072	0.5240956	0.4232137
	0.02071443	0.5081100	0.4711756
	0.02327503	0.6598561	0.3168689
5	0.02541607	0.5612708	0.4133132
	0.02213618	0.5523147	0.4255492
	0.02278944	0.5959058	0.3813048
15	0.02247579	0.5702555	0.4072687
	0.02247545	0.5702337	0.4072909
	0.02247664	0.5703184	0.4072050
20	0.02247595	0.5702692	0.4072548
	0.02247596	0.5702702	0.4072538
	0.02247591	0.5702665	0.4072576
25	0.02247594	0.5702686	0.4072554
	0.02247594	0.5702686	0.4072555
	0.02247594	0.5702688	0.4072553
28	0.02247594	0.5702687	0.4072554
	0.02247594	0.5702687	0.4072554
	0.02247594	0.5702687	0.4072554

Similarly, when we construct the above tables for father and respondent, we again find that the poverty status will be stable after 28 generations (table not shown).

Ⅶ. Statistical Inferences on Markov Chain and Stationarity of Transition Probabilities

To infer based on Markov chain model, we first assess the Markov dependence. For testing the null hypothesis that chain is of order zero (statistical independence) such that

$$H_0 : P_i^0 = P_j \text{; for all i, j = 1, 2, m}$$

against the alternative hypothesis (H_A) that the chain follows the first order dependence, the estimated value of test statistic is

$$-2\log\Lambda = 2\sum_{i=1}^{m}\sum_{j=1}^{m} n_{ij}\log\frac{(N)n_{ij}}{(n_i)(n_j)} = 106.7851$$

which has an asymptotic χ^2 distribution with $(3-1)^2=4$ degrees of freedom. This implies that $P\{\chi^2 \geq 0.5620953\}$ is rejected even at 1 percent level of significance. Thus, it may be concluded that the three-generation transition probability matrix suggests prevalence of Markov dependence, implying that the transitions between poverty states follow Markov chain model.

Subsequently, stationarity of poverty distribution of the three generations has been tested with the help of the following test-statistic:

$$-2\ln\Lambda = [L(\hat{P}^t) - L(P)] = 2\sum_{1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{m} n_{ij}^t \ln\frac{n_{ij}^t}{n_i^{t-1}p_{ij}}$$

Under the null hypothesis H_0,- 2 lnL has a χ^2 distribution with (T-1)[m(m-1)] degrees of freedom. The estimated value of the test-statistic is found to be 564.2039 that fall in the critical region $\chi^2 \geq \chi^2_{0.1(6)} = 16.81$ signifying that we can reject the null hypothesis of stationarity even at 1% level of significance and conclude that the distribution of poverty status is not stationary with three generations. It needs further generations to be stationary, which substantiates our interpretation on the distribution from the limiting behavior of the transition probability matrices. It is discovered from the limiting behavior of the probability matrices constructed from the poverty status distribution of grandfather and father, and from father to son/daughter (respondent) that the distribution will be stationary at the succeeding 28th generation each. As we have dealt with only three generations and further generations are not available, our test result goes in favor of non-stationarity of the distribution.

Ⅷ. Measurement of Mobility of Poverty State

For measuring mobility of poverty state, the predicted equilibrium distribution of poverty states obtained from the limiting distribution of transition probabilities can be compared with the actual distribution of poverty states obtained from the mobility matrix of Table 1. The estimated results are presented in Table 5.

Table 5: Actual and Equilibrium Distribution of Poverty State

Poverty State	Distribution at initial generation (father)	Distribution after one generation (son/daughter)	Predicted equilibrium distribution
State-1	0.1078431	0.009803922	0.00853653
State-2	0.6509804	0.733333333	0.7561535
State-3	0.2411765	0.256862745	0.23531

Table 6: The Expected Stay and Measures of Mobility in Each Poverty State

Poverty State	$i_i=(1-P_{ii})^{-1}$	$(1-v_i)^{-1}$	$i_i^s=\frac{(1-v_i)}{(1-P_{ii})}$
State-1	1.018519	1.00861	1.009824
State-2	7.904763	4.100941	1.927549
State-3	2.510204	1.307719	1.919529

It appears from Table 5 that the difference between the three distributions exists and there is distinct shift from higher poverty state to lower state. This phenomenon indicates the lack of prevalence of equilibrium state of poverty in the rural society. The average number of generation spent by a continuing household in poverty state-i and its standardized value i_i^s have also been estimated from equations (4) and (5) and presented in Table 6.

The average staying period for a household in state-2 is the highest which is followed by state-3, while it is the lowest for a household in state-1. This phenomenon indicates that those households who live in state-2 and state-3 failed to promote their livelihood pattern for a long period of time. Conversely, households who live in state-1 are mobile and fall in deteriorating situation within a short period of time by being unable to stay in the same state of poverty. The estimated standardized value i_i^s also supports this finding. The higher the value of departure of i_i^s from unity indicates a high degree of mobility in poverty state, while no departure (value hovering around zero) indicates a high degree of mobility [Bartholomew (1982)]. The value of i^s for state-1 is the lowest and the departure from unity is almost zero characterizes the most frequent movements of a household from state-1 to state-2 or state-3. On the other hand, the value of i^s for a household in state-2 and state-3 departs from unity to approximately same extent, showing slower change in boosting up their state of poverty.

Ⅸ. Conclusions

Analysis of intergeneration mobility of poverty status for chronically poor households indicated that 48 grandfathers as well as fathers of the respondents were non-poor and they could provide adequate food and bear educational and medical expenses has further deteriorated. But in the process, the poverty situation when the respondents were household

heads only one household, could maintain that state of poverty of grandfather and father. From the emergence of Bangladesh, different policies and programs have been designed to ameliorate the well being of the poor. Besides, increased allocation in the Annual Development Program (ADP), government has implemented several safety-net and target group oriented programs and projects for the poor. Wretchedly, these efforts have failed to abate the relentless poverty situation, especially for chronically poor and as evidence from this study shows, they have benefited least from economic growth and development. People, who endure poverty for longer period of time or all of their lives, typically defile their children by their own poverty. It circulates from one generation to another as if the offspring sucks it from the mother's breast. As a result, more than 70 percent of the chronically poor households maintained the same state of poverty and failed to upgrade their livelihoods.

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