

Self-preservation of a Turbulent Boundary Layer over d-type Roughness*

Shinsuke MOCHIZUKI**, Takatsugu KAMEDA** and Hideo OSAKA**

**Department of Mechanical Engineering, Yamaguchi University

2-16-1 Tokiwadai, Ube 755-8611, Japan

E-mail: shinsuke@yamaguchi-u.ac.jp

Abstract

Using a direct drag balance measurement for the local wall shear stress, self-preserving development of a turbulent boundary layer was achieved experimentally over a d-type rough surface without pressure gradients. The wall shear stress and mean velocity measurements confirmed the requirements for exact self-similarity and highly similar Reynolds stress profiles within the Reynolds number range for a constant skin friction coefficient. Under the condition that the boundary layer flow is completely independent of Reynolds number, the effect of wall roughness was investigated with respect to the similarity laws for the wall layer as well as the outer layer. Experimental observation reveals the wall similarity $\partial U/\partial y = u_\tau/\kappa y$ to be applicable to the present rough wall boundary layer remaining the accepted value of *Kármán* constant to be $\kappa = 0.41$. Otherwise, investigation of the wake strength in the mean velocity and Reynolds stress profiles reveals that the wall roughness does affect the outer layer structure. Reynolds stress measurements indicate that the primary effect of wall roughness on turbulence properties is in the component normal to the wall.

Key words: Turbulent Flow, Boundary Layer, Law of the Wall, d-Type Roughness

1. Introduction

Self-similarity is commonly observed in jets, wakes, and boundary layers and provides a fundamental basis for calculating the evolution of thin shear or approximated thin shear turbulent flows using a set of similar solutions and an ordinary differential equation (e.g. Schlichting-Gersten⁽¹⁾). The law of the wall and the velocity defect law are essential in constructing a universal representation of the mean velocity profile, as well as any statistical quantities for wall turbulence. As $Re \rightarrow \infty$, statistical independence between the inner and outer parts of the boundary layer guarantees the applicability of these laws and yields the logarithmic mean velocity profile in the overlap region (Millikan⁽²⁾).

If a self-preserving boundary layer emerges at zero pressure gradient and under the limited condition of linear growth of boundary layer thickness and constant skin friction coefficient, then the boundary layer equations can be reduced to an ordinary differential equation (Rotta⁽³⁾). In general, a constant skin friction coefficient has not been realized over a smooth surface, but rather appears over a particular roughness. Perry et al.⁽⁴⁾ and Wood-Antonia⁽⁵⁾ reported experimental evidence of a self-preserving boundary layer over the two-dimensional grooved rough surface,

which they called d-type roughness. Perry et al. ⁽⁴⁾ observed that the requirement is satisfied if effective roughness height is proportional to boundary layer thickness. Their experimental data confirmed that error-in-origin, as the relevant length scale for d-type roughness, is proportional to the boundary layer thickness and that both length scales grow linearly downstream. However, in their experiment the wall shear stress was estimated only from the integral of the wall static pressure measurement around a rectangular roughness element. More precise verification is desired in order to confirm that the skin friction coefficient remains constant, because the wall shear stress must be the sum of pressure drag and viscous shear stress over a rough surface.

Arguments as to whether the log law or the power law is more appropriate for use in determining similarity of the mean velocity profile in turbulent boundary layers have been made based on experimental data corrected over smooth surfaces (e.g., Barenblatt et al. ⁽⁶⁾, George-Castillo ⁽⁷⁾). However, the outer layer structure is always subject to the Reynolds number effect in turbulent boundary layers over a smooth surface; that is, the skin friction coefficient continuously decreases in the streamwise direction no matter how large the Reynolds number. Incomplete similarity should be considered in such a turbulent boundary layer, in which the skin friction coefficient decreases continuously as a function of Reynolds number over smooth ⁽⁸⁾ and k-type rough surface ⁽⁹⁾, $dC_f/dx \neq 0$. Whereas, in a self-preserving boundary layer, in which the skin friction coefficient must be completely independent of Reynolds number, the outer layer structure will exactly attain Reynolds number similarity at finite Reynolds numbers, as observed in laboratory experiments. In other words, a self-preserving boundary layer allows viscosity to be removed from dimensional arguments on the outer layer structure and would provide an example of complete similarity for the argument on the self-similarity of mean velocity profile.

First, we will confirm self-preserving development over d-type rough surface by experimental investigation using a direct drag balance measurement for the wall shear stress. Second, similarity of the mean velocity and Reynolds stresses will be verified using the carefully corrected experimental data obtained for the self-preserving boundary layer.

2. Experimental Set-up and Measurement Techniques

Figure 1 shows a schematic of the flow field, coordinate system, and nomenclature. The experimental study was performed in a low-turbulence wind tunnel having a cross section of 300 x 500 mm² and a length of four meters. The free-stream turbulence level was maintained below 0.3%, and the pressure coefficient $C_p \equiv 2(p - p_0) / \rho U_1^2$ varied within 0-0.5% (p and p_0 are the free-stream static pressure and atmospheric pressure, respectively, and $\rho U_1^2 / 2$ is the free-stream dynamic pressure).

Each two-dimensional rectangular bar roughness element was produced by machining a 18-mm-thick Bakelite plate such that the roughness height kr , element width b , and groove width w are all

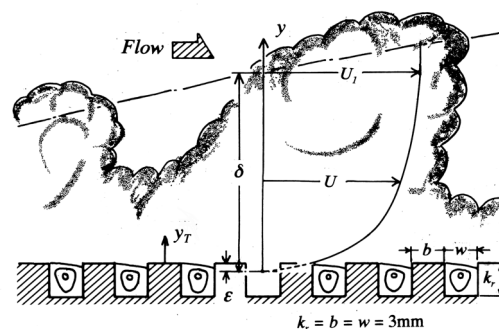


Fig.1 Flow field, coordinate system and nomenclature.

equal to 3 mm. The test plate was placed behind an initial smooth plate of 480 mm in length so that the boundary layer was laminar upon entering the rough surface section.

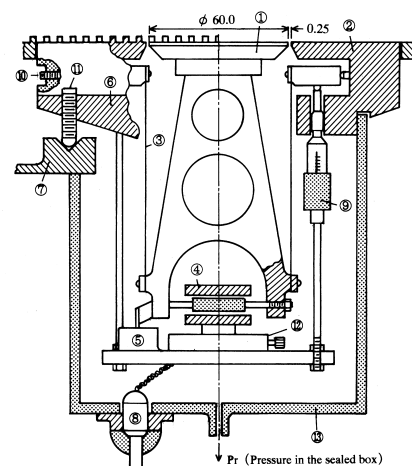
A direct measurement device, as shown in Fig.2, was used to measure the wall shear stress acting on the rough surface. A circular floating element of 60 mm in diameter and containing 10 roughness elements was held in place by two bronze parallel links that formed a uniform gap with the surrounding plate of 0.25 mm. Since the gap size is 2.5-7.5 times larger than the viscous wall length ν / u_τ (ν is kinematical viscosity and u_τ is friction velocity), the flow disturbance due to the gap is negligible (Gaudet-Winter⁽¹⁰⁾). A force-displacement relationship was examined using a stationary procedure, and the obtained calibration curve was found to have extremely good linearity. The output of a linear differential transducer was averaged over 40 seconds using a 12-bit AD converter and a personal computer. The linear differential transducer used in the present experiment is able to detect displacements as small as $0.1 \mu\text{m}$, which corresponds to 1% accuracy for the wall shear stress measurement.

The velocity measurement was performed using constant temperature anemometers (KANOMAX model 1011) and a single probe or a crossed hot-wire probe. A tungsten filament sensor of $5 \mu\text{m}$ in diameter and a sensor length, ℓ , of 1 mm, which satisfies the criterion $\ell^+ = \ell u_\tau / \nu < 30$ for reasonable spatial resolution of Reynolds stress measurement based on (Ligrani-Bradshaw⁽¹¹⁾, Mochizuki-Nieuwstadt⁽¹²⁾). The two sensors welded onto prong tips of the crossed hot-wire probe were spaced at 0.5 mm and crossing angle of 90 degrees. Over k-type mesh rough walls, a large crossing angle of 120 degrees is recommended for obtaining accurate measurements in high-intensity turbulence normal to the wall (Perry et al.⁽¹³⁾, Krogstadt et al.⁽¹⁴⁾).

However, the crossing angles of 90 to 120 degrees were found to have no effect on measured turbulence quantities for the skimming flow over the d-type rough wall. Velocity signals from the anemometers were passed through low-pass filters, in which the cut-off frequency was set to 20 kHz, and were digitized using a 12-bit AD converter at a 10 kHz sampling rate before being stored on a hard disk. The mean velocity and Reynolds stresses were calculated from the stored signals. The maximum measurement uncertainties for the hot-wire measurement close to the wall $y/\delta = 0.068$ (δ is

Table 1: Experimental Uncertainties (%)

| C_f | U | $\overline{u^2}$ | $\overline{v^2}$ | \overline{uv} |
|-------|-----|------------------|------------------|-----------------|
| 5 | 3 | 4 | 30 | 14 |



- ① Floating element
- ② Surrounding plate
- ③ Bronze leaf spring
- ④ Linear differential transformer
- ⑤ Silicon oil damper
- ⑥ Supporting base
- ⑦ Frame of the wind tunnel
- ⑧ Connecting tap
- ⑨ Micrometer-head
- ⑩ Clearance adjusting screw
- ⑪ Misalignment adjusting screw
- ⑫ Adjusting slider block for the transformer
- ⑬ Sealed cover box

Fig.2 Direct measurement device for the wall shear stress.

the boundary layer thickness, the height of which corresponds to $yu_\tau / \nu = 20$) and that for the wall shear stress measurement at $R_\theta = 700$ ($=U_1\theta/\nu$, θ is the momentum thickness) are summarized in Table 1 (Yavzkurt⁽¹⁵⁾).

3. Requirement for a Self-preserving Boundary Layer

Rotta⁽³⁾ presented six possibilities for the self-preserving boundary layer based on an analysis in which compatibility between the presumed local similarity and the equations of motion is required. In this section the self-preserving condition will be derived for evolving turbulent boundary layers with no pressure gradients.

In general, the governing equations for stationary turbulent flow in a two-dimensional boundary layer under zero pressure gradient can be expressed as

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad \text{and} \quad U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{\partial}{\partial x}(\overline{u^2} - \overline{v^2}) - \frac{\partial}{\partial y}(\overline{uv}) + \nu \frac{\partial^2 U}{\partial y^2}. \quad (3.1)$$

The similarity forms for mean velocity and Reynolds stress distributions are assumed to be

$$U_1 - U = u_\tau F(\eta, \omega), \quad \overline{u^2} = u_\tau^2 g_1(\eta, \omega), \\ \overline{v^2} = u_\tau^2 g_2(\eta, \omega), \quad \text{and} \quad \overline{uv} = u_\tau^2 g_{12}(\eta, \omega) \quad (3.2)$$

where $\eta \equiv y/\delta$ and $\omega = u_\tau/U_1$. Substituting these similarity forms into Eq. (3.1) yields a dimensionless equation,

$$\omega \left(\eta - \omega \int_0^\eta F d\eta' \right) F' \frac{d\delta}{dx} - \left[F - \omega \left(F^2 - F' \int_0^\eta F d\eta' \right) \right] \delta \frac{d\omega}{dx} = \\ \omega^2 g'_{12} - 2\omega\delta(g_1 - g_2) \frac{d\omega}{dx} + \omega^2 \eta (g'_1 - g'_2) \frac{d\delta}{dx} - \nu \left(\frac{\delta}{U_1} \right) \omega F''' . \quad (3.3)$$

Here, with available experimental evidences on flat plate boundary layer, dependence of these functions on the friction parameter ω is sufficiently small so that derivatives with respect to ω can be neglected. In addition, the momentum integral equation under the constant free-stream velocity

$$\frac{d\theta}{dx} = \frac{C_f}{2} + \frac{1}{U_1^2} \frac{d}{dx} \int_0^\infty (\overline{u^2} - \overline{v^2}) dy \quad (3.4)$$

is transformed using the dimensionless forms as follows:

$$\left(\delta \frac{d\omega}{dx} + \omega \frac{d\delta}{dx} \right) \int_0^1 F(1-\omega F) d\eta - \delta \frac{d\omega}{dx} \omega \int_0^1 F^2 d\eta = \\ \omega^2 + \left(2\delta\omega \frac{d\omega}{dx} + \omega^2 \frac{d\delta}{dx} \right) \int_0^1 (g_1 - g_2) d\eta . \quad (3.5)$$

The last term in Eq. (3.5) is usually negligible in turbulent boundary layers at points far distant from the flow separation point. When $d\omega/dx=0$, that is when $\omega=\text{const.}$, Eq. (3.5) yields $d\delta/dx=\text{const.}$ Eq. (3.3) then becomes an ordinary differential equation with respect to the single independent variable η . In boundary layers, $d\omega/dx=0$ means that the local skin friction coefficient is independent of Reynolds number, and this requirement can never be met for flow over a smooth wall.

If the surface is covered by roughness, the self-preserving condition for evolving turbulent boundary layers may be satisfied. The universal mean velocity profile for the wall layer above uniform roughness can be written as

$$\frac{U}{u_\tau} = \frac{1}{\kappa} \ln \frac{u_\tau y}{\nu} + C - \frac{\Delta U}{u_\tau} \quad (3.6)$$

where κ is Kármán constant, C a numerical constant determined over a smooth surface, and $\Delta U/u_\tau$ is a roughness function that is the velocity defect from the standard distribution over a smooth wall, which indicates the additional increment in wall shear stress due to the roughness. Since the near wall flow is independent of viscosity, i.e. in the fully rough state, the roughness function can be expressed in terms of representative roughness height ℓ_k only,

$$\frac{\Delta U}{u_\tau} = \frac{1}{\kappa} \ln \frac{u_\tau \ell_k}{\nu} + B \quad (3.7)$$

where B is a numerical coefficient depending on the specific roughness geometry. Substituting Eq. (3.7) into Eq. (3.6) and setting $y = \delta$ yields the following relationship:

$$\sqrt{\frac{2}{C_f}} = \frac{U_1}{u_\tau} = \frac{1}{\kappa} \ln \frac{\delta}{\ell_k} + C - B. \quad (3.8)$$

If $\omega = \text{const.}$, then the length scale ratio δ/ℓ_k must be constant. Since the boundary layer thickness grows in proportion to the boundary layer thickness $\delta \propto x$, the representative roughness height must be proportional to the streamwise distance $\ell_k \propto x$. Consequently, self-preserving development of a turbulent boundary layer is accomplished at a constant skin friction coefficient, and the boundary layer thickness and representative roughness length scale are proportional to the streamwise travel distance:

$$\omega = u_\tau/U_1 = \text{const.}, \text{ where } \delta \propto x \text{ and } \ell_k \propto x. \quad (3.9)$$

4. Self-preserving Development over d-type Roughness

4.1 Skin Friction Coefficient and Boundary Layer Thickness

In this section, we will determine whether the boundary layer attains a self-preserving state satisfying the constant skin-friction coefficient requirement for linear growth of boundary layer thickness. The skin friction coefficient, which gives definition of the friction parameter $\omega \equiv u_\tau/U_1 = \sqrt{C_f/2}$ in Rotta's analysis, is defined as

$$C_f = \tau_w / (\rho U_1^2 / 2), \quad (4.1)$$

where τ_w is the wall shear stress. The skin friction coefficient is plotted with respect to the momentum thickness Reynolds number R_θ in Fig.3. All experimental data over smooth and d-type roughness were obtained experimentally by direct wall shear stress measurement in the same wind tunnel facility (Osaka et al. ⁽⁸⁾). For a two-dimensional smooth wall boundary layer under zero pressure gradient, a frequently used empirical formula is Kármán-Schoenherr's formula (e.g., Granville ⁽¹⁶⁾):

$$1/C_f = 17.08(\log R_\theta)^2 + 25.11(\log R_\theta) + 6.012, \quad (4.2)$$

and a more accurate formula for low and moderate Reynolds numbers (Osaka et al. ⁽⁸⁾) is the following:

$$1/C_f = 20.03(\log R_\theta)^2 + 17.24(\log R_\theta) + 3.71. \quad (4.3)$$

The skin friction coefficient of the d-type rough wall flow has a constant value of $C_f = 0.0039$ when $R_\theta > 2000$ (the skin friction coefficient estimated by the momentum balance $2d\theta/dx$ agrees well with the values obtained by direct measurement). The present experiment yields direct evidence that the constant skin friction coefficient evaluated from the total wall shear stress includes both pressure drag and viscous shear stress. Otherwise, for the lower Reynolds numbers $R_\theta < 2000$ the skin friction coefficient gradually increases as R_θ decreases and approaches approximately the same value as that of the smooth wall boundary layer, $C_f = 0.0048$ at approximately $R_\theta = 800$.

Figure 4 shows the development of boundary layer thickness, displacement thickness, momentum thickness, and error-in-origin, which is the representative roughness length scale of the d-type rough wall (see Fig.6). All three length scales develop along straight lines $\sim x^1$ over a considerably long streamwise distance. The experimentally obtained skin friction coefficient and the development of the boundary layer thickness satisfy the conditions required for the exact self-preservation of the boundary layer. According to the skin friction measurement, the boundary layer attains the self-preserving state over the Reynolds number range of $R_\theta > 3000$, corresponding to $x > 1700$ mm and $\delta^+ \equiv \delta u_\tau / \nu > 1000$.

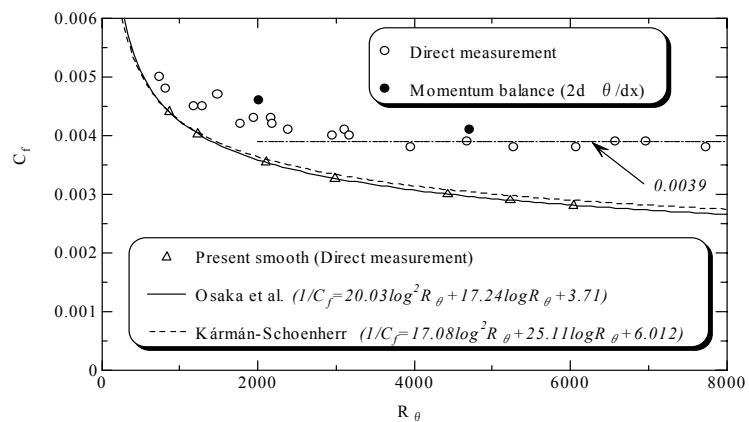


Fig.3 Local skin friction coefficient as a function of momentum thickness Reynolds number.

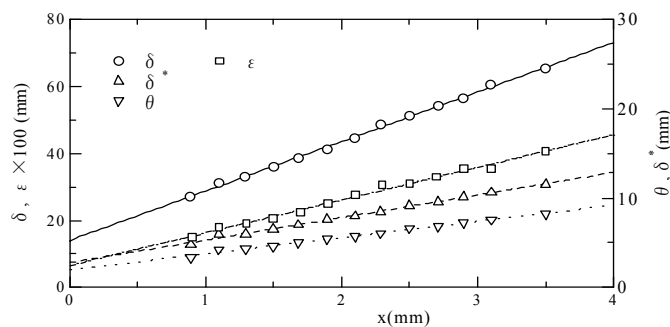


Fig.4 Development of length scales: boundary layer thickness δ , displacement thickness δ^* , momentum thickness θ , and error-in origin ϵ .

4.2 Similarity of the Mean Velocity and Reynolds Stress Profiles

The extensive survey on available experimental data of turbulent boundary layer in Stanford Conference shows the logarithmic velocity profiles shown in Fig.5 using the standard log-law lines proposed in 1968 (Coles-Hirst⁽¹⁷⁾):

$$\frac{U}{u_\tau} = 5.62 \log \frac{yu_\tau}{\nu} + 5.0. \quad (4.4)$$

The distance from the wall y is measured from the virtual origin, $y = y_T + \varepsilon$, where error-in-origin ε is determined using the method proposed by Monin-Yaglom⁽¹⁸⁾. The mean velocity profile of the present smooth boundary layer matches that predicted by the standard log-law at both ends of the examined range of Reynolds numbers, $R_\theta = 860$ and 5230. Over the higher Reynolds number range of $R_\theta > 3000$, at which good similarity can be expected in the boundary layers over the d-type rough wall, the log layer is immediately recognized and the velocity gradient coincides with that of the standard log-law, that is, the

Kármán constant κ remains at the accepted value of 0.41 over the roughness. This experimental observation confirms the wall similarity $\partial U / \partial y = u_\tau / \kappa y$ to be applicable to the present rough wall boundary layer. The logarithmic velocity profile having the same velocity gradient as the standard log law is recognizable at somewhat smaller Reynolds numbers of $R_\theta = 1230$ and 2000. This result indicates that the wall similarity is sustained regardless of exact self-preservation of the outer layer structure, and this behavior is similar to that formerly observed over smooth surfaces. However, the validity of the log law is difficult to demonstrate using $\kappa = 0.41$ for the experimental evidence at much smaller Reynolds numbers of $R_\theta = 700$ and 790, at which the skin friction coefficient depends strongly on the Reynolds number. Because the Reynolds number

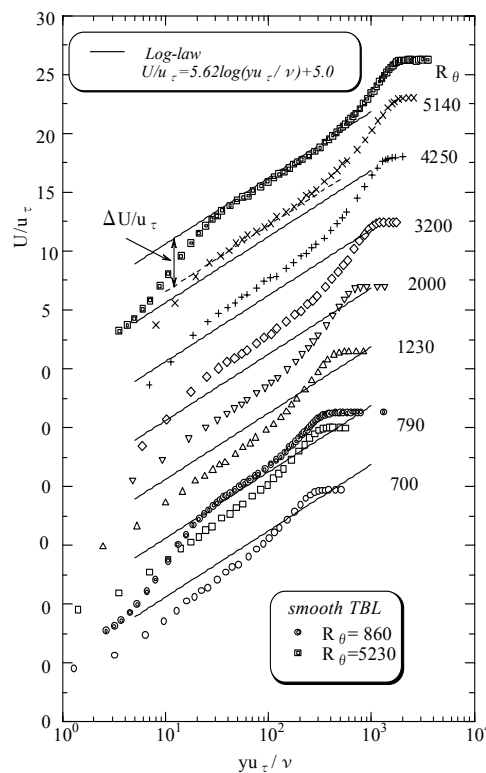


Fig.5 The logarithmic mean velocity profiles over d-type rough wall at different Reynolds numbers.

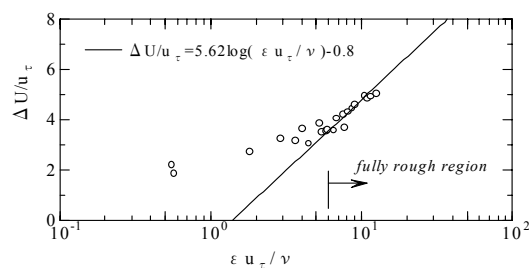


Fig.6 Correlation of the roughness function with the roughness Reynolds number based on error-in-origin.

$\delta^+ \equiv u_\tau \delta / \nu$ is smaller than 500, even if the wall layer remains, the thickness of the log layer is not sufficient to easily locate its existence.

In order to examine whether the error-in-origin is acceptable as representative length scale for d-type roughness, the roughness function is plotted against the roughness Reynolds number based on the error-in-origin ε , as shown in Fig.6. In the fully rough state defined by the constant skin friction coefficient, the experimental data is reasonably correlated by the logarithmic function having the same gradient as the mean velocity profile in the wall layer. This implies that $\ell_k \sim \varepsilon$, and the alternative expression for the mean velocity profile in the fully rough state becomes:

$$\frac{U}{u_\tau} = 5.62 \log \frac{y}{\varepsilon} + C_\varepsilon \quad (4.5)$$

where C_ε is a numerical constant, and the error-in-origin indicates the effective length scale of wall roughness.

Let us examine self-similarity in the outer layer mean velocity profile in detail using the wake law proposed by Coles⁽¹⁹⁾, in which a possible formula for rough wall turbulent boundary layers is defined as

$$\frac{U}{u_\tau} = 5.62 \log \frac{yu_\tau}{\nu} + 5.0 - \frac{\Delta U}{u_\tau} + \frac{\Pi}{0.41} W\left(\frac{y}{\delta}\right), \quad (4.6)$$

where Π is a wake parameter and $W(y/\delta)$ is the universal wake function. Figure 7 shows the wake function obtained using the two semi-empirical formulas as proposed by Coles⁽¹⁹⁾ and Lewkowicz⁽²⁰⁾, respectively, for smooth wall boundary layers:

$$W\left(\frac{y}{\delta}\right) = 1 - \cos\left(\frac{\pi y}{\delta}\right) \quad (4.7)$$

$$W\left(\frac{y}{\delta}\right) = 2\left(\frac{y}{\delta}\right)^2 \left(3 - 2\frac{y}{\delta}\right) - \frac{1}{\Pi} \left(\frac{y}{\delta}\right)^2 \left(1 - \frac{y}{\delta}\right) \left(1 - 2\frac{y}{\delta}\right) \quad (4.8)$$

The first equation by Coles was later improved by Lewkowicz in order to satisfy the condition $\partial U / \partial y = 0$ at $y = \delta$. The wake function of the d-type rough wall boundary layer is reasonably similar, except for the minimum limit on Reynolds number, $R_\theta = 700$. Lewkowicz's formula better fits the experimental data of the d-type rough wall, especially in the outer half layer of $y/\delta > 0.6$. The slight upward deviation of the experimental data from the formulas in the inner half layer of $y/\delta < 0.6$ is associated with the slightly smaller thickness of the wall layer in the present rough wall boundary layer. The ratio of the wall layer thickness, which is defined as the upper height of the logarithmic layer, to the boundary layer thickness is approximately 0.15 at large Reynolds numbers, at which the self-preserving development is observed. Otherwise, the ratio is from 0.15 to 0.20 over the smooth surfaces (Purtell et al.⁽²¹⁾, Sabramanian-Antonia⁽²²⁾). The small deviation in the wake function over the rough surface would cause an error in the estimation of the wall shear stress using the universal profile. However, the universal wake function is an acceptable means by which to represent the mean velocity profile in the outer layer for the d-type rough wall flow at low Reynolds numbers, at which the requirement is not satisfied for the self-preserving developments.

The wake parameter is plotted against the Reynolds number $\delta^+ \equiv \delta / (\nu / u_\tau)$,

defining the length scale ratio of the largest eddy to the smallest eddy in Fig.8. As $\delta^+ \rightarrow \infty$, the wake parameter increases rapidly and approaches to an asymptotic value within the range of $\Pi = 0.72-0.73$ over $\delta^+ = 800-1400$, at which the self-preserving development is observed at constant skin friction coefficient over the corresponding Reynolds number range of $R_\theta = 2000-4000$. The self-preserving development over the d-type roughness can be confirmed by the wake strength $\Delta U / u_\tau = 2\Pi / \kappa$ to be constant at finite Reynolds numbers. The solid line represents the semi-empirical formulation by Coles' survey on carefully selected experimental data of smooth wall boundary layers having no pressure gradients (Coles⁽²³⁾):

$$\Pi = 0.62 - 1.21 \exp(-\delta^+ / 290) \quad (4.9)$$

This is an improved formulation of Π as a function of δ^+ , and the present experimental data obtained in a two-dimensional smooth wall boundary layer under zero pressure gradient agree well with Eq.(4.9). Note that the asymptotic value of the wake parameter over the d-type rough surface is greater than that for smooth wall flow. The larger value of wake strength, being independent of the Reynolds number effect by virtue of the constant skin friction coefficient, proves that the velocity defect law depends on the wall roughness, and a parameter describing the effect of wall roughness can never be completely neglected in a dimensional argument of the outer layer structure. If the wall shear stress determined in the matching process of the measured mean velocity profile to the defect law is assumed to be universal, the quantitative difference in the wake strength causes skin friction coefficients over the d-type rough wall to be overestimated by approximately 30%.

Similarly, Furuya et al.⁽²⁴⁾, Perry et al.⁽¹³⁾ and Dijedini et al.⁽²⁵⁾ have reported the effect of wall roughness on the outer layer structure in thick turbulent boundary layers. Furuya et al.⁽²⁴⁾ suggested the dependence of the additive constant upon wall roughness, and proposed a procedure by which to determine the error-in-origin using the velocity defect law. In the experimental study by Perry et al.⁽¹³⁾, the Reynolds shear stress in the constant stress layer, which was carefully obtained by hot-wire measurement, was found to be considerably smaller than the wall shear stress determined by velocity fitting to the universal velocity defect shape, casting some doubt on the velocity defect law being independent

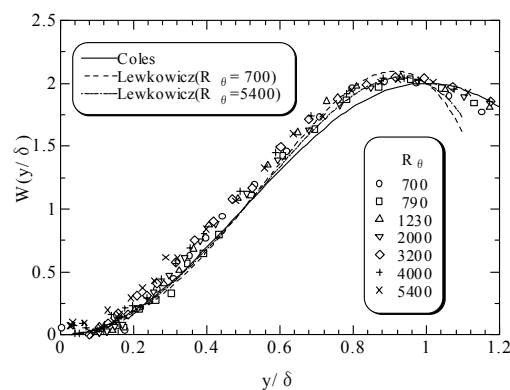


Fig.7 Wake functions of the d-type rough wall boundary layer.

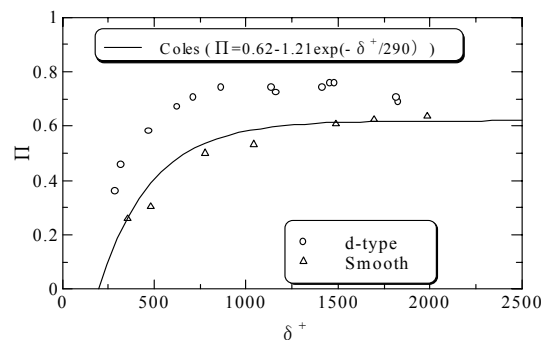


Fig.8 Developments of the wake parameter and comparison with the smooth wall results.

of wall roughness. Dijedini et al. ⁽²⁵⁾ and Krogstadt et al. ⁽¹⁴⁾ recognized a clear distinction in Reynolds stress profiles or turbulent properties in the outer layer of the boundary layer that had developed over rough walls. The universal velocity defect law is often used as a convenient method by which to determine the wall shear stress over non-smooth surfaces, such as Riblets surface (Choi ⁽²⁶⁾). However, the present experimental findings indicate that this method leads to significant quantitative error in estimating wall shear stress.

4.3 Profile of Reynolds Stresses

Figures 9-11 show the turbulent intensity and Reynolds shear stress profiles normalized with the velocity and length scales employed for self-similarity, namely, friction velocity u_τ and thickness $\Delta \equiv \delta^* U_1 / u_\tau$, as introduced by Rotta's analysis for similarity argument. Similarity of the turbulent intensity and Reynolds shear stress profiles is fairly good over the Reynolds number range of $R_\theta = 3200-5140$, at which self-preserving development is observed at constant skin-friction coefficient. At low Reynolds numbers of $R_\theta = 700, 790$ and 1230 , at which the wake strength grows faster in the downstream direction, the magnitude of normalized Reynolds stresses increases around the outer edge of the layer. Compared with the smooth surface, an effect of the wall roughness can be seen in the magnitude of the normal component of turbulent intensity v_{rms} / u_τ throughout the layer. The magnitude of v_{rms} / u_τ in the inner layer over the present rough surface is 1.2-1.25, whereas that over a smooth surface is approximately 1.0. Dijedini et al. ⁽²⁵⁾ also find greater magnitude of v_{rms} / u_τ in the LDV measurement performed over d-type roughness. Over three-dimensional k-type rough walls, for example, three-dimensional arrays of isolated roughness elements (Raupach et al. ⁽²⁷⁾) and a mesh rough surface (Krogstadt et al. ⁽¹⁴⁾), a much greater value of v_{rms} / u_τ , up to 1.35, was observed in the wall layer. Consequently, these experimental findings indicate that the roughness effect on the turbulent structure appears primarily in the turbulent intensity component normal to the wall. Considering the experimental uncertainty of the wall shear stress and Reynolds shear stress measurements, we can conclude that a constant stress layer (Townsend ⁽²⁸⁾) exists in the d-type rough wall boundary layer over the entire

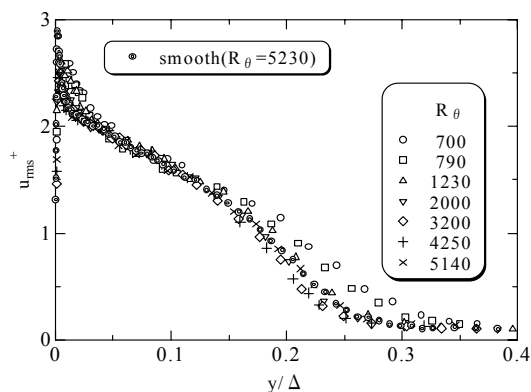


Fig.9 Similarity of the streamwise turbulent intensity profile normalized with the similarity variables.

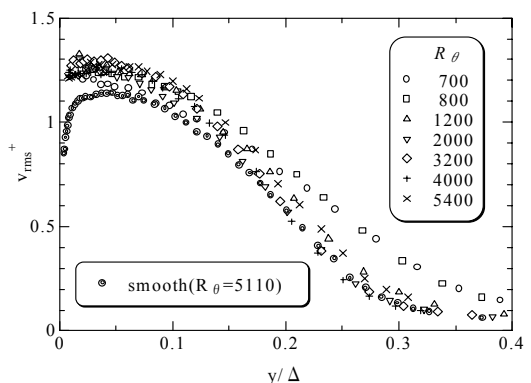


Fig.10 Similarity of the normal to the wall turbulent intensity profile normalized with the similarity variables.

Reynolds number range of $R_\theta = 700-5400$. This experimental evidence supports the friction velocity as a relevant scale in the wall layer over the d-type roughness for the Reynolds number range examined.

5. Conclusions

- (1) Based on the skin friction measurement using the direct drag balance, the boundary layer attains the self-preserving state over the Reynolds number range of $R_\theta > 3000$, corresponding to $x > 1700$ mm and $\delta^+ \equiv \delta u_\tau / \nu > 1000$.
- (2) The experimental observation of mean velocity profiles confirms the wall similarity $\partial U / \partial y = u_\tau / \kappa y$ to be applicable to the present rough wall flow, and the Kármán constant κ remains at the constant value of 0.41. The larger value of wake strength compared with the smooth wall boundary layer, being independent of Reynolds number, proves that the velocity defect profile depends on wall roughness, then a parameter describing the effect of wall roughness can never be completely neglected in a dimensional argument of the outer layer structure.
- (3) Similarity of the turbulent intensity and Reynolds shear stress profiles is fairly good over the Reynolds number range of $R_\theta = 3200-5140$, at which the self-preserving development is observed at constant skin-friction coefficient. The Reynolds stress measurements indicate that the roughness effect on a turbulent structure appears primarily in the normal turbulent intensity.

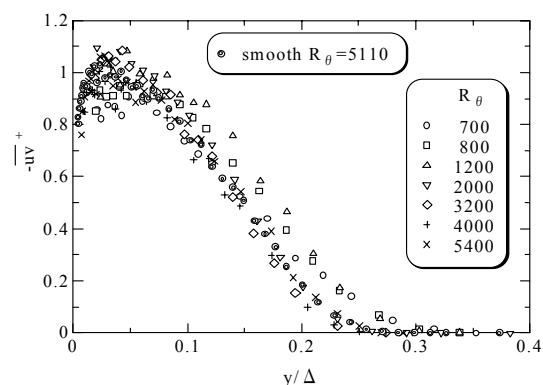


Fig.11 Similarity of the Reynolds shear stress profile normalized with the similarity variables.

References

- (1) Schlichting, H. and Gersten, K., Boundary layer theory, Springer, 8th ed, (2000), pp.578-586.
- (2) Millikan, C.B., A critical discussion of turbulent flow in channels and circular tubes, Proceedings of 5th International Congress Applied Mech., Cambridge Massachusetts, Wiley, (1938), pp.386-392.
- (3) Rotta, J.C., Turbulent boundary layers in incompressible flow, Progress Aero Sci 2, Pergamon Press, (1962), pp.105-114.
- (4) Perry, A.E., Schofield, W.H. and Joubert, P.N., Rough wall turbulent boundary layers, J Fluid Mech., Vol.37, (1969), pp.383-413.
- (5) Wood, D.H. and Antonia, R.A., Measurements in a turbulent boundary layer over a d-type surface roughness, Trans ASME, J Applied Mech., (1975), pp.591-597.
- (6) Barenblatt, G.I., Chorin, A.J. and Prostokishin, V.M., Self-similar intermediate structures in turbulent boundary layers at large Reynolds numbers, J Fluid Mech., Vol.410, (1999), pp.263-283
- (7) George, W.K. and Castillo, L., Zero-pressure-gradient turbulent boundary layer, Applied Mech. Rev., Vol.50, No.11, (1997), pp.689-729.
- (8) Osaka, H, Kameda, T. and Mochizuki, S., Re-examination of the

- Reynolds-number-effect on the mean flow quantities in a smooth turbulent boundary layer, *JSME Int. J, Ser.B*, Vol.41, No.1, (1998), pp.123-129.
- (9) Kameda, T., Osaka, H. and Mochizuki, S., Mean flow quantities for the turbulent boundary layer over a k-type rough wall, *Proceedings of 13th Australian Fluid Mechanics Conference*, Dec., Melbourne, (1998), pp.357-360.
 - (10) Gaudedt, L. and Winter, K.G., Measurements of the drag of some characteristic aircraft excrescences immersed in turbulent boundary layer, *AGARD CP124*, (1973).
 - (11) Ligrani, P.M. and Bradshaw, P., Spatial resolution and measurement of turbulence in the viscous sublayer using subminiature hot-wire probes, *Exp Fluids*, Vol. 5, (1987), pp.407-417
 - (12) Mochizuki, S. and Nieuwstadt, F.T.M., Reynolds-number-dependence of the maximum in the streamwise velocity fluctuations in wall turbulence, *Exp Fluids*, Vol.21, (1996), pp.218-226.
 - (13) Perry, A.E., Lim, K.L. and Henbest, S., An experimental study of the turbulence structure in smooth- and rough-wall boundary layers, *J Fluid Mech.*, Vol.177, (1987) , pp.437-466.
 - (14) Krogstad, P.-Å. , Antonia, R.A. and Browne, L.W., Comparison between rough- and smooth-wall turbulent boundary layers, *J Fluid Mech.*, Vol.245, (1992), pp.599-617.
 - (15) Yavuzkurt, S., A guide to uncertainty analysis of hot-wire data, *Trans. ASME, J Fluids Eng.*, Vol.106, (1984), pp.181-186.
 - (16) Granville, P.S., Drag and turbulent boundary layer of flat plates at low Reynolds numbers, *J Ship Res.*, Vol.21, No.1, (1977), pp.30-39.
 - (17) Coles, D.E. and Hirst, E.A., *The Young Person's Guide to the Data, AFOSR-IFP-Stanford*, Vol.2, (1968)
 - (18) Monin, A.S. and Yaglom, A.M., *Statistical fluid mechanics of turbulence*, The MIT Press, (1973), pp.284-295.
 - (19) Coles, D., The law of the wake in the turbulent boundary layer, *J Fluid Mech.*, Vol.1, (1956), pp.191-226.
 - (20) Lewkowicz, A.K.Z., An improved universal wake function for turbulent boundary layers and some of its consequence, *ZFW*, Vol.6, (1982), pp.261-266.
 - (21) Purtell, L.P., Klebanoff, P.S. and Buckley, F.T., Turbulent boundary layer at low Reynolds number, *Phys Fluids*, Vol.24, (1981), pp.802-811.
 - (22) Sabramanian, C.S. and Antonia, R.A., Effect of Reynolds Number on a Slightly Heated Turbulent Boundary Layer, *International J Heat Mass Transfer*, Vol.24, No.11, (1981), pp.1833-1846.
 - (23) Coles, D.E., Coherent structures in turbulent boundary layers, *Perspectives in turbulence studies*, Springer-Verlag, (1987), pp.93-114.
 - (24) Furuya, Y, Miyata, M. and Fujita, H., Turbulent boundary layer and flow resistance on plates roughed by wires, *J Fluid Eng.*, Vol.98, (1976), pp.635-644.
 - (25) Dijedini, L., Elavarasan, R. and Antonia, R.A., The turbulent boundary layer over transverse square cavities, *J Fluid Mech.*, Vol.395, (1999), pp.271-294.
 - (26) Choi, K.-S., Near-wall structure of a turbulent boundary layer with riblets, *J Fluid Mech.*, Vol.208, (1989), pp.417-458.
 - (27) Raupach, M.R., Antonia, R.A. and Rajagopalan, S., Rough wall turbulent boundary layers, *Applied Mech. Rev.*, Vol.11, (1991), pp.1-25.
 - (28) Townsend, A.A., *The structure of turbulent shear flow*, Cambridge University Press, 2nd ed., (1976), pp.139-143.