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Dynamic Pictures of Tidal Movements Predicted by the Theory of Forced Oscillation

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ABSTRACT：A tide –generating force has two dynamic properties. One of them is a property as a vector with magnitude and direction, the other is a periodicity of force acting with a cycle of about 12 hours. If the tide-generating force is a periodic external force, the tides must be a kind of forced oscillation phenomena.

Three proposals regarding the origin of tides will be suggested in this article.

(1) The ocean tides can be divided into three types by the depth. They are '**out of phase**' type (shallower than 22,000m), '**in phase**' type (deeper than 22,000m) and '**resonance**' type (equal to 22,000m). The actual ocean tides are classified into the 'out of phase' type. On the other hand, the Earth tides occurring on the solid Earth are classified into the 'in phase' type.

(2) Dynamic movement pictures of each type of tides have been determined. Unlike the widely accepted explanation, the global form of the ocean surface repeats an oval and a circle in turn according to the Moon's position. As a result, the tidal movements of the ocean will occur in fixed four areas with an angular distance of 90 degrees apart along the equator, and four spots escaping any tidal movements will appear between them.

(3) A new concept of '**tide burst**' is deduced and its scientific meanings are discussed. Judging from the delay of Earth's rotation period, the tide burst will happen in the ocean about 4.5 billion years from the present.

Key Words: tide, forced oscillation, tide burst, out of phase, in phase

1. Introduction

The long history of studies of tides from the ancient to the present is briefly summarized by Deparis, V. et al. (2013). According to them, the modern history of tidal studies can be divided into two stages as follows.

The first stage is from the mid-17th century, when Newton published the famous *Principia*, to the mid-18th century, when his successors such as Bernoulli, D. and Euler, L. and others further developed Newton's theory. From this stage on, it was a common way of thinking that the tides are a kind of dynamic phenomena caused by an interaction of two forces, such as the tide-generating force and centrifugal force. Therefore, this type of tidal theory is referred to as the 'static theory of tides.' In the majority of science textbooks and web sites, the tides are explained by this theory. The static theory can explain some important characteristics of tides, for example, spring and neap tides, and the fact that tides always happen twice a day. But it is also true that there are some serious contradictions between the theoretical statements and the observations of actual tides. The disagreement of the time between high tides and the culmination of the Moon is one of the examples well known from the past.

The second stage is the time from Laplace, who worked on in the 18th century, to the present day. For the mathematical studies focused on the three- dimensional propagation of tidal waves on the real Earth, Laplace, P. I. (1749-1829) based his work on some fundamental

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suppositions. One of his targets was to solve the problem of how the ocean responds to the tide-generating forces acting on it. In this study, he introduced analytical mechanics and hydrodynamics. So, the theory in this stage is now referred to as the 'dynamic theory of tides.'

Both theories depend on a similar way of thinking about the origin of tides. This way of thinking takes almost none of the periodicity of tide-generating forces into account.

The periodicity of tide-generating force was first taken into account only 20 years ago. The first scientist who pointed out the importance of the periodicity was Fukuzumi, Y. (Fukuzumi, 2001). He solved the equation of motion, into which the periodic external force was introduced, and he proved theoretically that the tides should be a kind of vibrating movement of the ocean caused by forced oscillation.

After him, some studies of tides using the theory of forced oscillation came to be published by Butikov, E. I. (Butikov, 2002 and 2017), Tokieda, T. (Tokieda, 2013) and Ikeda, Y. (Ikeda, 2019). In these papers, they pointed out that the theory of forced oscillation may have a possibility to change our common sense about the origin of tides and tidal phenomena.

This article is comprised of three parts. In the first, we classify the ocean tides into three types based on the depth of the water. These are '**in phase**, '**out of phase**' and '**resonance**.' In the second, according to the concept proposed by Butikov (2017), the dynamic pictures for each type of tides will be discussed. In the third, the mechanism and scientific meanings of a new concept '**tide burst**' (Ikeda, 2019) caused by the resonance, especially in relation to Earth science and astronomy, will be discussed in detail.

2. Free oscillation of the ocean

In order to find out a simple dynamic principle of tides behind the complicated actual tides, it is necessary to simplify the ocean into as simple a model as possible. For this purpose, in this study we will employ a canal model which was introduced first by Airy about 200 years ago. In the hypothetical canal, it is assumed that the canal goes around the Earth with a uniform depth and width along the equator. By using the canal model, it becomes easy to examine the problems behind some complexities and difficulties caused by various geophysical and geographical conditions.

Using this canal model, a Russian scientist Butikov, E. I. has clarified some dynamic images of the free oscillation movement of the ocean (Butikov, 2002, 17). One period of schematic illustration about the free oscillation of the ocean is shown in Fig. 1. This illustration was made based on a concept of Butikov (2017). He assumed that the water surface in the canal is transformed into an ellipse in shape, with its major axis parallel to the line B-D, by some kinds of the external force (Fig. 1-a), and that at this moment the external force suddenly disappeared. After all the external forces have vanished, only the gravity of the Earth will act on the water. As a result, the water surfaces at B and D in Fig. 1a will begin to fall down vertically and to rise up at A and C. Hence, after a quarter of the period (t = T/4), the water surface will become circular in shape (Fig. 1b).

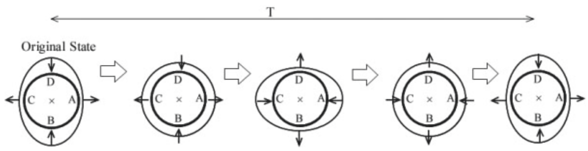

Fig. 1 One cycle of free oscillation in a canal

Black arrows show the moving direction of the water and T is one period of free oscillation.

(an illustration based on Butikov's cocept).

Although this circle coincides with an equilibrium surface related to the gravity of the Earth, the vertical motion of the water continues further. And after the next quarter of the period, judging from the whole Earth, the form of the ocean surface will become the ellipse again. But, unlike the first ellipse, its major axis will be perpendicular to the first (see Fig. 1c). The water level at A and C will begin to fall down vertically and to rise up at B and C, because all of the kinetic energy will be converted into static energy at this moment. After the next quarter period of time, the water surface will become circle in shape again (Fig.1d) and after another a quarter of period it will return to the elliptical shape same as in the initial state (Fig. 1e). The series of movements (a –e in Fig. 1) is equivalent to one period of free oscillation T. In the perspective of physics, the period of free oscillation is equal to the eigenperiod of the object. Therefore, we can conclude that T is equal to the eigenperiod T_0 of the hypothetical canal.

In the next step, we will examine the relations between eigenperiod T_0 and the depth of the canal h. According to wave mechanics, the period of wave T is proportional to the wavelength λ and inversely proportional to the velocity v. Therefore, this relation can be expressed in an equation $T = \lambda / v$. In the case of free oscillation occurring in the hypothetical canal shown in Fig.1, the wavelength λ is equal to the perimeter of the Earth's semicircle. Then we can write $\lambda = \pi R$, where R is the radius of the Earth. Substituting this equation for the former one, we will get,

$$T_0 = \pi R/v. \qquad (1)$$

According to hydrodynamics, the velocity of long wave v is proportional to the square root of the product of depth h and the acceleration of gravity g. So we can express $v = \sqrt{(hg)}$. Substituting this equation for (1), we get,

$$T_0 = \pi R / \sqrt{(hg)}. \qquad (2)$$

By solving this equation for h, we will get a curve as shown in Fig. 2. This curve indicates the relation of the eigenperiod T_0 and the depth of the canal h.

According to the RIKA NEMPYOU 2019 (Japanese Chronological Scientific Table) edited by the National Astronomical Observatory of Japan, the mean depth of the Pacific Ocean is about 4,200m (4,188m in precision). Substituting this value for equation (2), we will get a value of 27.5 hours for the eigenperiod of the Pacific Ocean. In the following chapters, we will discuss the origin of ocean tides by using the theory of forced oscillation.

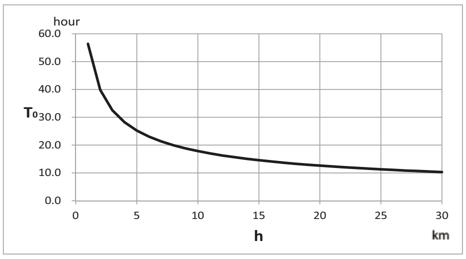

Fig. 2 Theoretical relations with eigenperiod T_0 and the depth of canal h

3. Classification of the oceanic tides

According to the theory of forced oscillation, amplitudes of forced oscillation L caused by the periodical external forces is inversely proportional to $(T^2 - T_0^2)$, so we can express L in the following expression,

$$L \propto 1/(T^2 - T_0^2). \qquad (3)$$

Where T is the period of external force and T_0 is the eigenperiod of the object. The response of objects to a periodic external force can be theoretically divided into three types, that is, 'in phase', 'out of phase' and 'resonance'.

Table 1 Ocean's responses to the tide-generating force

Period	Depth	Response
T > T0	h>22,000m	in phase
T < T0	h< 22,000m	out of phase
T = T0	h= 22,000m	resonance

In the case of ocean tides, the tide-generating force and the ocean are equivalent to the external force and the object respectively. Therefore, the period of external force T equals a half of the rotation period of the Earth (12 hours).

As mentioned in the previous chapter, the dynamic condition that the resonance happens is $T=T_0$. So, substituting $T=T_0=4.3\times10^4$ seconds (=12 hours) for the equation (2), we can obtain h= 22,000m. This value indicates the depth that the resonance occurs in the ocean. Based on the above description, we can divide the ocean tides into three types as shown in Table 1 and Fig. 3. In this paper, we call these types 'in phase' type, 'out of phase' type and 'resonance' type respectively.

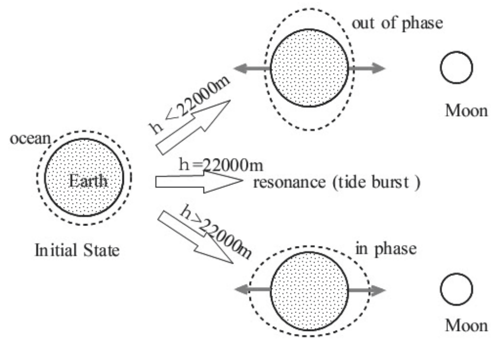

Fig. 3 Three types of the ocean tides

Gray arrows indicate the directions of maximum tide-generating force.

The schematic view illustrating the tidal responses of the ocean is shown in Fig. 3. As shown in the figure, when the water depth is less than 22,000m, the response of the ocean should be 'out of phase' with the tide-generating force. In this case, the time gap between the culmination of the Moon and the high tide is about six hours. It is the reason of this time lag that the traveling velocity of tidal waves is slower than that of the Moon moving over the Earth. In the actual ocean, the velocity of tidal waves traveling across the ocean with the depth of 4,200m (the mean depth of the Pacific Ocean) is about

200m/s. This value is considerably slower than that of the Moon over the Earth (460m/s).

On the contrary, in the ocean deeper than 22,000m, the tides should be 'in phase', because the velocity of tidal wave becomes faster than 460m/s. In the case of 'in phase' tides, there are almost no time lags between high tide and the culmination of the Moon, because high tides certainly take place in accordance with the culmination of the Moon.

And further, when the depth becomes exactly equal to 22,000m, the tidal response of the ocean must be resonance. In this case, therefore, the amplitude becomes infinite in scale. The author called this kind of gigantic tide the **'tide burst'** (Ikeda, 2019). Some detailed explanations of this new concept will be carried out in the next chapter.

4. Dynamic pictures of tidal movements

In the previous chapter, it has been clarified that the ocean tides can be divided into three types depending on the depth of water. In this chapter, we will suggest some dynamic movement pictures for each type of tides.

To recognize the mechanical origin of tides more clearly, it is necessary to clarify the dynamic movement of tides. In an article published in 2017, Butikov, E. I. put forth some important and fruitful proposals about this problem (Butikov, 2017). With reference to his suggestions, the dynamic pictures of tides will be discussed in this chapter.

4-1 Tides of 'in phase' type

As mentioned before, in the hypothetical canal with a depth deeper than 22,000m, the traveling velocity of tidal wave will become faster than 460m/s (the velocity of the Moon moving over the Earth's surface). As the relationship between the eigenperiod of the canal T_0 and the period of tide-generating force T is expressed as $T > T_0$, the tidal response of water in the canal should be 'in phase' with the external tide-generating force.

By the action of tide generating force, the shape of the water surface is deformed into an ellipse with its major axis parallel to the Earth-Moon line (see Fig. 4a), and this ellipse rotates around the Earth in accordance with the Moon while keeping the elliptical shape (Fig.4b and c). This positional relationship between the elliptical shape of the ocean and the Moon is the same as the relationship that is widely accepted all over the world.

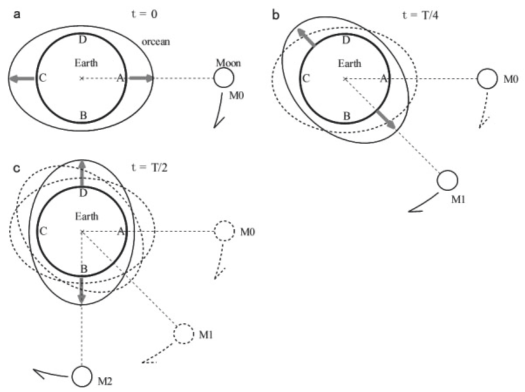

Fig. 4 Movement pictures of the 'in phase' tides
Gray arrows indicate the directions of maximum and minimum tide-generating forces.
(An illustration based on Bukikov's concept)

By the way, as the velocity of tidal waves traveling through the solid Earth is very fast, it is easy to say that the Earth tides belong to this type of tides.

4-2 Tides of 'out of phase' type

On the other hand, in the case shallower than 22,000m, the velocity of tidal waves traveling along the hypothetical canal will become slower than that of the Moon traveling over the Earth's surface (460m/s). Then, the relationship between the two periods T and T_0 is expressed as $T_0 > T$.

In this case, the tidal response of the water in the canal should be 'out of phase' with the tide-generating force, so that the angular gap between the high tide and the largest tide-generating force unequivocally becomes 90 degrees (Fig. 5). This angular gap is equivalent to about six hours in time.

Now, let us examine the problems of the dynamic pictures of 'out of phase' tides occurring in the hypothetical canal. Based on the dynamic analysis, Butikov (2002, 17) suggested a simple and very understandable explanation for this problem. The outline is as follows.

At first, for an initial state (T=0) he assumed that the surface of the canal water becomes an ellipse in shape, whose major axis is perpendicular to the Earth-Moon line (Fig. 5a). In this state, the tide-generating forces will act to raise the water surface upward vertically at A and C, and to lower downward at B and D (Fig.5a).

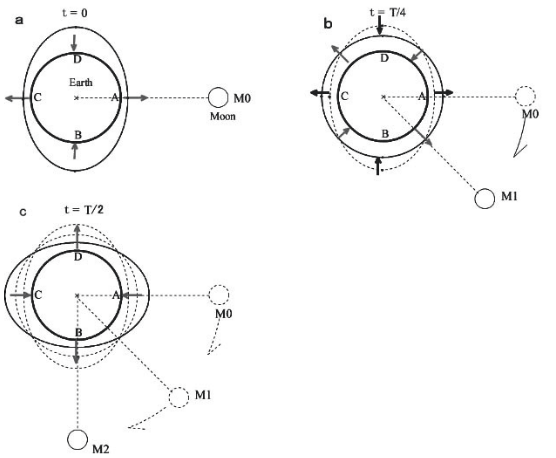

Fig. 5 Movement picture of the 'out of phase' tides
Gray arrows show the directions of maximum tide-generating forces, and black arrows show the moving directions of the water surface.
(An illustration based on Bukikov's concept)

After a quarter period of the tide-generating force (t=T/4), the water surface going around the Earth will become circular in shape (Fig. 5b). This shape is equivalent to an equilibrium surface owing to the Earth's gravity. However, the vertical movement of the water surface will continue further without stopping. It is the reason that the kinetic energy of water at this state becomes largest. After another one quarter period (t=T/4), the form of the water surface in the canal returns to an ellipse in shape, but its major axis is perpendicular to that of the initial one as shown in Fig. 5c. And after further another half period of time, the form of the water surface will return to an ellipse in the same form as in the initial state. The time span from t = 0 to t = T is equivalent to one period of tidal movement. In other words, the water surface of the canal surrounding the Earth deforms its form : ellipse → circle → ellipse → circle → ellipse (see Fig. 6).

It should be noted that the change of form shown in Fig. 5 and 6 is greatly different from our common sense image of tides that the tidal waves travel across the ocean from east to west in accordance with the Moon. It is one of our most important conclusions that the tidal movements in the canal will occur in fixed four domains with an angular distance of 90 degrees along the equator and that four spots escaping from any tidal movements should occur between them (see Fig. 5).

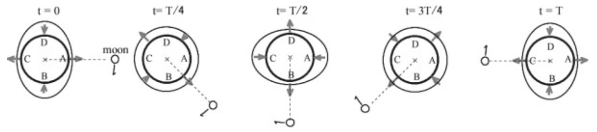

Fig. 6 Schematic illustration of one cycle of the tidal movement and the position of the Moon in the 'out of phase' tides

4-3 Tide Burst

The most scientifically interesting and important phenomenon relating to the forced oscillation is a resonance. In general, the resonance occurs under a dynamic condition that the period of external force equals the eigenperiod of object. If this condition has been satisfied, the object will surely come to vibrate violently with infinite amplitude.

Applying the theory of forced oscillation to the ocean tides, this resonance will happen when the period of tide-generating force comes to equal the eigenperiod of the ocean. Ikeda (2019) called this kind of extremely huge tide, predicted theoretically, the **'tide burst'**. Now, we will examine the tide burst in detail and clarify its scientific meanings, especially in relation to Earth science and astronomy. To simplify the problem, we will again assume a canal model encircling the whole Earth along the equator.

We may imagine two cases of condition that causes the tide burst. In the first, the depth of the ocean varies, but the Earth keeps a rotation period the same as at the present (24 hours a day). As mentioned before, the tide burst will happen at the depth of about 22,000m, because the eigenperiod of the ocean equals the period of the external tide-generating force. However, there is little scientific interest in this case, because the depth of the ocean never reaches 22,000m.

In the second, the ocean keeps its depth to be about 4,200m (the mean depth of the Pacific Ocean), but the rotation period of the Earth varies. In the real ocean, the possibility of this case is higher than the first.

Using modern instrumentation, it has become clear that the rotation period of the Earth becomes slightly longer with a constant ratio of about 0.0024s/100years (Stacey et al., 2008). Supposing that the ratio of delay does not change through the history of the Earth from 3.0 billion years ago to 5.0 billion years from now, the rotation period of the Earth will change as shown in a diagram of Fig. 7. The dynamic conditions just after the birth of the Earth were greatly different from that of the present Earth, so that the rotation periods before three billion years ago are not shown in the diagram. From the diagram, we are able to infer some interesting phenomena relating to the tide burst of the ocean and also of the solid Earth.

Using the diagram in Fig. 7, we are able to estimate that the rotation period of the Earth in 4.5 billion years from now will become 55 hours a day. If the Earth rotates with this period, the period of the tide-generating force is 27.5 hours (a half of the rotation period of the Earth). As a result, it is possible to conclude that the tide burst will occur and huge ocean tides with great amplitude will be generated at that time.

The tide burst occurring on the solid Earth should be an attractive concept relating to Earth science and astronomy. Owing to the recent developments of the observations of Earth's free oscillation, it has become clear that the most prominent oscillation period is about 54 minutes among various modes of the Earth's free oscillation (Stacey et al., 2008). From this observation, we may decide that the eigenperiod of the solid Earth T_0 is 54 minutes. This conclusion means that when the Earth rotates with a period of 108 minutes (double of the period of tide-generating force), the tide burst will certainly occur on the solid Earth and further that the solid Earth may collapse into pieces. The dotted line in the diagram of Fig. 7 shows this rotation period (about two hours) causing the tide burst of the solid Earth.

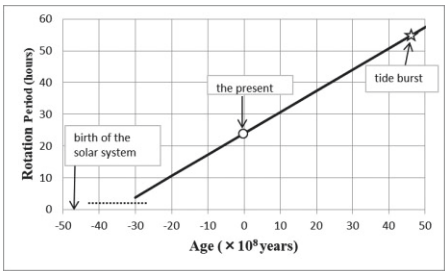

Fig. 7 Rotation period of the Earth from 3.0 billion years ago to 5.0 billion years from the present
Dotted line shows the rotation period of the Earth (about two hours) when the tide burst happens on the solid Earth.

Viewing Fig. 7, it is able to predict theoretically that the Earth has never rotated with a period shorter than about two hours. If we suppose that this conclusion can generally apply to the solid planet of our solar system, the solid proto-planets rotating with a period less than two hours might collapse by the tide burst.

5. Summary

Unlike the explanations accepted widely in the world, we have examined the ocean tides as a kind of forced oscillation phenomena caused by the periodicity of tide-generating forces. The important conclusions of this article are summarized as follows.

(1) The ocean tides can be divided into three types by the depth of the sea:

$h > 22{,}000$m, $T_0 < T$, in phase

$h < 22{,}000$m, $T_0 > T$, out of phase

$h = 22{,}000$m, $T_0 = T$, resonance.

Where, h and T_0 are the depth and the eigenperiod of the ocean respectively, and T is a period of the tide-generating force. Since the depth of the actual ocean is much shallower than 22,000m, all of the natural ocean tides are classified into the 'out of phase' type. On the other hand, the Earth tides occurring on the solid Earth are classified into the 'in phase' type.

(2) The dynamic movement pictures of each type of tides have been determined. In the tides of 'out of phase' type, the global form of the sea level repeats oval and circular shapes in accordance with the changing apparent position of the Moon. If we express these movements in a geocentric frame of reference, two important characteristics of ocean tides will be revealed. The first is an appearance of fixed four areas, where tidal movement begins to happen, with an angular distance of 90 degrees apart along the equator. And the second is an appearance of four spots escaping any tidal movements between them.

(3) A new concept of 'tide burst' is deduced and its scientific meanings have been discussed. The tide burst will occur at the time when the period of tide-generating force equals the eigenperiod of the canal (ocean). Judging from the delay of Earth's rotation period, the tide burst of the ocean will occur about 4.5 billion years from the present.

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Dr. Furukawa, H. (emeritus professor of Yamaguchi University). The author also received some useful advice through a productive discussion about tides from Professor Sakai, N. (Yamaguchi University). I sincerely express my appreciation to them.

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概要

強制振動理論から予測した潮汐の力学的運動像

池田　幸夫

起潮力はベクトルと周期性という2つの力学的特性をもつ。周期的な外力が物体に作用すると，一般にその物体は強制振動を起こす。起潮力は周期性をもつ外力であるので，潮汐は強制振動現象でなければならない。

この論文では，潮汐の起源について3つの提案を行った。

(1)海洋潮汐は水深によって3つのタイプに分類される。すなわち，「逆位相型（水深22000m未満）」，「同位相型（水深22000m以上）」，および「共振型（水深22000m）」である。地球の海洋で発生する潮汐は逆位相型の潮汐である。一方，固体地球に起こる地球潮汐は同位相型である。

(2)それぞれの型の潮汐に見られる力学的運動像の違いを明らかにした。広く普及している潮汐論の説明とは違って，実際の潮汐に伴う海水面の変化は，月の位置に従って楕円形と円形を繰り返す楕円振動を行う。その結果，潮汐による海水面の上下動は赤道に沿って互いに90°離れた4領域で起こり，それらの間にはほとんど潮汐運動を起こさない無潮域が出現する。

(3)「潮汐バースト」という新しい概念を提唱し，その科学的意義について考察した。自転周期の遅れから推定すると，海洋に潮汐バーストによる巨大潮汐が発生するのは約45億年後と予想することができる。また，この理論を固体地球に適用すれば，原始地球の自転周期が約2時間より短いことはなかったと考えなければならない。