Growth Slow-Down in China: An Intertemporal Dynamic CGE Analysis

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Abstract

This paper introduces a dynamic intertemporal computable general equilibrium (CGE) model for China with forward-looking agents and intertemporal consumption and investment decisions. Some simple long-term growth scenarios are also run. The results imply that a long-run growth rate below 5% may have adverse macroeconomic impacts on the Chinese economy.

1. Introduction

Forward-looking intertemporal computable general equilibrium (CGE) models overcome a major shortcoming of static and recursive dynamic CGE models where investments, savings, and consumption decisions do not reflect intertemporal decision-making.

Intertemporal CGE models have been used to examine a wide range of economic policies and issues. In the case of China, dynamic intertemporal CGE models have been used to study various issues about the Chinese economy such as the economic impact of climate change and environment policies (Jin, 2002; Lu et al., 2010), energy policies (Hübler, 2011), fiscal policies (Zhang et al., 2013), and pension reform (Li and Mérette, 2005), among others. This paper introduces an intertemporal CGE model that can be used to simulate macroeconomic policy options for the Chinese economy. The model builds on Devarajan and Go (1998) with some modifications for the Chinese economy.

The rest of the paper is organized as follows. Section 2 summarizes the technical specifications of the model. Section 3 describes the data used and presents the results of the simulations. Finally, Section 4 concludes the paper.

2. Model Specification

For the purpose of this paper, we utilize the intertemporal CGE model developed by Go (1994) and Devarajan and Go (1998) with some modifications. The model is based on a Walrasian general equilibrium setting where the producers seeking profit maximization determine their demand for production factors, and institutions in the economy (households, firms, and government) base their expenditure decisions on preferences and budget constraints. The model specifies the dynamics of an open economy by taking into account the intertemporal decisions of the forward-looking consumers and the intertemporal investment decisions of forward-looking producers with the attached adjustment costs. The continuous functions are transformed into discrete ones for the purpose of numerical solution. The dynamic model involves discounting future values to present values. In what follows, we explain the salient features of the CGE model.

2.1. Static Within-Period Model

The static part of the CGE model shares the characteristics of the generic class of static CGE models, which can be found in Dervis et al. (1982) and Lofgren et al. (2002). The general structure of the model in the form of an extended circular flow diagram is depicted in Figure 1, and the structure of output and consumption is presented in Figure 2. Here, we briefly explain the structure of the static part of the model.

Figure 1. The structure of the CGE model

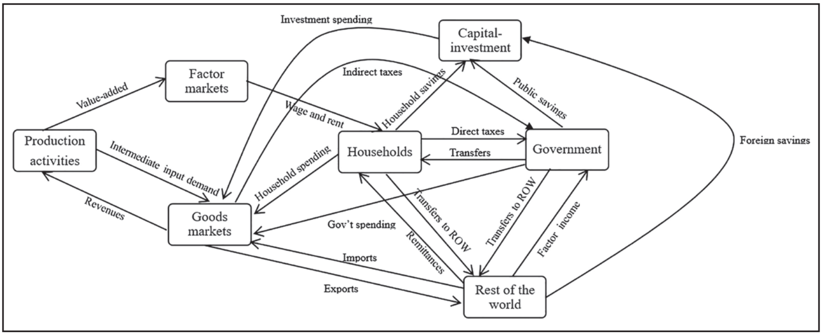
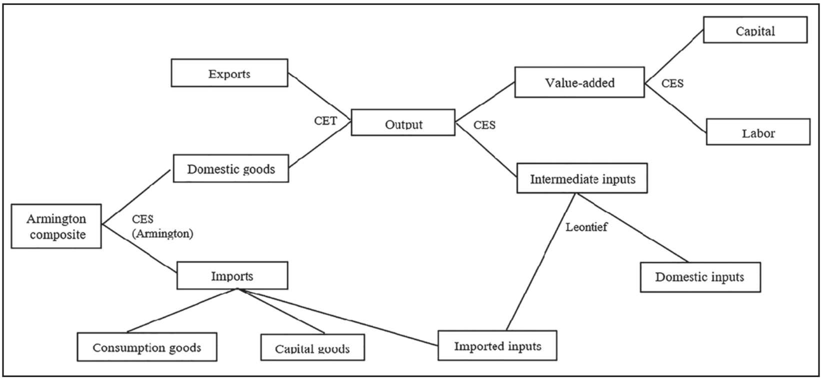


Figure 2. The structure of production



Output is represented by a nested production with two stages. At the lower stage, firms use intermediate inputs in fixed proportions of output. In addition, capital and labor are aggregated using a constant elasticity of substitution (CES) aggregation function to produce value added. At the second stage, value added and intermediate inputs are aggregated using a CES aggregation function to produce output. Optimal amounts of labor and capital are derived from the first-order conditions of the value-added function. These conditions yield the wage-rent ratio as well.

In trade block, imports and domestic goods are assumed to be imperfect substitutes. Output is distributed between domestic goods and exports with a constant elasticity of transformation (CET) aggregation function. Households consume a composite good made up of domestic goods and imported consumer goods using an Armington-type CES function. Optimal levels of exports, imports, and domestic supply are calculated from the first order conditions of Armington CES and CET functions. We specify three different types of imported goods: intermediate goods, capital goods, and consumer goods. It is important to understand the dependency of the economy on imported inputs. Because of the difference in the specifications of exports and imports, the real exchange rates faced by export goods and imports goods also differ. The real exchange rate for imported consumer goods equals the ratio of the import price level of consumer goods to domestic good price level. Similarly, the real exchange rate for exported goods equals the ratio of the export price level to domestic good price level. The prices of exports are exogenously determined, and the prices of imports for each type of good are inclusive of tariffs.

Government is subject to a budget constraint. Government revenues are made up of direct taxes paid by households, import tariffs, and indirect taxes. Government also borrows from the rest of the world. It allocates these revenues among public consumption, subsidies, and transfers to households, payments on foreign debt, and public savings. The behavior of the represented household is discussed in the following subsection about intertemporal consumption.

By Walras' Law, total savings (government, household, and foreign savings) equal total investments and capital consumption allowances (depreciation). The depreciation rate is a fixed ratio of capital, and household and government saving rates are exogenous.

2.2. Intertemporal Consumption Decision and Household Behavior

Households own both capital and labor and rent the capital they own to firms. The representative household in the model maximizes lifetime utility (U_t). The utility function is as follows:

$$U_t = \frac{C_t^{1-\theta}}{1-\theta} \tag{1}$$

where the subscript t denotes time, C_t is consumption and θ is the constant risk aversion factor.

The discounted value of lifetime utility is as follows:

$$U_t = \int_{t=0}^{\infty} e^{-\rho t} \frac{C_t^{1-\theta}}{1-\theta} \tag{2}$$

where ρ is the rate of time preference. In discrete time, the present value of lifetime utility is expressed as $U_t = \sum_t \left(\frac{1}{1+\rho}\right)^{t+1} \frac{C_t^{1-\theta}}{1-\theta}$.

The representative household also faces a budget constraint where the change in accumulated wealth is made up of the discounted value of lifetime capital investments and after-tax revenues from labor services and net transfers from the government and the rest of the world. Total value of future consumption (C_t) is constrained by wealth accumulation. The problem of the consumer can be summarized as follows:

$$\max\ U_t = \sum_t \left(\frac{1}{1+\rho}\right)^{t+1} \frac{C_t^{1-\theta}}{1-\theta} \tag{3}$$

$$\text{subject to} \sum_t \left(\frac{1}{\prod_{\tau=0}^{t}(1+r_{C,\tau})}\right) P_t C_t \leq \sum_t \left(\frac{1}{\prod_{\tau=0}^{t}(1+r_{C,\tau})}\right) Y_t \tag{4}$$

where r_C is the interest rate attached to capital investments, P_t is the price level of consumption, and Y_t is income in period t. As shown below, r_C is determined by the international interest rate and the change in the real exchange rate in an open-economy setting.

The well-known condition for consumption between two consecutive periods, arising from the solution of the Lagrangian equation is as follows:

$$\frac{C_{t+1}}{C_t}=\left(\frac{P_{t+1}(1+\rho)}{P_t(1+r_{C,t+1})}\right)^{-\frac{1}{\theta}} \tag{5}$$

Households receive income from capital and labor services, government transfers, and the rest of the world (in the form of remittances and net factor income). We also deduct investment spending and foreign debt service payments from household income. They allocate their income to consumption, direct taxes, household savings, and payments to the rest of the world.

2.3. Intertemporal Investment Decision of the Firm

The representative firm's decision to invest is based on a maximization problem where the objective is to maximize the value of the firm (V) subject to the constraint for the accumulation of capital stock (K). The firm's objective function is as follows:

$$\max\ V_t=\int_{t=0}^{\infty}\left(\frac{1}{\prod_{\tau=0}^{t}(1+r_{C,\tau})}\right)R_t \tag{6}$$

$$\text{subject to } K_t=I_{t-1}+(1-\delta_K)K_{t-1} \tag{7}$$

where I_t is investment spending by firm, r_Q is real interest rate faced by the firm (i.e., international interest rate plus the change in real exchange rate for exports), and δ_K is the capital depreciation rate. R_t is the value of the marginal product of capital, and it refers to the discounted values of total future profits after investment expenditures are deducted. The firm maximizes the present values of future profits which are specified as the value of total investments exclusive of the investment support by the government in the form of investment tax credits ($t_{I,t}$) and inclusive of adjustment cost of investment (ADJ_t). Adjustment cost of investment implies that changing the capital stock of a firm involves a cost, which is specified as a function of investment to capital ratio. The adjustment cost implies that the firm's adjustment of its capital stock is not instantaneous and takes time. This adjustment cost is a function of investment and capital stock, and is convex in investment to capital ratio. Accordingly, total investment expenditures (V) of a firm which aims to increase its capital stock by I units is as follows: $V_t = (1 - t_{I,t} + ADJ_t) I_t$, where t_I is the investment tax credit.

The solution of the maximization problem above, arising from the Hamiltonian equation, produces two well-known results. First, the firm invests until the marginal cost of investing equals the shadow price of capital ($\widetilde{P_{K,t}}$): $q_t = \dfrac{\widetilde{P_{K,t}}}{P_{K,t}} - 1 + t_{I,t}$. The term $\dfrac{\widetilde{P_{K,t}}}{P_{K,t}} - 1$ refers to the excess of the shadow price of capital with respect to the replacement cost of capital. When this is greater than unity, i.e., $\widetilde{P_{K,t}} > P_{K,t}$, the firm will choose to increase its capital stock by investing. Intertemporal conditions for investments also dictate that the firm will invest until the cost of investment is equal to the shadow price of capital. Tobin's q, which is the market value of the unitary amount of the firm's capital, is calculated as the ratio of the shadow price of capital to the replacement cost of capital ($P_{K,t}$), inclusive of the investment tax credit per unit of capital. Second, the user cost of capital ($r_{Q,t}\widetilde{P_{K,t}}$) is equal to the value of the marginal product of capital (R_t), potential earnings from investments $\widetilde{\Delta P_{K,t}} = \left(\dfrac{\widetilde{P_{K,t+1}} - \widetilde{P_{K,t}}}{\widetilde{P_{K,t}}}\right)$, and the depreciation of capital: $r_{Q,t}\widetilde{P_{K,t}} = R_t + \dfrac{\widetilde{P_{K,t+1}} - \widetilde{P_{K,t}}}{\widetilde{P_{K,t}}} - \delta_K \widetilde{P_{K,t+1}}$.[1)]

1) The transversality condition yields a solution for the shadow price of capital. Following Summers (1981) and Go (1994), we specify the investment to capital ratio as a function of Tobin's q (q_t) and exogenous parameters a and b as follows: $I_t/K_t = bq_t$. Adjustment cost of investment is as follows: $ADJ_t = (I_t/K_t)/2b$.

2.4. Equilibrium Conditions

The solution of the model requires satisfying two conditions: fulfilling intertemporal equilibrium conditions and achieving within-period equilibrium. Within-period equilibrium requires some equilibrium conditions to be fulfilled.

1. Equilibrium must be achieved in the goods markets, i.e., total demand (the sum of household and government consumption spending, investments, and exports) equals total supply (Armington composite good of domestic goods and imported consumer goods).
2. The equilibrium in the factor markets must be achieved, i.e., labor supply equals labor demand through wage adjustments.
3. Balance of payments equilibrium is achieved, i.e., the current account balance equals saving-investment gap. This gap translates into adjustments in the capital account by way of inflows or outflows.
4. Savings-investment equilibrium is achieved.[2)]

Intertemporal equilibrium conditions dictate that the consumption and investment conditions across periods are satisfied. The intertemporal decisions of households and firms yield a steady-state equilibrium for the economy where, in the terminal period, the transversality conditions of utility, wealth, and investment prevail.

2.5. Calibration and Solution of the Model

Our calibration procedure for the within-period model follows the conventional CGE models. The model is solved through both within-period

2) Savings are made up of household, public, and foreign savings. Foreign savings affect this equilibrium because foreign savings adjust for the savings-investment gap. Increase in foreign savings means increasing debt, and an increase in debt payments reduces the household budget, which eventually affects government budget as households transfer part of their income to the government as tax. Therefore, foreign savings affect public sector balances.

and intertemporal prices ensuring general equilibrium conditions. Certain parameters of the model are set exogenously (see Table 1) and the remaining parameters are calibrated using behavioral equations. We replicate the model for the benchmark year and calculate the exogenous parameters such as tax rates, tariff rates, saving rates, and parameters obtained from the first order conditions in the production and trade blocks. Our benchmark year is 2012, the year for which the latest input-output tables are available. We organize our data in the form of a social accounting matrix (SAM) to ensure consistency with the within-period CGE model which shares common characteristics of the SAM-based CGE models.

Table 1. Selected exogenous parameters of the model

a	0.00	β_{MK}	0.740	ρ	1.00	ρ_Q	0.60
b	0.50	δ_K	0.08	ρ_{CC}	0.50	θ	0.90
β_{MC}	0.106	ε	6.643	ρ_f	0.90	$\widetilde{P_{K,0}}$	1.90
β_{MI}	0.455						

Note: The remaining exogenous parameters are calibrated within the model. β_{MC}, β_{MI}, and β_{MK} are the shares of consumption goods, intermediate inputs, and capital goods, respectively, in total imports.

Calibration of the dynamic part of the model assumes that the economy is on a steady-state growth path in the benchmark year. In addition to the calibration of the parameters in the within-period (static) part of the model, the dynamic module requires calibration of the interest rate, the rate of return to capital, the shadow price of capital, and the parameters of the instantaneous utility and consumption functions.

We set a 30-year horizon for the dynamic module, based on a series of sensitivity analyses. The level of the capital stock and household consumption at various horizons (5, 10, 15, 20, 30, 40, and 50 years) reveal that, after 25-30 years the long-run levels of capital stock and consumption change only slightly. Furthermore, the level of utility in the objective function remains virtually unchanged after 30 years. For these reasons, we run our model for a 30-year period. An important parameter in the model is the long-run growth rate, which we set at 5%. This parameter enters the equation for steady-state level of I/K ratio in the terminal year. For the investment adjustment function, we assume that investment is somehow responsive to Tobin's q and set the parameter b at 0.5.

The solution of the model enables that intertemporal rates, such as the exchange rate, discount rate, interest rate, and the time preference of households, determine price levels[3] in the economy across periods, which then lead to equilibrium through behavioral equations and the general equilibrium conditions of the within-period and intertemporal modules. Intertemporal consumption and investment modules ensure that unique steady state equilibrium is reached in the terminal year, where the price levels cease to change and the relevant discount rate for assets is the world interest rate. In the solution process, we maximize the utility of the representative consumer.

3. Data and Results

The data used in the numerical solution were organized in the form of a social accounting matrix, built around the input-output tables for 2012. In addition, we used data about labor, tariff rates, exchange rate, and public balances, which were obtained from the China Statistical Yearbook.

For the purpose of exposition, some simple simulations are run. Specifically, an alternative set of long-run growth rates are used to calibrate the model and the consequent changes are reported. The changes in welfare are computed using the intertemporal utility function.

３）The relevant prices for the consumers are the prices of domestic goods and imports, and the prices of exports and domestic goods for the producers.

Before the simulations, first the results of the business as usual scenario are reported in Figure 3. In this scenario, the growth rate is the actual growth rate in the benchmark year and the parameters are as stated in section 2.5. The results show that real consumption increases by more than 40% at the end of the 30-year horizon and investment expenditures flatten after 10 periods. Capital stock growth and consumption growth trajectories are similar. The detailed results for some variables are also reported in Table 2. In addition, Table 3 shows the changes in welfare under different horizons. The changes in welfare, i.e. Hicksian equivalent variation, are fairly robust to the length of the period.

Figure 3. The business as usual scenario results

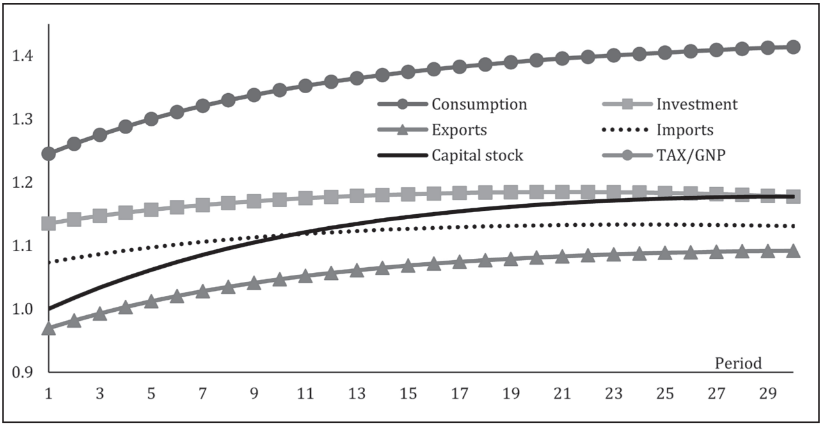


Table 2. The values of some variables in the business as usual scenario for different time horizons

t	C	I	E	M	D	P_M/P_D	q/P_K	K
1	1.2449	1.1348	0.9695	1.0734	1.0066	0.9393	0.9436	1.0000
2	1.2605	1.1412	0.9817	1.0803	1.0155	0.9451	0.9411	1.0175
3	1.2748	1.1468	0.9928	1.0865	1.0235	0.9504	0.9388	1.0336
4	1.2879	1.1519	1.0029	1.0921	1.0308	0.9552	0.9368	1.0483
5	1.2998	1.1564	1.0120	1.0972	1.0374	0.9595	0.9349	1.0618
10	1.3454	1.1726	1.0469	1.1158	1.0623	0.9759	0.9281	1.1133
15	1.3741	1.1811	1.0683	1.1265	1.0774	0.9861	0.9240	1.1453
20	1.3924	1.1844	1.0812	1.1319	1.0861	0.9925	0.9213	1.1642
25	1.4048	1.1831	1.0885	1.1331	1.0906	0.9968	0.9195	1.1742
30	1.4135	1.1775	1.0918	1.1308	1.0918	1.0000	0.9181	1.1775

Table 3. Change in welfare at the terminal period relative to welfare at the benchmark year for different time horizons

5-year	3.15 %	30-year	3.03 %
10-year	3.11 %	40-year	3.03 %
15-year	3.08 %	50-year	3.03 %
20-year	3.05 %		

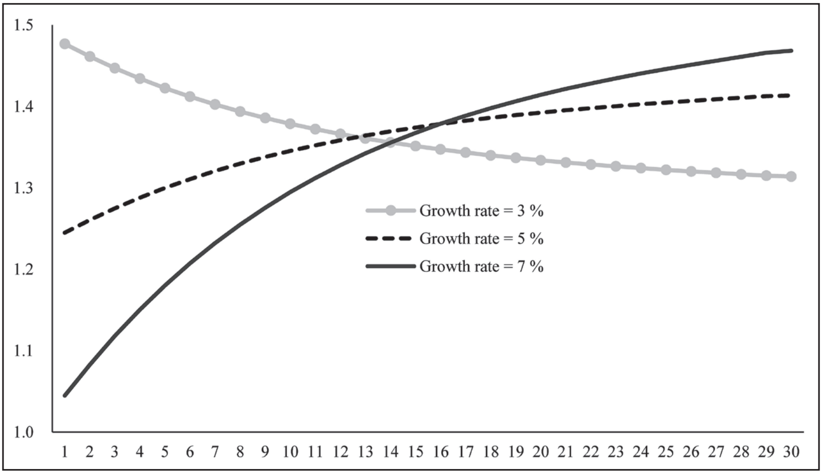
The results of the simulation exercises are presented in Table 4. Since the aim of the government since 2013 is transition to consumption-based growth rather than investment-based growth based on capital accumulation, we focus our attention on the consumption variable, C. In the simulations, we specify three long-run growth rates, namely, 3%, 5%, and 7%, and trace the long-run trajectory of consumption over the 30-year horizon.

Table 4. Simulation results under different long-run growth scenarios

Period	Consumption			Investments			Exports			Imports			PM/PD		
	g=3%	g=5%	g=7%	g=3%	g=5%	g=7%	g=3%	g=5%	g=7%	g=3%	g=5%	g=7%	g=3%	g=5%	g=7%
1	1.477	1.245	1.045	0.868	1.135	1.344	1.028	0.970	0.919	0.931	1.073	1.195	1.059	0.939	0.845
2	1.461	1.261	1.083	0.867	1.141	1.373	1.018	0.982	0.953	0.927	1.080	1.220	1.054	0.945	0.860
3	1.447	1.275	1.118	0.865	1.147	1.399	1.008	0.993	0.984	0.923	1.087	1.242	1.049	0.950	0.874
4	1.434	1.288	1.150	0.864	1.152	1.422	0.999	1.003	1.013	0.920	1.092	1.262	1.044	0.955	0.886
5	1.423	1.300	1.180	0.863	1.156	1.442	0.991	1.012	1.039	0.917	1.097	1.280	1.040	0.960	0.898
10	1.379	1.345	1.295	0.858	1.173	1.515	0.962	1.047	1.139	0.905	1.116	1.345	1.024	0.976	0.940
15	1.351	1.374	1.368	0.855	1.181	1.554	0.943	1.068	1.200	0.898	1.127	1.382	1.014	0.986	0.965
20	1.334	1.392	1.414	0.855	1.184	1.570	0.932	1.081	1.237	0.894	1.132	1.400	1.008	0.993	0.981
25	1.322	1.405	1.446	0.857	1.183	1.570	0.926	1.088	1.257	0.893	1.133	1.404	1.003	0.997	0.992
30	1.314	1.413	1.468	0.862	1.177	1.556	0.923	1.092	1.266	0.895	1.131	1.398	1.000	1.000	1.000

Consumption increases above the benchmark level for all growth rates, implying that the consumption-based growth is a viable option for the Chinese economy. The levels of the variables at the end of the 30-year horizon are much lower for the 3% long-run growth rate scenario, compared with 5% and 7% scenarios. As expected, if the Chinese economy grows by 7% in the long run, the macroeconomic gains are remarkable. In the case of consumption, its level at the end of the 30-year horizon is 47% higher than the benchmark level, comparable to 41% for the 5% growth scenario (see Figure 4). For investment, exports, and imports, the levels at the end of the 30-year period are lower than the benchmark levels. This implies that 3% long-run growth rate yields negative macroeconomic outcomes for the economy. 5% long-run growth rate is the baseline scenario, i.e., business as usual. 7% long-run growth rate yields far better macroeconomic outcomes compared to the business as usual scenario.

Figure 4. Results for consumption under different long-run growth scenarios (3%, 5%, and 7%)



Welfare increases naturally due to increasing consumption and positive GDP growth rate in all scenarios. In the case of 3% long-run growth rate, welfare increases by 2.6% whereas it increases by 3.0% for 5% growth rate, and 3.2% for 7% growth rate. The welfare change is not significantly different for 5% and 7% long-run growth rates. Therefore, the relatively low growth rate of 5%, which is expected to prevail in the near future, should not be much of a concern for policymakers.

4. Conclusion

This paper introduces a generic forward-looking dynamic CGE model based on Devarajan and Go (1998) for China. The general structure of the model is fit into macroeconomic data from China. While the purpose of this paper is to present long-run trajectories of the macroeconomic features of the economy, it can be enriched by disaggregating various institutional accounts to study the impact of various policy alternatives for China such as pension reform, changes in trade policies, and tax policies. The results from such analyses will be important not only for China but also for the world economy due to the strong linkages between China and major economies of the world.

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Appendix

Model Equations

Prices

(1) $PC_t \cdot CC_t = PD_t \cdot D_t + PM_t \cdot M_t$

(2) $PM_{C,t} = \varepsilon \cdot PWM_{C,t}(1 + t_{MC,t})$

(3) $PM_{I,t} = \varepsilon \cdot PWM_{I,t}(1 + t_{MI,t})$

(4) $PM_{K,t} = \varepsilon \cdot PWM_{K,t}(1 + t_{MK,t})$

(5) $PQ_t \cdot Q_t = PD_t \cdot D_t + PE_t \cdot E_t$

(6) $PE_t = \dfrac{\varepsilon \cdot PWE_t}{1 + t_{E,t}}$

(7) $P_t = PC_t(1 + t_{C,t})$

(8) $P_{VA,t} Q_t = \sum_f P_{f,t} F_{f,t}$

(9) $P_{VA,t} = \dfrac{PQ_t}{1 + s} - \beta_{MI} P_{MI,t}$

(10) $P_{K,t} = (\beta_{MK} PM_{K,t} + (1 - \beta_{MK}) PC_t)(1 + t_{C,t})$

(11) $R_{E,t} = \dfrac{PE_t}{PD_t}$

(12) $R_{M,t} = \dfrac{PM_{C,t}}{PD_t}$

Production and trade

(13) $Q_t = \gamma_{VA} \sum_f \beta_f F_{f,t}{}^{-\frac{1}{\rho_f}}$

(14) $\frac{K_t}{L_t} = \left(\frac{w_t}{r_t} \cdot \frac{\beta_K}{1-\beta_K}\right)^{\rho_f - 1}$

(15) $Q_t = \gamma_Q \left[\beta_E E_t^{\rho_Q} + (1 - \beta_E) D_t^{\rho_Q}\right]^{\frac{1}{\rho_Q}}$

(16) $\frac{E_t}{D_t} = \left(\frac{PE_t}{PD_t} \cdot \frac{1-\beta_E}{\beta_E}\right)^{\frac{1}{\rho_Q - 1}}$

(17)

(18) $CC_t = \gamma_{CC}\left[\beta_M M_{C,t}^{-\rho_{CC}} + (1 - \beta_M) D_t^{-\rho_{CC}}\right]^{-\frac{1}{\rho_{CC}}}$

(19) $\frac{M_{C,t}}{D_t} = \left(\frac{PD_t}{PM_{C,t}} \cdot \frac{\beta_M}{\beta_M}\right)^{\frac{1}{1+\rho_{CC}}}$

(20) $M_{I,t} = \beta_{MI} Q_t$

(21) $M_{K,t} = \beta_{MK} V_t$

System constraints

(22) $L_{D,t} = L_{S,t}$

(23) $CC_t = V_t(1 - \beta_{MK}) + C_t + G_t$

Institutions

(24) $Y_{f,t} = \sum_f P_{f,t} F_{f,t}$

(25) $Y_{H,t} = \sum_f YF_{f,t} + P_{C,t} TR_{GH,t} + \varepsilon(TR_{WH,t} - i \cdot DBT_t)$

(26) $Y_t = (1 - t_H)(YH_t - P_{K,t} V_t + \varepsilon B_t + S_{G,t})$

(27) $Y_{G,t} = \varepsilon(PWM_{C,t} t_{MC,t} M_{C,t} + PWM_{I,t} t_{MI,t} M_{I,t}$
$+ PWM_{K,t} t_{MK,t} M_{K,t} + PWE_t t_{E,t} E_t)$
$+ t_{C,t} PC_t (C_t + G_t + V_t)$
$+ t_H (YH_t - P_{K,t} V_t + \varepsilon B_t + S_{G,t})$

(28) $S_{G,t} = Y_{G,t} + \dfrac{s_t}{1 + s_t} PQ_t \cdot Q_t + PC_t TR_{GH,t} - P_t G_t$

(29) $PWM_{C,t} M_{C,t} + PWM_{I,t} M_{I,t} + PWM_{K,t} M_{K,t} + i \cdot DBT_t -$
$PWM_{E,t} E_t = B_t + TR_{WH,t}$

Intertemporal consumption

(30) $\dfrac{C_{t+1}}{C_t} = \left(\dfrac{P_{t+1}(1 + \rho)}{P_t(1 + r_{C,t+1})}\right)^{-\frac{1}{\theta}}$

(31) $U_t = \dfrac{C_t^{1-\theta}}{1 - \theta}$

Intertemporal investment

(32) $\dfrac{I_t}{K_t} = b q_t$

(33) $q_t = \dfrac{\widetilde{P_{K,t}}}{P_{K,t}} - 1 + t_{I,t}$

(34) $r_{Q,t} \widetilde{P_{K,t}} = R_t + \dfrac{\widetilde{P_{K,t+1}} - \widetilde{P_{K,t}}}{\widetilde{P_{K,t}}} - \delta_K \widetilde{P_{K,t+1}}$

(35) $V_t = (1 - t_{I,t} + ADJ_t) I_t$

(36) $ADJ_t = \dfrac{I_t / K_t}{2b}$

(37) $K_t = I_{t-1} + (1 - \delta_K) K_{t-1}$

(38) $R_t = MPK_t + P_{K,t} \cdot \dfrac{b}{2} \cdot (I_t / K_t)^2$

Growth dynamics

(39) $r_{Q,t} = i_t + \frac{R_{E,t+1} - R_{E,t}}{R_{E,t}}$

(40) $r_{C,t} = i_t + \dfrac{R_{M,t+1} - R_{M,t}}{R_{M,t}}$

(41) $DBT_t - DBT_{t-1} = B_t$

Terminal conditions

(42) $\dfrac{I_t^*}{K_t^*} = \delta_K + g_Y$

(43) $P_t^* C_t^* = Y_t^*$

(44) $r_{Q,t} = r_{C,t} = i$

Note: The terms with asterisk (*) refer to terminal period values.

Glossary

Time-varying variables and parameters		Exogenous parameters	
ADJ_t	Investment adjustment function	b	Parameter in investment adjustment function
B_t	Borrowing from abroad (net)	β_E	Share of exports in gross output
C_t	Private consumption	β_f	Value-added factor shares (β_K: K share), $\sum_f \beta_f = 1$
CC_t	Armington composite commodity	β_{MC}	Share of consumer good imports in composite good
D_t	Domestic sales	β_{MI}	Share of intermediate imports in composite good
DBT_t	Aggregate foreign debt	β_{MK}	Share of capital good imports in composite good
E_t	Exports	δ_K	Depreciation rate of capital
$F_{f,t}$	Factor endowment, $f = K$ (capital), L (labor)	ε	Nominal exchange rate
G_t	Government consumption	γ_{CC}	Shift coefficient for CES function
I_t	Real investments	γ_Q	Shift coefficient for CET function
K_t	Capital stock	γ_{VA}	Shift coefficient for CES value-added function
$L_{D,t}$	Labor demand	g_Y	Long-run potential growth rate
$L_{S,t}$	Labor supply	i	World interest rate
M_t	Imports: consumer goods ($M_{C,t}$), intermediate goods ($M_{I,t}$), capital goods ($M_{K,t}$)	ρ	Rate of time preference for consumers
MPK_t	Marginal product of capital	ρ_{CC}	Elasticity coefficient for Armington CES function
P_t	Consumer prices	ρ_f	Elasticity coefficient for value-added function
$\widetilde{P_{K,t}}$	Shadow price of capital	ρ_Q	Elasticity coefficient for the CET function
$P_{VA,t}$	Price of value added	θ	Discount factor in the utility function
PC_t	Price of composite commodity produced		
PD_t	Price of domestic sales		
PE_t	Export price		
$PF_{f,t}$	Return to production factors f (w_t: wage, r_t: rate of return to capital)		
PM_t	Import prices: consumer goods ($PM_{C,t}$), intermediate goods ($PM_{I,}$), capital goods ($PM_{K,t}$)		
PQ_t	Gross output price		
PWE_t	World prices of exported goods		
PWM_t	World prices: consumer goods ($PWM_{C,t}$), intermediate goods ($PWM_{I,}$), capital goods ($PWM_{K,t}$)		
Q_t	Gross output		
q_t	Tobin's q		
$r_{C,t}$	Discount rate for consumption		
$r_{Q,t}$	Discount rate for output		
R_t	Rate of return to capital		
$R_{E,t}$, $R_{M,t}$	Real exchange rate for exports (E) and imports (M)		
s_t	Rate of subsidy		
$S_{G,t}$	Public savings		
$t_{C,t}$	Sales tax rate		
$t_{E,t}$	Taxes on exports		
$t_{I,t}$	Investment tax credit		
$TR_{GH,t}$	Government transfers to households		
$TR_{WH,t}$	Foreign transfers to households		
U_t	Utility		
V_t	Investment spending including adjustment costs		
Y_t	Total household income		
$Y_{f,t}$	Aggregate income of factor f		
YG_t	Government income		
YH_t	Private income		

Note: f: production factors: K: capital, L: labor), t: time. The terms with asterisk (*) refer to terminal period values.