DYNAMICS MODEL OF MOVABLE BODY-TYPE WAVE ENERGY CONVERTER CONSIDERING TWO DIMENSIONAL MOTIONS OF THE FLOAT

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Dynamics model has been proposed for the float-counterweight type wave energy converter which takes into account the vertical and horizontal forces on the energy extracting float. The model consists of the equation of the generator, force balance at stationary free state, equation of the float motion in operation, and the equation for the driving pulley motion. Second order simultaneous differential equations for the vertical and horizontal displacement of the float have been obtained as the equations to be solved. Components of the flow force have been evaluated from the linear progressive wave theory. Examination of the model using the experimental data shows that the model underestimates the heaving and surge, overestimates the wire tension, and gives relatively good agreement to energy gain.

Key Words : *heave, surge, mechanical work rate, float, counterweight, wire tension*

1. INTRODUCTION

A rising demand for energy coupled with the problem of environmental pollution has led to investigations into potential new energy resources. Wave energy represents one of the most dependable and predictable sources of renewable energy available, which is free from the variations present in wind or solar energy. Takahashi et. al. have evaluated that the wave energy available in the coast of Japan is about 35 millions kW in average.[1] Various mechanisms for extracting wave energy have been developed but not fully realized due to structural strength and economical problems. The comprehensive explanation has been given by several authors[2),3),4)].The major systems of wave energy conversion seem to be the OWC and the movable body type. The OWC system has air chamber in which water surface elevation and air pressure vary by the action of the water wave. The energy of air pressure is converted into unidirectional rotation of a generator using bi-directional turbines. This system has been developed actively by many investigators (Evans[5]), Malmo & Reiten[6]), Folley et al.[7]), Suzuki et al.[8])). The low efficiency of energy conversion and the high cost of construction, particularly of the air chamber, are pointed out as major problems. On the other hand, movable body type extracts wave energy through the motion of the body caused by the wave motion. Gustafson & Loqvist[9]), Solell[10]), Tornabene[11]) etc. have proposed different movable type devices. The structural strength problem is the major drawback of this system.

Hadano et al.[12]) have proposed a movable body type, which utilizes the heaving motion of a float to rotate a shaft and drive an electric generator shown in **Fig.1**. This system extracts wave energy through the weight of the float. The major advantage of this system is flexibility, which is due to the utilization of a cable as the medium for energy transfer from the waves to the energy conversion component located above water level. Also with this, a major part of the structural strength problem has been solved. In **Fig.1**, the energy extracting float is located in a water column formed by vertical fence constructed in front of the vertical harbor structure in order to avoid the collision with the structure. The hint for this idea was given by Nakamura et al.[4)]

The authors were interested in the working condition of the device including energy gain and safety of the structure. Safety of the structure specifically concerns with preventing the collision of the float with the wall of the structure which supports the device when it is in operation. As a physical phenomenon, standing wave will occur in front of the vertical structure such as wave breakers. In this case, the normal component of the velocity of water particle will diminish near the vertical structure, so the collision of the float and the structure does not seem to occur. But the proper understanding has not been given as for the phenomenon which occurs in real environment. In order to consider this phenomenon systematically, the authors have developed a dynamics model which considers the heave and surge motions of the float due to the movement of water generated by the plane progressive waves. It is essentially the same as the previous one[12)], and is constructed of the physics of the generator, force balance at stationary free state, equation of float motion in two directions and driving pulley motion. The equation of the float considers various situations of float submergence. The validation of the model is checked by the result of experiment performed in a wave tank.

2. DYNAMICS MODEL

For the full understanding of the dynamics model, we describe the operational mechanism of the device here. As shown in **Fig.1**, the device mainly consists of a float, counterweight, cable, driving pulley, ratchet, gearbox and generator. The mechanism of energy transfer is basically the conversion of the motion of the float mass into a rotational motion of the shaft connected to the electric generator. The ratchet mechanism converts the bi-directional rotation of the driving pulley into a unidirectional rotation of the shaft which is then accelerated by the gearbox. In principle, the system can extract energy both when the float is moving up and down corresponding to the rise and fall of the water level. But as the weight of the counterweight is much less than that of the float, it does not generate enough torque to rotate the driving pulley connected to electric generator when the water level is rising. This causes the cable on the float side to slacken. As a result of this,

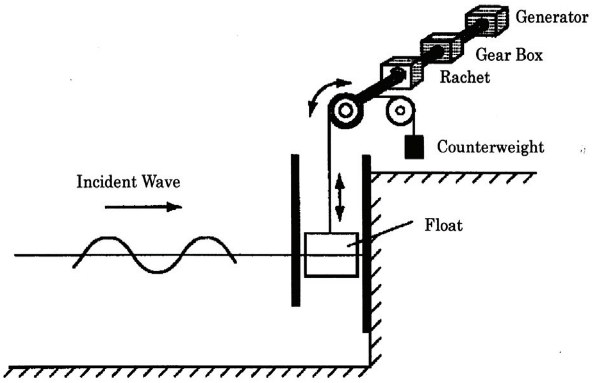
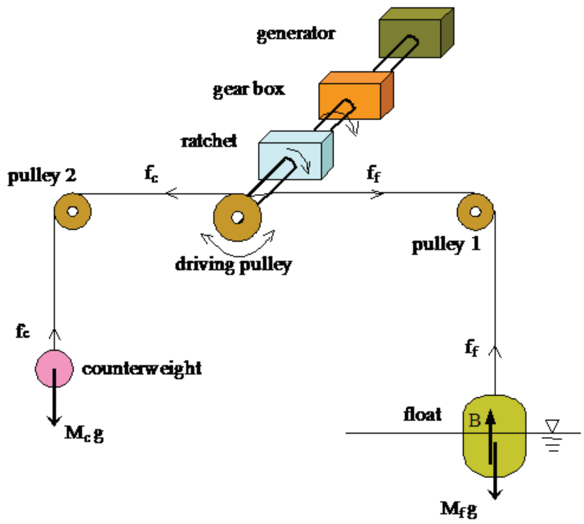
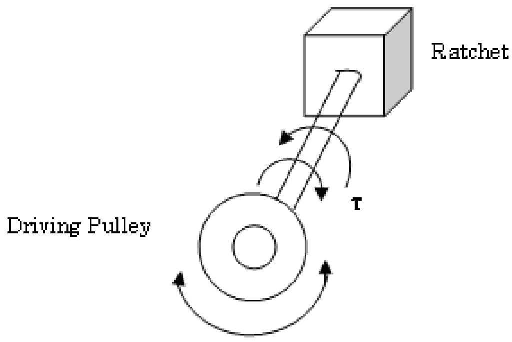


Fig.1 Schematic diagram of the device.



(a) Sketch of the device



(b) Definition of the sense of the torque

Fig. 2 Schematic figure for mathematical modeling.

the device experiences a sudden and a very high tensile force due to the weight of the float when the float turns to fall subsequently. This affects the over all safety of the device as well as brings large variation in the power output.Therefore the device is designed such that the generator works only when the float is falling and the shaft moves freely when the float is rising. Description for energy gain presented in this paper are only for those cases where the float is falling. Although not discussed in this paper, the authors have also proposed the use of tension pulley, a combination of a spring and pulley to counter the problem of the occurrence of the sudden tensile force and to stabilize the output power[13]. **Fig.2** shows the schematic figure for mathematical modeling.

In this section, a mathematical model of the physical process of energy conversion is discussed considering the heave and the surge motions of the float. Although heave is the primary source of energy generation for the proposed system, the authors are interested to calculate the surge to determine the minimum clearance between the float and the wall of the structure necessary to avoid collision. Effects of pitch, roll, yaw and sway are assumed small compared to the heave and the surge and therefore not included in the analysis owing to the complexity they bring .

(1) Equations of the generator

If θ is the angle of rotation of the driving pulley in anticlockwise direction, the torque that the driving pulley receives from the generator in anticlockwise direction, τ , and the potential difference between the two terminals of the generator, e, are given by Eq. (1) and Eq. (2) respectively.

$$\tau = Gk_\tau i \cdot \frac{1}{2}\left\{\left|\mathrm{sgn}(\dot{\theta})\right| - \mathrm{sgn}(\dot{\theta})\right\} \tag{1}$$

$$e = Gk_e \dot{\theta} \cdot \frac{1}{2}\left\{\left|\mathrm{sgn}(\dot{\theta})\right| - \mathrm{sgn}(\dot{\theta})\right\} \tag{2}$$

where, $\dot{\theta}$ is the angular velocity of the driving pulley, i is the current flowing in the coil of the generator, sgn(x) =1 for x > 0 and -1 for x < 0, G is the total gear ratio from the driving pulley to the generator, k_τ is the torque constant, k_e is the induced voltage constant. Here the term in the brackets multiplied by half describes the effect of ratchet mechanism. Its value is one when the rotation of the pulley is clockwise and zero when the rotation of the pulley is counterclockwise. Eqs. (1) and (2) indicate that the driving pulley receives an anticlockwise torque from the generator when the float is falling.

As shown in **Fig. 2(b)**, positive torque is the one which occurs when the driving pulley has rotational force to rotate the shaft in clockwise direction, and as its reaction the ratchet box has a rotational force to rotate the shaft in anticlockwise direction.

(2) Force balance at stationary free state

The left part of **Fig.3** shows the situation of float and water surface at stationary condition without work, and the right part of the figure shows their situation at an arbitrary time when the system is working indicating the heave and the surge motions.

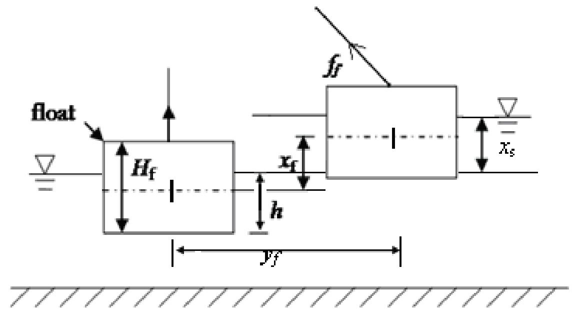


Fig. 3 Sketch of the partially submerged float.

Eq.(3) gives the equilibrium equation of the force in non wavy stationary condition for the circular cylindrical float chosen for this work.

$$M_c g + \frac{1}{4}\pi d_f^2 \rho_w h g = M_f g \tag{3}$$

Where, M_f: mass of the float, M_c: mass of the counterweight, d_f: diameter of the float, h : submerged height of the float in this equilibrium and ρ_w : density of water.

(3) Equations of float motion

The total fluid force acting on the cylinder has been calculated using the modified Morrison's formula as given below[12),14)] .

$$F = \frac{1}{2}\rho_w C_d A_p \left|U - \dot{x}_B\right|(U - \dot{x}_B) - \rho_w V C_a \ddot{x}_B \tag{4}$$

Where, C_d: drag coefficient, C_a: added mass coefficient, U: fluid velocity, x_B: position of body with each dot representing its time derivative, V: submerged volume of the cylinder.

Depending on the wave conditions and system dimensions, the float will be either partially submerged, fully submerged or completely out of the water, necessitating that the mathematical model be able to respond to all these conditions as and when they arise. For simplicity, each of these conditions is dealt separately and the corresponding equations are developed, rather than a single universal equation covering all the conditions.

a) Float partially submerged $(0 \le h + x_s - x_f \le H_f)$

The equation of motion in the vertical direction is given by

$$M_f \ddot{x}_f = f_f \cos\alpha + \frac{1}{4}\pi d_f^2 \rho_w (h + x_s - x_f) g$$

$$-M_f g + \frac{1}{8} C_d \rho_w \left| \dot{x}_w - \dot{x}_f \right| \left(\dot{x}_w - \dot{x}_f \right) \pi d_f^2$$
$$-\frac{1}{4} C_a \pi d_f^2 \rho_w \left(h + x_s - x_f \right) \ddot{x}_f \qquad (5)$$

The equation of motion in the horizontal direction is given by

$$M_f \ddot{y}_f = -f_f \sin\alpha + \frac{1}{4} \pi d_f^2 \rho_w (h + x_s - x_f) g$$
$$-M_f g + \frac{1}{8} C_d \rho_w \left| \dot{y}_w - \dot{y}_f \right| \left(\dot{y}_w - \dot{y}_f \right) \pi d_f^2$$
$$-\frac{1}{4} C_a \pi d_f^2 \rho_w \left(h + x_s - x_f \right) \ddot{y}_f \qquad (6)$$

b) Float wholly submerged $(h + x_s - x_f > H_f)$

Equation for vertical motion is given by

$$M_f \ddot{x}_f = f_f \cos\alpha + \frac{1}{4} \pi d_f{}^2 H_f \rho_w g - M_f g$$
$$+\frac{1}{8} C_d \rho_w \pi d_f{}^2 \, | \dot{x}_w - \dot{x}_f \, | (\dot{x}_w - \dot{x}_f)$$
$$-\rho_w \frac{\pi}{4} d_f{}^2 H_f C_a \ddot{x}_f \qquad (7)$$

Equation for horizontal motion is given by

$$M_f \ddot{y}_f = -f_f \sin\alpha + \frac{1}{2} C_d \rho_w H_f d_f$$
$$\cdot | \dot{y}_w - \dot{y}_f \, | (\dot{y}_w - \dot{y}_f) - \rho_w \frac{\pi}{4} d_f{}^2 H_f Ca \, \ddot{y}_f \qquad (8)$$

c) Float suspended in air $(h + x_s - x_f < 0)$

Equation for vertical motion is given by

$$M_f \ddot{x}_f = f_f \cos\alpha - M_f g \qquad (9)$$

Equation for horizontal motion is given by

$$M_f \ddot{y}_f = -f_f \sin\alpha \qquad (10)$$

where, H_f: height of the float, C_d: drag coefficient, C_m: (1+C_a), f_f: tensile force in the cable supporting the float, x_f , y_f: vertical and horizontal displacement of the float measured from the stationary free state as shown in **Fig. 3**, x_s: vertical displacement of the water surface, and x_w, y_w: vertical and horizontal displacement of the water particle.

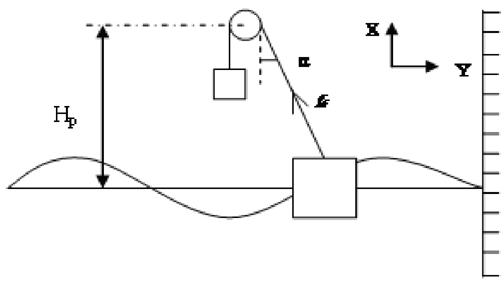


Fig.4 Schematic of the float motion.

(4) Equations for the driving pulley motion

The equation of the rotation of the driving pulley considering all the rotating components is as follows

$$I \frac{d^2\theta}{dt^2} + C \frac{d\theta}{dt} = \tau + (f_c - f_f) R_m \qquad (11)$$

where, I: the mass moment of inertia of rotating bodies, C : viscous damping coefficient caused by friction between the mechanical parts, R_m : radius of driving pulley, f_c: tensile force of the wire supporting the counterweight evaluated from Eq. (12).

$$f_c = M_c \left(g + \ddot{x}_c \right) \qquad (12)$$

where, $\ddot{x}_c$ is the upward acceleration of the counterweight.

Eq. (1) and Eq. (2) can be combined to write the torque as follows

$$\tau = -\frac{G^2}{r} k_\tau k_e \frac{d\theta}{dt} \qquad (13)$$

where, the expression $e = i \cdot r$ is applied, in which r is the internal resistance of the generator.

(5) Combination of equations

Eqs. (1),(2) (12) and (13) need to be expressed in terms of the float motion components x_f and y_f before combining with the equations of float motion. From **Fig.4**, the change in the length of the wire, ΔS , from the stationary free state at any instant due to the motions of the float can be written as

$$\Delta S = S' - S_0 = \sqrt{(H_p - x_f)^2 + y_f^2} - H_p \qquad (14)$$

Where, S_0: length of the wire on the float side at stationary free condition equal to H_p, S': length of the wire at any instant in operation. Also, stretching of the wire is ignored, i.e. the wire is assumed to have an infinitely large value of elastic modulus. Thus, the angle between the vertical and the direction of the wire tension can be written as Eq.(15).

$$\alpha = \tan^{-1}\left(\frac{y_f}{H_p - x_f}\right) \quad (15)$$

Finally, the angle of rotation of the driving pulley counterclockwise and the vertical displacement of the counterweight can be expressed as Eq.(16) and (17) respectively.

$$\theta = -\frac{\Delta S}{R_m} = -\frac{1}{R_m}\left[\sqrt{(H_p - x_f)^2 + y_f^2} - H_p\right] \quad (16)$$

$$x_c = \Delta S = \left[\sqrt{(H_p - x_f)^2 + y_f^2} - H_p\right] \quad (17)$$

To obtain the equation for the wire tension, f_f, in terms of x_f and y_f, Eqs.(12) and (13) are substituted into Eq. (11) first and then Eq.(16) is used for θ.

$$f_f = \left(M_c + \frac{I}{R_m^2}\right)\left[\left\{(H_P - x_f)^2 + y_f^2\right\}^{-\frac{1}{2}}\right] \cdot \left[\left\{\dot{x}_f^2 + \dot{y}_f^2 - (H_P - x_f)\ddot{x}_f + y_f\ddot{y}_f\right\}\right] - \left(M_c + \frac{I}{R_m^2}\right)\left[\left\{(H_P - x_f)\dot{x}_f - y_f\dot{y}_f\right\}^2\right] \cdot \left\{(H_P - x_f)^2 + y_f^2\right\}^{-\frac{3}{2}} - \left(\frac{C}{R_m^2} + \frac{G^2 k_\tau k_e}{R_m^2 r}\right)\left\{(H_P - x_f)\dot{x}_f - y_f\dot{y}_f\right\} \cdot \left\{(H_P - x_f)^2 + y_f^2\right\}^{-\frac{1}{2}} + M_c g \quad (18)$$

Substitution of Eqs.(18) and (15) into Eqs.(5)~(10) results in the corresponding equations of motion which will have x_f, y_f and their derivatives as the only unknowns. These equations take the form of a pair of simultaneous equations as given below.

$$a_1\ddot{x}_f + b_1\ddot{y}_f = C \quad (19)$$

$$a_2\ddot{x}_f + b_2\ddot{y}_f = C_2 \quad (20)$$

Where a_1, a_2, b_1 and b_2 contain terms of x_f and y_f, and C_1 and C_2 further contain their time derivative terms. The equations for the partially submerged float can also be applied to the wholly submerged float and the suspended float with a little modification, and duplications of these equations are not necessary. However for the purpose of clarity of presentation, these coefficients are expressed individually for each condition of float submergences, instead of writing the equations (19) and (20) in their entirety.

a) Float partially submerged

$$a_1 = M_f - \left(M_c + \frac{I}{R_m^2}\right)\left\{(H_P - x_f)^2 + y_f^2\right\}^{-1} (H_P - x_f)^2 + \rho_w\frac{\pi}{4}d_f^2(h + x_s - x_f)C_a \quad (21)$$

$$b_1 = -\left(M_c + \frac{I}{R_m^2}\right)y_f(H_P - x_f) \left\{(H_P - x_f)^2 + y_f\right\}^{-1} \quad (22)$$

$$C_1 = \left(M_c + \frac{I}{R_m^2}\right)\left[\left(\dot{x}_f^2 + \dot{y}_f^2\right)(H_P - x_f)\right] \cdot \left[\left\{(H_P - x_f)^2 + y_f^2\right\}^{-1}\right] - \left(M_c + \frac{I}{R_m^2}\right)\left[\left\{(H_P - x_f)\dot{x}_f - y_f\dot{y}_f\right\}^2\right] \cdot \left[(H_P - x_f)\left\{(H_P - x_f)^2 + y_f^2\right\}^{-2}\right] - \left(\frac{C}{R_m^2} + \frac{G^2 k_\tau k_e}{R_m^2 r}\right)\left\{(H_P - x_f)\dot{x}_f - y_f\dot{y}_f\right\} \cdot (H_P - x_f)\left\{(H_P - x_f)^2 + y_f^2\right\}^{-1} + M_c g(H_P - x_f)\left\{(H_P - x_f)^2 + y_f^2\right\}^{-1} - M_f g + \frac{1}{4}\pi d_f^2(h + x_w - x_f)\rho_w g + \frac{1}{8}C_d\rho_w\pi d_f^2\left|\dot{x}_w - \dot{x}_f\right|(\dot{x}_w - \dot{x}_f) \quad (23)$$

$$a_2 = -\left(M_C + \frac{I}{R_m^2}\right)(H_P - x_f)y_f \cdot \left\{(H_P - x_f)^2 + y_f^2\right\}^{-1} \quad (24)$$

$$b_2 = M_f + \rho_w\frac{\pi}{4}d_f^2(h + x_s - x_f)C_a +$$

$$\left(M_C+\frac{I}{R_m^2}\right)y_f^2\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-1} \tag{25}$$

$$C_2=-\left(M_c+\frac{I}{R_m^2}\right)\left(\dot{x}_f{}^2+\dot{y}_f{}^2\right)(y_f)\cdot\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-1}$$
$$+\left(M_c+\frac{I}{R_m^2}\right)\left\{(H_P-x_f)\dot{x}_f-y_f\dot{y}_f\right\}^2\cdot\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-2}(y_f)$$
$$+\left(\frac{C}{R_m^2}+\frac{G^2k_\tau k_e}{R_m^2.r}\right)\left\{(H_P-x_f)\dot{x}_f-y_f\dot{y}_f\right\}^{-\frac{3}{2}}y_f$$
$$-M_c g\cdot y_f\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-\frac{1}{2}}$$
$$+\frac{1}{2}C_d\rho_w d_f(h+x_w-x_f)\left|\dot{y}_w-\dot{y}_f\right|(\dot{y}_w-\dot{y}_f) \tag{26}$$

b) Float wholly submerged

$$a_1=M_f-\left(M_c+\frac{I}{R_m^2}\right)(H_P-x_f)^2\left\{(H_P-x_f)^2+y_f^2\right\}^{-1}+\rho_w\frac{\pi}{4}d_f^2(H_f)C_a \tag{27}$$

$$b_1=-\left(M_c+\frac{I}{R_m^2}\right)y_f(H_P-x_f)\left\{(H_P-x_f)^2+y_f\right\}^{-1} \tag{28}$$

$$C_1=\left(M_c+\frac{I}{R_m^2}\right)\left[\left(\dot{x}_f{}^2+\dot{y}_f{}^2\right)(H_P-x_f)\right]\cdot\left[\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-1}\right]$$
$$-\left(M_c+\frac{I}{R_m^2}\right)\left[\left\{(H_P-x_f)\dot{x}_f-y_f\dot{y}_f\right\}^2\right]\cdot\left[(H_P-x_f)\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-2}\right]$$
$$-\left(\frac{C}{R_m^2}+\frac{G^2k_\tau k_e}{R_m^2 r}\right)\left\{(H_P-x_f)\dot{x}_f-y_f\dot{y}_f\right\}$$
$$+M_c g(H_P-x_f)\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-1}\cdot(H_P-x_f)\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-1}$$
$$-M_f g+\frac{1}{4}\pi d_f{}^2(H_f)\rho_w g+\frac{1}{8}C_d\rho_w\pi d_f{}^2\left|\dot{x}_w-\dot{x}_f\right|(\dot{x}_w-\dot{x}_f) \tag{29}$$

$$a_2=-\left(M_c+\frac{I}{R_m^2}\right)(H_P-x_f)y_f\cdot\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-1} \tag{30}$$

$$b_2=M_f+\rho_w\frac{\pi}{4}d_f{}^2(H_f)C_a+\left(M_c+\frac{I}{R_m^2}\right)y_f^2\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-1} \tag{31}$$

$$C_2=-\left(M_c+\frac{I}{R_m^2}\right)\left(\dot{x}_f{}^2+\dot{y}_f{}^2\right)\cdot(y_f)\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-1}$$
$$+\left(M_c+\frac{I}{R_m^2}\right)\left\{(H_P-x_f)\dot{x}_f-y_f\dot{y}_f\right\}^2\cdot\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-2}(y_f)$$
$$+\left(\frac{C}{R_m^2}+\frac{G^2k_\tau k_e}{R_m^2.r}\right)\left\{(H_P-x_f)\dot{x}_f-y_f\dot{y}_f\right\}^{-\frac{3}{2}}(y_f)$$
$$-M_c g y_f\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-\frac{1}{2}}$$
$$+\frac{1}{2}C_d\rho_w d_f H_f\left|\dot{y}_w-\dot{y}_f\right|(\dot{y}_w-\dot{y}_f) \tag{32}$$

c) Float suspended in air

$$a_1=M_f-\left(M_c+\frac{I}{R_m^2}\right)(H_p-x_f)^2\cdot\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-1} \tag{33}$$

$$b_1=-\left(M_c+\frac{I}{R_m^2}\right)(H_p-x_f)y_f\cdot\left\{(H_P-x_f)^2+y_f{}^2\right\}^{-1} \tag{34}$$

$$C_1=\left(M_c+\frac{I}{R_m^2}\right)\left[\left(\dot{x}_f{}^2+\dot{y}_f{}^2\right)(H_P-x_f)\right]$$

$$\cdot\left[\left\{(H_P - x_f)^2 + y_f^{\,2}\right\}^{-1}\right]$$
$$-\left(M_c + \frac{I}{R_m^2}\right)\left[\left\{(H_P - x_f)\dot{x}_f - y_f\,\dot{y}_f\right\}^2\right]$$
$$\cdot\left[(H_P - x_f)\left\{(H_P - x_f)^2 + y_f^{\,2}\right\}^{-2}\right]$$
$$-\left(\frac{C}{R_m^2} + \frac{G^2 k_\tau k_e}{R_m^2 r}\right)\left\{(H_P - x_f)\dot{x}_f - y_f\,\dot{y}_f\right\}$$
$$(H_P - x_f)\left\{(H_P - x_f)^2 + y_f^{\,2}\right\}^{-1} + M_c g (H_P - x_f)$$
$$\cdot\left\{(H_P - x_f)^2 - y_f^{\,2}\right\}^{-\frac{1}{2}} - M_f g \qquad (35)$$

$$a_2 = -\left(M_c + \frac{I}{R_m^2}\right)(H_p - x_f)y_f$$
$$\cdot\left\{(H_P - x_f)^2 + y_f^{\,2}\right\}^{-1} \qquad (36)$$

$$b_2 = M_f + \left(M_c + \frac{I}{R_m^2}\right)y_f^2$$
$$\cdot\left\{(H_P - x_f)^2 + y_f^{\,2}\right\}^{-1} \qquad (37)$$

$$C_2 = -\left(M_c + \frac{I}{R_m^2}\right)\left(\dot{x}_f^{\,2} + \dot{y}_f^{\,2}\right)(y_f)$$
$$\cdot\left\{(H_P - x_f)^2 + y_f^{\,2}\right\}^{-1}$$
$$+\left(M_c + \frac{I}{R_m^2}\right)\left\{(H_P - x_f)\dot{x}_f - y_f\,\dot{y}_f\right\}^2$$
$$\cdot\left\{(H_P - x_f)^2 + y_f^{\,2}\right\}^{-2} y_f$$
$$+\left(\frac{C}{R_m^2} + \frac{G^2 k_\tau k_e}{R_m^2 r}\right)\left\{(H_P - x_f)\dot{x}_f - y_f\,\dot{y}_f\right\}^{-3/2} y_f$$
$$-M_c g y_f\left\{(H_P - x_f)^2 + y_f^{\,2}\right\}^{-\frac{1}{2}} \qquad (38)$$

Equations (19) and (20) are first rearranged so that each contains only one of the second order differential variables, then solved numerically using Runge-Kutta fourth order method to obtain the time series of the heave and surge displacements. Time series of the angle of rotation of the driving pulley and its derivative can now be calculated using Eq.(16) and used to estimate the electric power output from the following equation.

$$P_G = r\cdot i^2 = r\left(\frac{-Gk_e}{r}\dot{\theta}\right)^2 \qquad (39)$$

The mechanical work rate or power is calculated as the product of the wire tension and the velocity of the float in the direction of the wire tension and is given as follows.

$$W.R. = f_f \,.\, \dot{x}_c \qquad (40)$$

where *W.R.* stands for the work rate.

3. EXPRESSIONS OF THE WATER MOTION AND INITIAL CONDITIONS

(1) Expression of the water motion

Here, we define the terms indicating the water motion in Eqs.(5)~(38). Equations given by the linear wave theory, which is applicable to the case where the wave height is small, are used for calculations as the linear theory gives good representation for many features of water wave phenomenon.

The water surface elevation x_s is given by Eq.(41) for progressive waves with wave height H, angular wave number k and angular frequency ω,

$$x_s = \frac{H}{2}\cos(ky - \omega t) \qquad (41)$$

where, y is the horizontal distance and t is time. Horizontal component of the water particle velocity v is given by

$$v = \frac{\pi H}{T}\frac{\cosh k(h_1 + z)}{\sinh kh_1}\cos(ky - \omega t) \qquad (42)$$

This is related to $\dot{y}_w$ in Eq.(26) and (32), but in these equations the value averaged over the submerged depth of the float, h, should be used. The equation is

$$\dot{y}_w = \frac{\pi H}{T}\cdot\frac{1}{kh}\frac{\sinh kh_1 - \sinh k(h_1 - h)}{\cosh kh_1 \sinh kh_1}$$
$$\cdot\cos(ky - \omega t) \qquad (43)$$

Vertical component of the water particle velocity u is given by

$$u = \frac{\pi H}{T}\frac{\sinh k(h_1 + z)}{\sinh kh_1}\sin(ky - \omega t) \qquad (44)$$

This is related to $\dot{x}_w$ in Eq.(23) and (29), but in these equations too the value averaged over the submerged depth of the float should be used. The form is

$$\dot{x}_w = \frac{\pi H}{T} \cdot \frac{1}{kh} \frac{\cosh kh_1 - \cosh k(h_1 - h)}{\cosh kh_1 \sinh kh_1} \cdot \sin(ky - \omega t) \quad (45)$$

In Eqs.(41)~(45), T: wave period, h_1 : water depth, and z: position upward from the stationary water surface level.

(2) Initial conditions

Stationary free conditions of the float are chosen as the initial conditions and are given by the following.

$$x_f(0) = 0, \quad \dot{x}_f(0) = 0 \quad (46)$$

$$y_f(0) = 0, \quad \dot{y}_f(0) = 0 \quad (47)$$

4. VALIDATION OF THE MODEL

(1) Experimental setup[12)]

Experiments were conducted in an artificial wave tank at the Research and Development Center of Mitsubishi Heavy Industries LTD, Nagasaki, Japan. The wave tank used was 3.2 m deep, 30m wide and had an effective length of 160 m. At one side of the longitudinal direction was a wave maker and the model was set at the opposite side so that the float received waves directly. **Fig.5** shows the model set in the wave tank. Floats were supported by idler pulleys mounted at an intermediate position of a beam supported by vertical columns. The experimental apparatus consisted of two pairs of floats and counterweights with the dimensions specified in **Table 1**. The shafts rotate with the same speed everywhere but individual torques are accumulated at points where they are connected to the driving pulleys. Regular waves were produced and the clutch was turned on at some proper time when the wave crest reached the floats. The water level, heave and surge displacements, pitch and roll angles, speed of the float motion in the vertical direction, wire tensile force and torque were simultaneously measured. Wave period and wave heights for which experiments were performed are as follows:1.8s/0.32m, 2.0s/0.25m, 3.0s/0.14m, 3.5s/0.24m, 4.0s/0.27m, 4.5s/0.15m and 5.0s/0.10m.

(2) Inertia and drag coefficients

The present experiment was carried out using two floats of circular cylinders. The central distance between the two floats was 2.5 meters and the floats

Fig.5 Experimental setup.

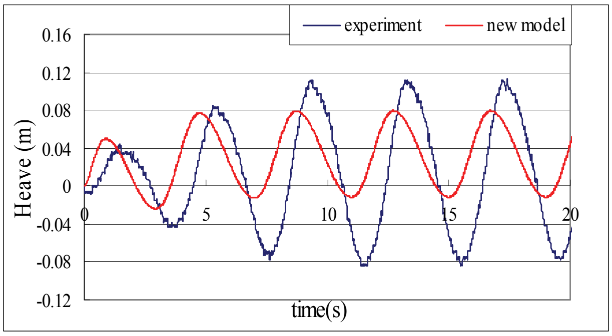
Table 1 Dimensions for experiment.

Float	Density (kg/m^3)	745.7
	Height (m)	0.7
	Diameter (m)	2
	Submergence ratio	0.5714
	Mass (kg)	1680
Driving pulley	Radius (m)	0.18
Inertia	(kg.m$^{2)}$	0.1234
Counterweight	Mass (kg)	150
Gearbox	Gear ratio	41.36
Hp	(m)	1.6

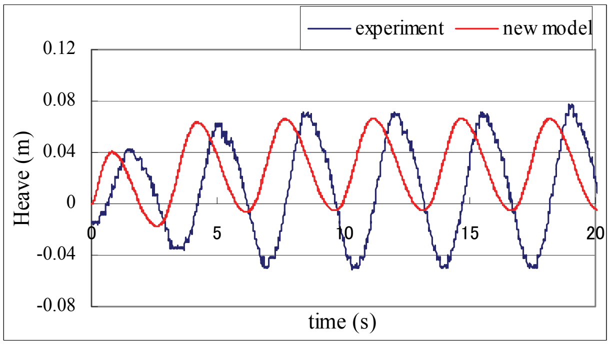
Table 2 Experimental conditions for significant energy conversion.

Experiment No.	Wave Height	Time Period	Average Energy conversion rate
	H(m)	T(s)	(Σwire tension*float speed*dt)/time(W)
1	0.27	4	80
2	0.24	3.5	60

were arranged normal to the wave direction. The ratio of the float diameter to spacing is 0.8. As for the inertia and drag coefficients, several investigators have reported experimental works on the fluid forces acting on the fixed body with the presence of interference with neighboring bodies. Spring & Monkmeyer[15)] have shown that the force on a cylinder is significantly affected by the presence of neighbouring cylinders and their spacing. Chakrabarti[16)] has calculated force ratios for different diameter to spacing ratios in an infinite row of cylinders normal to the wave direction, where the force ratio is the ratio of the maximum force on a cylinder in the row to the corresponding maximum force on the single cylinder. Gibson & Wang[17)] reported experimental works on the added mass of pile group for several arrangements and showed that the added mass coefficient,C_a, increases with increase of the diameter to spacing ratio. From their plot (figure 9 in their paper), the C_a value for diameter to spacing ratio 0.8 is about 4. Therefore C_a value was set to 4 in the calculations for both horizontal and vertical motions. The drag coefficient, C_d, was set to 1 for both vertical and horizontal motions. [18)]

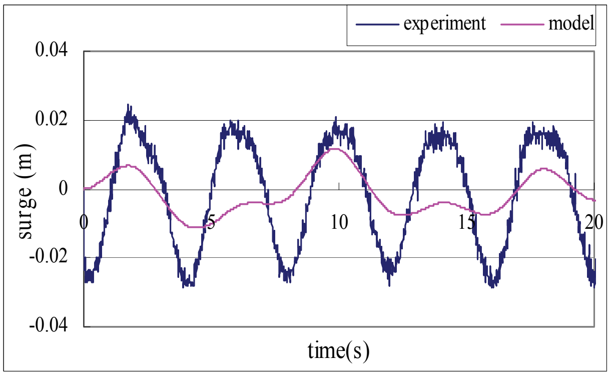


(a) Experiment 1

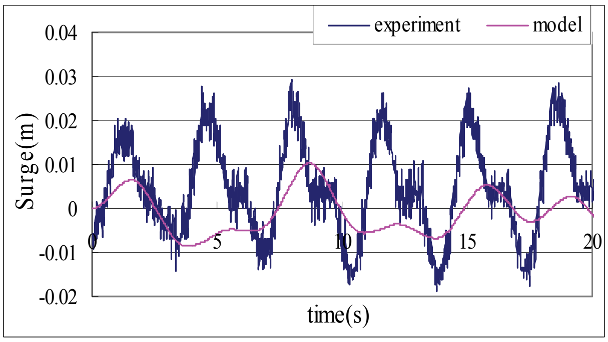


(b) Experiment 2

Fig.6 Time series of heave displacement.

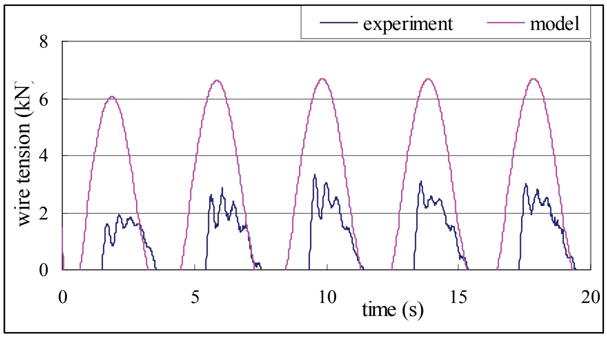


(a) Experiment 1

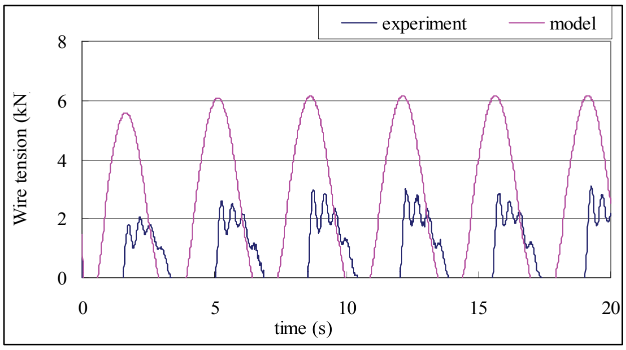


(b) Experiment 2

Fig.7 Time series of surge displacement.

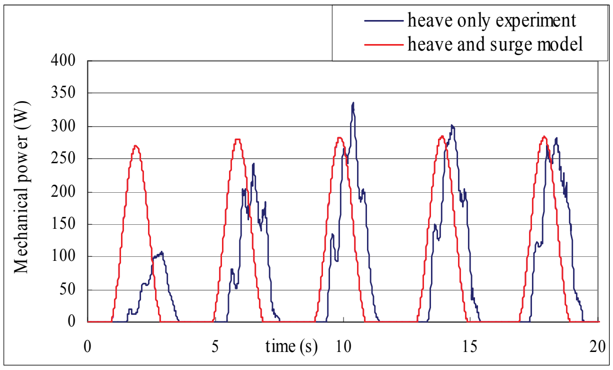


(a) Experiment 1

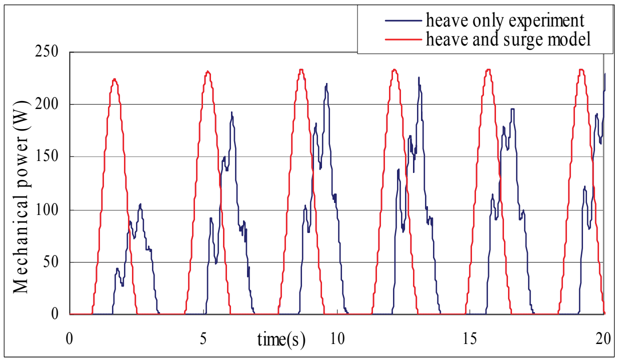


(b) Experiment 2

Fig.8 Time series of wire tension.



(a) Experiment 1



(b) Experiment 2

Fig.9 Time series of work rate.

(3) Comparison of experiment and model results

As a result of experiments when wave period was shorter than 3 seconds, the device could not convert energy significantly due to the large pitching motion of the floats. However for wave conditions 3.5s/0.24m and 4.0s/0.27m, significant power output was observed on account of higher amplitude of heaving of the float. The surge was much smaller than the heave for both conditions.

Figs. 6,7, 8 and **9** show the comparison between the calculated and experiment results of the time series of heave and surge displacements of the float, wire

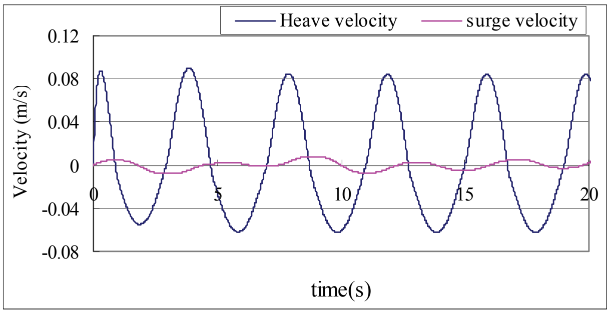


Fig.10 Time series of surge and heave velocities for Exp. 2.

tension and the work rate of the device respectively. These figures show that the model underestimates the heave and the surge by half and overestimates the wire tension. As for the work rate of the device, the model shows a fairly good agreement with experimental results. All the figures show that the model gives a temporal variation of these quantities similar to the experimental results for them. **Fig.10** shows the temporal variation of heave and surge velocities obtained from the model for experiment number two. This figure shows that the velocity of surge is much less than that of heave in this experiment.

5. EFFECTS OF WAVE HEIGHT AND PERIOD IN WORKING CONDITION

In this section, we examine in brief the effects of the wave height and period on the working condition of the device. The dimensions of the device examined are given in **Table.3.** The wave condition is changed as a combination of the wave height (0.5m, 1.0m, 1.5m) and the wave period (5s, 7s, 10s, 12s). The added mass coefficient, C_a=1 and the drag coefficient, C_d=1 are used for both heave and surge assuming the device is isolated and the float receives no interference. **Figs. 11~14** show the result of calculations of maximum heave displacement, maximum surge displacement, maximum wire tension and average electric power respectively as a function of wave height. From the figures, it is found that the maximum value of heave increases almost linearly with the increase of the wave height and that the maximum surge displacement increases with increase of wave height. The increment of the surge increases with increase of wave height. Also the surge for wave period 7s is the largest in the cases tested here. So it seems that this period is near the natural period of surge for this device. From **Fig.13**, it is found that the maximum value of wire tension also increases with increase of wave height.The dependence of the wire tension on the wave height is similar to **Fig. 12**. **Fig. 14** which indicates the relation between electric power and wave height shows the similar tendency with that of **Fig. 13** reflecting

Table 3 Dimensions for prototype.

Items	Values
Mass of float (kg)	10367
Diameter of float (m)	2
Height of float (m)	3
Mass of counterweight (kg)	4570
Radius of driving pulley (m)	0.14
Gear ratio	10
Damping coefficient (N.m.s)	567
Internal resistance (ohm)	0.26
Voltage constant (V/rpm)	1.2838
Torque constant (N.m/A)	1.2838
Heave added mass coefficient	1
Surge added mass coefficient	1
Heave drag coefficient	1
Surge drag coefficient	1
Wave height (m)	1.5
wave period (s)	7
Hp (m)	3

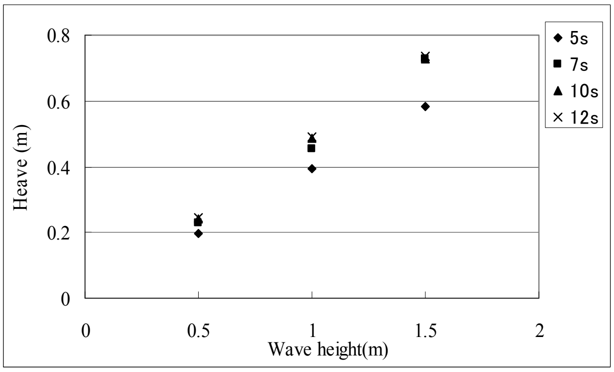


Fig.11 Max. heave displacement at different wave conditions.

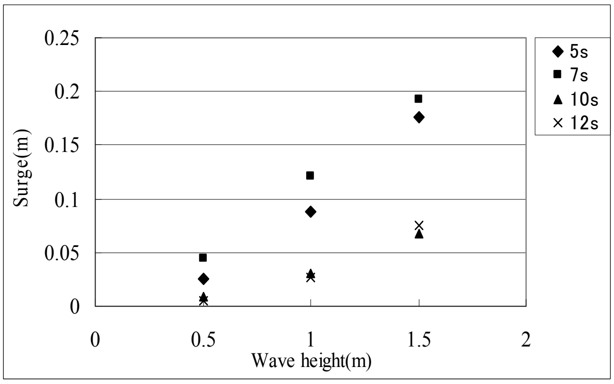


Fig.12 Max. surge displacement at different wave conditions.

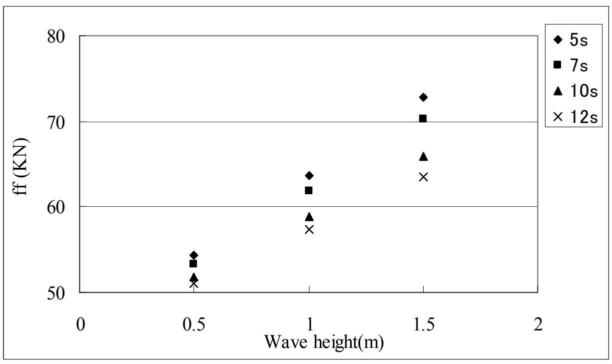


Fig.13 Max.wire tension at different wave conditions.

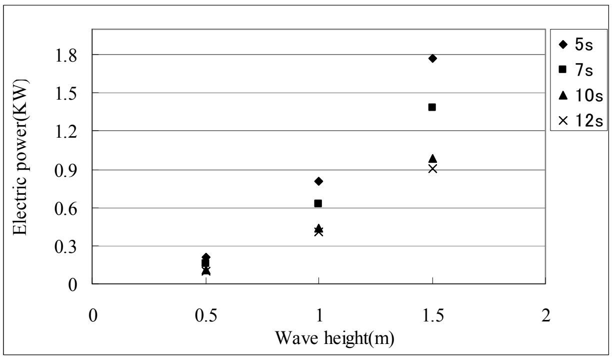


Fig.14 Averaged electric power at different wave conditions.

the strong relation between wire tension and power output.

6. CONCLUSIONS

In this paper, the authors have proposed dynamics model of float counterweight type wave energy converter which takes into account both vertical and horizontal forces on the float due to the water flow in wavy condition. The model consists of the combination of the equation of the generator, force balance at stationary free state, equations of float motion in operation, equation for the driving pulley motion. Finally, simultaneous second order differential equations for vertical and horizontal displacements of the float have been obtained. Vertical and horizontal components of the water particle velocity averaged over the float submergence depth have been evaluated from the linear progressive wave theory in order to give the components of the force acting on the float. The applicability of the model has been examined by comparing with experimental data. As a result, the model underestimates the heave displacement and overestimates the wire tension and gives a relatively good agreement for the energy gain.This feature is the same as the result of the author's previous model. As for the surge, the model gives an underestimation. Though we have used the same values for C_d and C_a both in the vertical and horizontal directions as the first step, these values should be carefully selected referring to experimental works in order to improve the applicability of the model. Also the model should be improved to include the wave force more correctly. As for the case of standing wave occurring near the vertical wall, the authors have started the numerical calculations.

This investigation has been planned as a fundamental to examine the operational condition of the proposed device when it is set in front of the vertical harbour structure where standing wave occurs. The authors have a plan to make experiment for the operation in this situation in the near future.

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