OPTIMAL DIVISION OF SIGNAL-COORDINATED ARTERIAL STREET INTO SUBAREAS

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This study proposes a method for optimally dividing the signal-coordinated arterial street into subareas with different cycle length and describes the features of optimal division showing some examples of calculation. The optimal division into subareas and the optimal cycle length for each subarea are obtained by the DP technique. The optimal offsets for each subarea are also obtained by DP. Therefore, DP is applied two times. Among the three parameters of cycle length, split and offset, the splits are treated as given. A weighted sum of delay and stops is used as a performance index for optimization. Traffic conditions are assumed to be undersaturated and a platoon to be represented by a single rectangular wave. From some examples of calculation, the following has been made clear about optimal division into subareas: if there are no large differences in splits, road and traffic conditions among intersections, division into subareas should not be made. If there are differences in these condition, areas including signals with smaller splits or areas with heavier traffic should be controlled with a longer cycle length than those of the other areas, and the whole street should be divided into subareas according to link lengths and that arrangement.

Key Words : cycle length,offset,delay,Dynamic Programming

1 INTRODUCTION

In coordinated signal control, if the control area is extensive and the number of signals is large, road and traffic conditions lack uniformity. In such cases, to secure the whole control efficiency, it is important to divide the whole control area into subareas and to control them according to the conditions of each subarea. Therefore, a method for optimally dividing the whole control area into subareas needs to be developed [1),2),3)]. This study, which has been made on signal-coordinated arterial street, proposes a method for optimally dividing the whole arterial street into subareas which cycle lengths are different from those of adjacent subareas. Furthermore, the features of optimal division are described showing some examples of calculation. A weighted sum of delay and stops is used as a performance index for optimization. The optimal division into subareas and the optimal cycle length for each subarea are obtained by the DP technique, and the offsets are also optimized by the DP technique [4)]. Therefore, DP is applied two times. Among the three parameters of cycle length, split and offset, the splits are treated as given. Traffic conditions are assumed to be undersaturated and a platoon to be represented by a single rectangular wave.

2 TRAFFIC FLOW MODEL

2.1 Rectangular Model

For optimal division into subareas and offset optimization, a performance index, namely, delay and the number of stops needs to be formulated beforehand. For that purpose, the traffic flow must be modeled.

At signalized intersections, the traffic flow is bunched, forming a platoon. This platoon is a cyclic function with the signal cycle as a cycle. Here, it is assumed that a platoon within a cycle can be represented by a single rectangular wave. **Fig.1** shows a comparison of the platoon flow represented by a single rectangular wave with a uniform flow. In the figure, q is the average flow rate (veh/s), s is the saturation flow rate (veh/s), and T is the cycle length(s). If the platoon length in seconds is represented by L and the flow rate in the platoon by a (veh/ s), the traffic volume Q (veh/cycle) is as follows:

$$Q = qT = aL \qquad (1)$$

where $a \leqq s$ and $L \leqq T$.

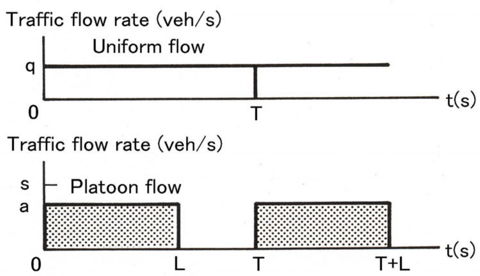

Fig.1 Platoon Flow and Uniform Flow

To take platoon dispersion into consideration, the platoon length L arriving at an intersection is assumed as follows:

$$L = c_c D + L_0 \qquad (2)$$

where L_0 is a departure platoon length in seconds at the adjacent upstream intersection and $L_0 \leqq T$. D is a link length (m) and c_c is the dispersion coefficient (s/m) representing a platoon dispersion. $c_c \geqq 0$. However, in the case of no dispersion, $c_c = 0$

2.2 Transformation Process of Platoon Pattern

A signalized intersection may be regarded as a converter that transforms an arrival platoon pattern into a departure platoon pattern. The platoon is transformed in sequence each time it passes each intersection, but is assumed to always be a single rectangular wave.

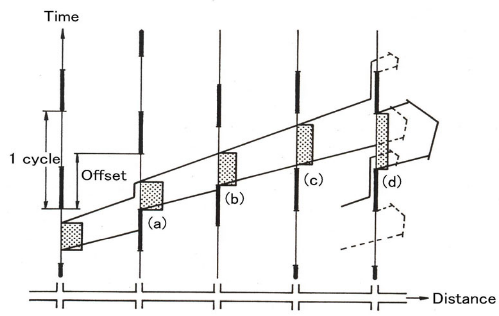

Fig.2 Transformation of Platoon Pattern

As rules for platoon pattern transformation, the 4 types of (a) to (d) are assumed as shown in **Fig.2**. A departure platoon pattern at the k-th intersection is represented by the platoon length λ_k , the ratio of L to the cycle length T, and the position x_k within the green time. The height, traffic flow rate, of the rectangular wave is $a = q / \lambda_k$. The transformation rule (a) is applied to a case where the whole arrival platoon is forced to stop at the red signal and the departure platoon is saturated (a = s). The transformation rule (b) is applied to a case where the front part of the arrival platoon is affected by a red signal and the actual departure platoon is a two-step staircase wave. According to the assumption, the platoon is replaced by a single rectangular wave with the same area. The transformation rule (c) is applied to a case where the platoon does not meet with a red signal. In this case, the arrival platoon pattern becomes a departure platoon pattern unchanged. The transformation rule (d) is applied to a case where the platoon is divided into two parts. According to the assumption of a single rectangular wave, the departure platoon length is set equal to the green time.

Thus, an arrival platoon pattern at an intersection is determined by a departure platoon pattern at the adjacent upstream intersection and a relative offset between both intersections, and the transformation from the arrival platoon pattern into the departure platoon pattern is determined by the transformation rule. The transformation rule in the outbound direction is the same as in the inbound direction.

2.3 Modeling of Right/Left Turn Traffic

In the case where right/left turn traffic exists at intersections, it is modeled as follows: if there is inflow traffic entering from crossing streets by right/left turn at an intersection, this traffic is distinguished as secondary traffic from the straight through main traffic. It is also assumed that this secondary traffic stays with a red signal of the adjacent downstream intersection regardless of the offset and departs from the intersection prior to the main traffic immediately after the green signal begins. Therefore, after the secondary traffic departs in saturation, the main traffic passes the intersection being affected by this secondary traffic, and the main traffic and the secondary traffic joins into new main traffic to move toward the next downstream intersection. In this case, the departure platoon is assumed to always be a single rectangular wave starting from the beginning of green time regardless of the offset.

On the other hands, if there is outflow traffic to crossing streets by right/left turn at an intersection, the traffic flow rate of departure platoon toward downstream is reduced by the amount equal to the outflow rate. It is assumed that the intersection has no right/left turn lane and allows free right/left turn without being interrupted by pedestrians and opposing straight through traffic.

3 DELAY AND NUMBER OF STOPS

3.1 Delay

Since a platoon is represented by a single rectangular wave, the delay function is easily derived. The delay function is derived for the case where the traffic flow rate of the platoon is smaller than the saturation flow rate ($a < s$, $\lambda < 1$). **Fig.3** shows an example of an arrival platoon, departure platoon and queue length, and the relationship among them. Letting $q_1(t, \xi)$ denote the number of vehicles in a queue at time t, the delay for a very short time interval from t to $(t + dt)$ is $q_1(t, \xi)dt$. Therefore, the delay per cycle can be obtained by integrating $q_1(t, \xi)$ over a cycle. This is a function of the arrival position of platoon.

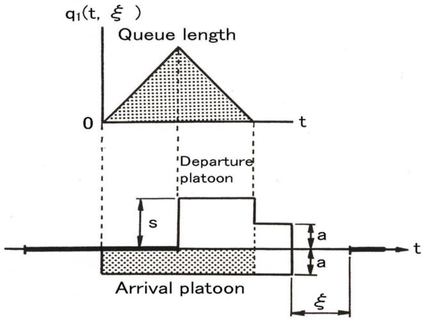

Fig.3 Relationship among Arrival Platoon, Departure Platoon and Queue Length

For example, the delay in Fig.3 is as follows:

$$w(\xi) = \frac{as}{2(s-a)}(\xi + \lambda - g)^2 \quad (3)$$

where

w : Delay per cycle

ξ : Arrival position of platoon

λ : Arrival platoon length

g : Green time

In the expression (3), the time values of ξ , λ and g are ratios to the cycle length.

The delay function must be obtained for ξ in the range of $0 \leqq \xi \leqq 1$. As described above, the delay function is obtained by integrating the queue length. The queue length depends not only on the ξ value but also on the relationship between the λ and g values. For this reason, the delay functions in every case are obtained and those delay functions are summarized in six function of ξ [4]. The delay in veh-s/s can be obtained by multiplying w by the cycle length.

For the case where the flow rate of a platoon is equal to the saturation flow rate ($a = s$, $\lambda \leqq g$), the delay functions can be also obtained.

3.2 Number of Stops

Since a platoon is represented by a single rectangular wave, the function of the number of stops is easily derived. For the case where the flow rate of a platoon is smaller than the saturation flow rate ($a < s$, $\lambda \leqq 1$), the number of stops is specified as the number of arrival vehicles during a red signal or the number of arrival vehicles while a queue exists. **Fig.4** shows an example of relationship between an arrival platoon and a departure platoon. In this case, the number of stops is equal to the area indicated by shadow as follows:

$$S(\xi) = \frac{as}{s-a}(\xi + \lambda - g) \quad (4)$$

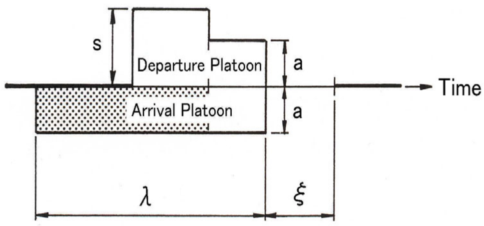

Fig.4 Arrival Platoon and the Number of Stops

The number of stops is the function of ξ . Therefore, the number of stops must be obtained for ξ in the range of $0 \leqq \xi \leqq 1$. The number of stops depends not only on the ξ value but also on the relationship between the λ and g value. For this reason, like the delay function, the function of the number of stops is obtained in every case, and those functions are summarized in six function of ξ . The number of stops per cycle is obtained by multiplying S by the cycle length.

For the case where the flow rate of a platoon is equal to the saturation flow rate

($a = s$, $\lambda \leqq g$), the function of the number of stops is also obtained.

3.3 Calculation for the Case Where Secondary Traffic Exists

The delay and the number of stops for secondary traffic formed by right/left turn at an intersection are not calculated at the adjacent downstream intersection. However, the delay and the number of stops for the main traffic at the adjacent downstream intersections are calculated by assuming that the green time is shortened by the time required for the secondary traffic to depart in saturation flow rate.

If there is outflow traffic to the crossing streets by right/left turn at an intersection, the delay and the number of stops at the adjacent downstream intersection should be calculated for the platoon which flow rate is reduced by the given outflow rate.

4 OFFSET OPTIMIZATION

4.1 Formulation of Offset Optimization

For a coordinated arterial street with undersaturated traffic conditions, the offsets are optimized with a weighted sum of delay and stops as a performance index.

Offset optimization is performed by the DP technique. The application of DP is based on the following thought. If the coordinated arterial street has many signals that are in close vicinity and the traffic conditions are undersaturated , a relative offset of an upstream link affects the platoon pattern of the downstream link . The signalized intersection is a converter that transforms an arrival platoon pattern into a departure platoon pattern and the platoon patterns are transformed in turn according to the offset. Therefore, the problem of offset optimization of the coordinated arterial street may be considered as a multi-stage decision process with the platoon patterns as the state variables and the offsets as the decision variables. For this reason, it is possible to apply the DP technique to solve the problem of offset optimization of a coordinated arterial street.

A study section of arterial street for offset optimization consists of N_s intersections or ($N_s - 1$) links as shown in **Fig.5**.

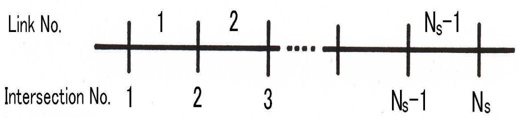

Fig.5 Study Section for Offset Optimization

The performance index , PI , for optimization is a total of the weighted sum of delay and stops, traffic loss, of each link.

The PI can be represented as follows:

$$PI = \sum_{k=1}^{N_s-1} \{ w_k(\lambda_k, x_k, \xi_k) + w'_k(\lambda'_{k+1}, x'_{k+1}, \xi'_k) \} \tag{5}$$

Its purpose is to search for the optimal decision ξ_k^* (k = 1, 2, … $N_s - 1$) that minimizes the PI. In the expression of PI, w_k is the traffic loss caused on the platoon in the inbound direction of the k-th link at the (k+1)-th intersection as shown in **Fig.6** and w'_k is in the outbound direction at the k-th intersection . The traffic loss w_k is defined as follows:

$$w_k = \alpha_w W_k + \alpha_s S_k \qquad (6)$$

where

W_k : Delay time (veh-s/s)

S_k : Number of stops (veh/cycle)

α_w : Weight for delay

α_s : Weight for the number of stops

The delay includes the following random term of delay (in veh-s/s).

$$\text{Random term of delay} = \frac{x^2}{4(1-x)}$$

where

$$x = \frac{q}{sg}$$

$$g = \pi(1 - A/T)$$

π : split

A :loss time(s)

w_k is the function of λ_k, x_k and ξ_k. λ_k is the departure platoon length at the k-th intersection, x_k is the position of departure platoon, and ξ_k is the position of arrival platoon to the (k+1)-th intersection. Each of them is represented by a ratio to the cycle length. w'_k is the function of λ'_{k+1}, x'_{k+1} and ξ'_k.

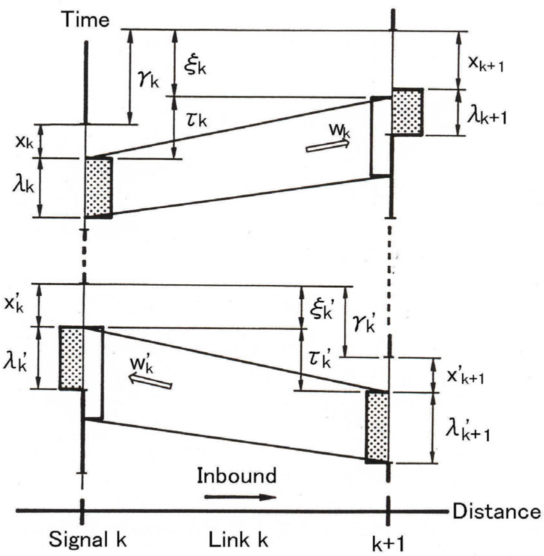

Fig.6 Platoon Patterns and Offsets of Link k

The expression (5) is represented by a functional equation of DP as follows:

$$f_1(\lambda_1, x_1, \lambda'_1, x'_1) = 0 \qquad (7)$$

$$f_{k+1}(\lambda_{k+1}, x_{k+1}, \lambda'_{k+1}, x'_{k+1}) = \min_{\xi_k}\{w_k(\lambda_k, x_k, \xi_k)\xi_k + w'_k(\lambda'_{k+1}, x'_{k+1}, \xi'_k) + f_k(\lambda_k, x_k, \lambda'_k, x'_k)\} \qquad (8)$$

$(k=1, 2, \ldots N_s-1)$

where $f_k(\lambda_k, x_k, \lambda'_k, x'_k)$ is the minimal value of the total traffic loss of (k-1) links from signals 1 to k that can be obtained when the optimal decision ξ_k^* (k = 1, 2, …, k−1) are obtained for the given $\lambda_k, x_k, \lambda'_k$ and x'_k values.

Since the linear expression holds between ξ_k and ξ'_k, it is enough to minimize the expression (8) only about ξ_k. After the optimal policy ξ_k^* (k = 1, 2, …, N_s-1) is obtained from the result of DP calculation, the optimal offset γ_k^* can be found by the following expression:

$$\gamma_k^* = \xi_k^* + \tau_k - x_k \pmod 1 \qquad (9)$$

$(k = 1, 2, \ldots, N_s-1)$

where τ_k is the travel time of the tail end of the platoon in the inbound direction on the k-th link. The expression (9) can be derived from the geometric relationship in Fig.6.

4.2 Calculation Method

DP calculation is made in the order of $f_1, f_2, \ldots, f_{Ns}$ by the expression (7) and (8). Generally, in the calculation of the k-th intersection, the 4 variables of λ_k, λ'_k, x_k, and x'_k are given by the discrete quantity.

In the calculation of the expression (7), f_1 is put the value of 0 for every combination of platoon patterns (λ_1, x_1) in the inbound direction and platoon patterns (λ_1', x_1') in the outbound direction.

In the calculation of the expression (8), f_{k+1} is calculated for every combination of λ_{k+1}, x_{k+1}, λ_{k+1}' and x_{k+1}'. In this case, among every combination of λ_k, x_k, λ_k' and x_k' that both transformation from the platoon pattern (λ_k, x_k) in the inbound direction into (λ_{k+1}, x_{k+1},) and transformation from the platoon pattern (λ_{k+1}', x_{k+1}') of the outbound direction into (λ_k', x_k') may satisfy the platoon transformation rule, ξ_k is determined among the possible ξ_k values so that $\{w_k + w_k' + f_k\}$ may be minimized, and the minimal value is put into f_{k+1}. ξ_k is optimized as a discrete quantity.

After DP calculation is completed up to f_{Ns}, the minimal value is found in $f_{Ns}(\lambda_{Ns}, x_{Ns}, \lambda_{Ns}', x_{Ns}')$ and the optimal decision ξ_{Ns}^* corresponding to it is found. After that, by reversely tracing $f_{Ns-1}, f_{Ns-2}, \ldots, f_2, f_1$, the optimal policy ξ_k^* (k=N_s-1, …, 2, 1) is determined.

4.3 Inflow Patterns from Boundary Intersections and Dispersion Coefficient

The optimal solution is finally determined by giving inflow platoon patterns from the boundary intersections at both ends of the arterial street. Suppose that the inflow patterns from the boundary intersections are as shown in **Fig.7**. In this case, the platoon lengths λ_1 and λ_{Ns}' are assumed to be two times as long as the length between the beginning of the green signal and the center of gravity of the staircase wave. This staircase wave is the departure wave corresponding to uniform arrival traffic. Thus, these inflow patterns are used to obtain the offsets that minimize the weighted sum of delay and stops.

Regarding the dispersion coefficient c_c, though it is better to be determined by observing the traffic flow at the site, a tentative value is given for the present. If the dispersion of the platoon is not so large, c_c may be given the value of 0.

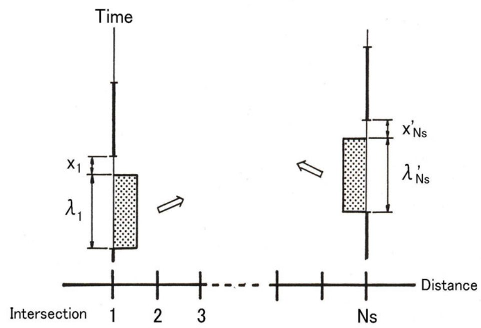

Fig.7 Inflow patterns from Boundary Intersections at Both Ends

In optimizing offsets by DP, various conditions can be rather easily applied to the optimal resolution. Therefore, the offset optimization can be carried out under the following conditions

(1) that the platoon division should be avoided at red signals.

(2) that two consecutive stops of the forefront vehicle at red signals should be avoided, and

(3) that the alternate offset at short link should be avoided.

In addition to the total traffic loss minimization offsets, the through band width maximization offsets, which are the special case of minimizing the performance index PI, can be obtained as the offsets that maximize the traffic volume under the condition of PI= 0 and c_c=0.

4.4 Examples of Optimal Offset Calculation

Fig.8 and **Fig.9** show examples of optimal offsets. The principal calculation conditions are set as follows:

weight for delay $\alpha_w = 1.0$,
weight for the number of stops $\alpha_s = 0.0$,
right/left turn rate = 0.

Fig.8 shows the traffic loss minimization offsets obtained under the condition that platoon division should be avoided. Fig.9 shows the through band width maximization offsets. A through band width of 0.340 is obtained.

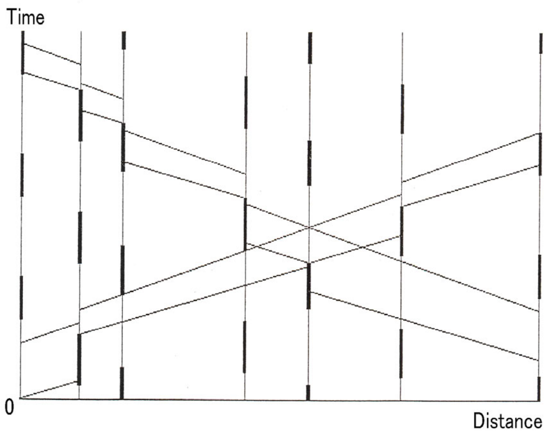

Fig.8 Example of Traffic Loss Minimization Offsets (T = 100 s, q = q'= 0.300 vec/s, c_c = 0.015 s/m)

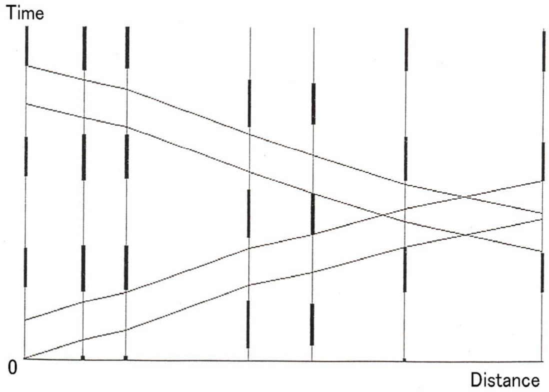

Fig.9 Example of Through Band Width Maximization Offsets (T = 100 s, q = q'= 0.340 veh/s)

5 OPTIMAL DIVISION INTO SUBAREAS

5.1 Formulation of Optimal Division

Suppose that a study section of arterial street to be divided into subareas consists of N_s signalized intersections or (N_s − 1) links as shown in **Fig.10**. The length of each link, overall speed, split(or green time) and minimal cycle length of each signal, and maximal cycle length common to all signals are assumed to be given as calculation conditions. Therefore, it is a question to find the optimal division into subareas and the optimal cycle length and optimal offsets of each subarea. Among them, the optimal offsets are obtained by the DP technique described in Chapter 4.

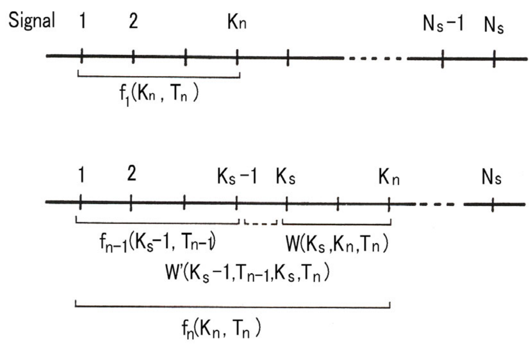

Fig.10 Study Section for Optimal Division into Subareas

The purpose is to optimally divide the arterial street into N_d subareas . The performance index PI for this purpose is a total of weighted sum of delay and stops of each link . The performance index is represented by the following expression:

$$PI = \sum_{k=1}^{N_s-1} \{w_k + w'_k\} \qquad (10)$$

where w_k is the traffic loss in the inbound direction of the k-th link and w'_k is in the outbound direction. The expression (10) can be represented by the functional equations of DP as the expression (11) and (12).

The functional equation for the first stage (the first subarea or n=1) is as follows:

$$f_1(K_n, T_n) = W(K_s, K_n, T_n) \qquad (11)$$

$$(K_n = 2,\ 3,\ \ldots,\ N_s)$$

where K_s and K_n are the signal numbers at both ends of the n-th subarea (K_s = 1 if n = 1) , and $f_1(K_n, T_n)$ and $W(K_s, K_n, T_n)$ are the minimal values of total traffic loss, which can be obtained by optimizing the offset of the subarea with K_n signals from signal K_s to K_n with the cycle length T_n. The cycle length is given in units of 10 s and have to be larger than the maximum of minimal cycle lengths of the signal K_s to K_n and less than the maximal cycle length common to all signals. The offsets are optimized for each cycle length.

For the n-th stage or n subareas, the equation is as follows:

$$f_n(K_n, T_n) = \min_{\substack{2n-1 \leqq K_s \leqq K_n - 1 \\ T_{n-1}(\neq T_n)}} \{f_{n-1}(K_s - 1, T_{n-1}) + W'(K_s - 1, T_{n-1}, K_s, T_n) + W(K_s, K_n, T_n)\} \qquad (12)$$

$$(K_n = 2n,\ 2n+1,\ \ldots,\ N_s)$$

where $f_n(K_n, T_n)$ is the minimal value of total traffic loss that can be obtained by optimizing the cycle length and offsets of each subarea after signal 1 to K_n are optimally divided into n subareas under the condition that the cycle length of the n-th subarea should be T_n. The expression (12) is used for optimization of K_s and T_{n-1}. The range of the n-th subarea is optimized by K_s and the cycle length of the (n-1)-th subarea is optimized by T_{n-1}. $W'(K_s - 1, T_{n-1}, K_s, T_n)$ on the right-hand side of the expression is a traffic loss of the uncoordinated link between the adjacent subarea. The traffic loss can be obtained when the signal ($K_s - 1$) is controlled by the cycle length T_{n-1} and the signal K_s by the cycle length T_n, respectively. In this case, the T_{n-1} value must be different from the T_n value. The delay of the uncoordinated link is calculated by the Webster's delay formula. The number of stops is found on the assumption that the arrival traffic to intersections should be uniform . $W(K_s, K_n, T_n)$ is the minimal value of total traffic loss, which can be obtained by optimizing the offset of the subarea with ($K_n - K_s + 1$) signals from signal K_s to K_n with the cycle length T_n. f_n is found for the cycle length T_n in units of 10s and for K_n in the range of $2n \leqq K_n \leqq N_s$. For the f_{n-1} value, the one which is obtained in the previous stage is used. The relationship among f_{n-1}, W', W and f_n is as shown in Fig.10.

5.2 *Calculation Method*

An arterial street consisting of 7 signals is used as an example of calculation. Road and traffic conditions are given. The minimal cycle length of each signal is supposed to be 80s and the maximal cycle length common to all signals is supposed to be 100s. The optimal cycle length of each subarea is searched in units of 10s. The maximal number of subarea is set 3. The result of the calculation is shown in **Table 1**. The calculation is carried out from first stage (n=1) to the third stage (n= N_d =3) which is the last stage. The optimal solution can be found by reversely tracing the stage from the N_d -th stage .

For coordinated control by uniting all the signals on the arterial street, the optimal solution can be found from the table of n = 1. From the three lines of K_n = 7 in the table, it can be found that the total traffic loss f_1 is 12.317 which is the minimal value when the cycle length is 80 s.

Table 1 Calculation Table for Optimal Division into Subareas by DP

n	K_n	T_n	$f_n(K_n)$	n	K_n	T_n	$f_n(K_n)$	K_{n-1}	T_{n-1}
1	2	80.0	1.326	2	4	80.0	7.682	2	90.0
	2	90.0	0.237		4	90.0	10.517	2	100.0
	2	100.0	0.237		4	100.0	13.023	2	90.0
	3	80.0	5.068		5	80.0	8.963	2	90.0
	3	90.0	4.596		5	90.0	8.973	2	100.0
	3	100.0	3.751		5	100.0	10.559	2	90.0
	4	80.0	5.223		6	80.0	8.813	2	90.0
	4	90.0	5.638		6	90.0	10.624	2	100.0
	4	100.0	6.660		6	100.0	14.669	4	80.0
	5	80.0	9.598		7	80.0	10.020	2	90.0
	5	90.0	7.779		7	90.0	14.232	5	100.0
	5	100.0	6.312		7	100.0	15.711	4	80.0
	6	80.0	9.644	3	7	80.0	15.367	5	90.0
	6	90.0	9.634		7	90.0	16.296	5	80.0
	6	100.0	10.125		7	100.0	18.170	4	80.0
	7	80.0	12.317						
	7	90.0	15.881						
	7	100.0	15.939						

If the arterial street should be divided into two subareas, the optimal solution can be found as follows. First, from the three lines of K_n = 7 in the table of n = 2, select the minimal value of 10.020 of the total traffic loss f_2. Read T_2=80, K_1 = 2 and T_1 = 90. With these results, it is optimal to control signal 1 to 2 with a cycle length of 90s and signal 3 to 7 with a cycle length of 80s, respectively.

For division into three subareas, the optimal solution can be found in the same way. From the three lines of K_n = 7 in the table of n = 3, select the minimal value of 15.367 of f_3. Read T_3=80, K_2 = 5 and T_2 = 90. Next, from the lines of K_2 = 5 and T_2 = 90 in the table of n = 2, obtain K_1 = 2 and T_1 = 100. Therefore, it is optimal to control signal 1 to 2 with a cycle length of 100 s, signal 3 to 5 with a cycle length of 90 s and signal 6 to 7 with a cycle length of 80 s, respectively. In this case, the total traffic loss is f_3 = 15.367.

After a coordinated arterial street is divided into subareas, how to determine the optimal number of divisions is another question . The relationship between the number of divisions N_d and total traffic loss f_n is shown below by using the above example of calculation:

Number of divisions	Total traffic loss
1	12.317
2	10.020
3	15.367

Judging from the above, two divisions are found to be optimal.

5.3 *Example of Calculation for Optimal Division into Subareas*

Fig.11 shows another results of calculation for optimal division into subareas. The calculation conditions are as follows:

Saturation flow rate = 1.0 veh/s

Dispersion coefficient c_c = 0 s/m

Loss time A =12 s

Average speed = 43.2 km/h (common to all links)

Maximal cycle length = 90 s (common to all signals)

Minimal cycle length = 50 s (common to all signals)

Weight for delay α_w = 1.0

Weight for the number of stops α_s = 0. 0

Fig.11 shows the division into subareas and the optimal cycle length of each subarea. The link length (m), split, inbound traffic volume and outbound traffic volume (veh/h) are as shown in the figure. Inflow/outflow traffic by right/Left turn is taken into consideration. The inflow/outflow rates are set to the values that are required to satisfy the traffic volume conditions for each intersection.

Case 1 is an arterial street consisting of 15 signals. In this case, the traffic volume and split is uniform. If the traffic conditions and signal conditions for each intersection are the same as seen in this case, it is found to be optimal to control all signals with a coordinated cycle length without making any division into subareas.

Case 2 is the same street as case 1. In this case, however, there is a difference in traffic volume between the left half and right half of the street. It may be supposed that the left half of the street is in a central area with heavy traffic and the right half is in a suburban area. From the result of the calculation, it is found to be optimal to control the area with heavy traffic with a longer cycle length and the area with light traffic with a shorter cycle length.

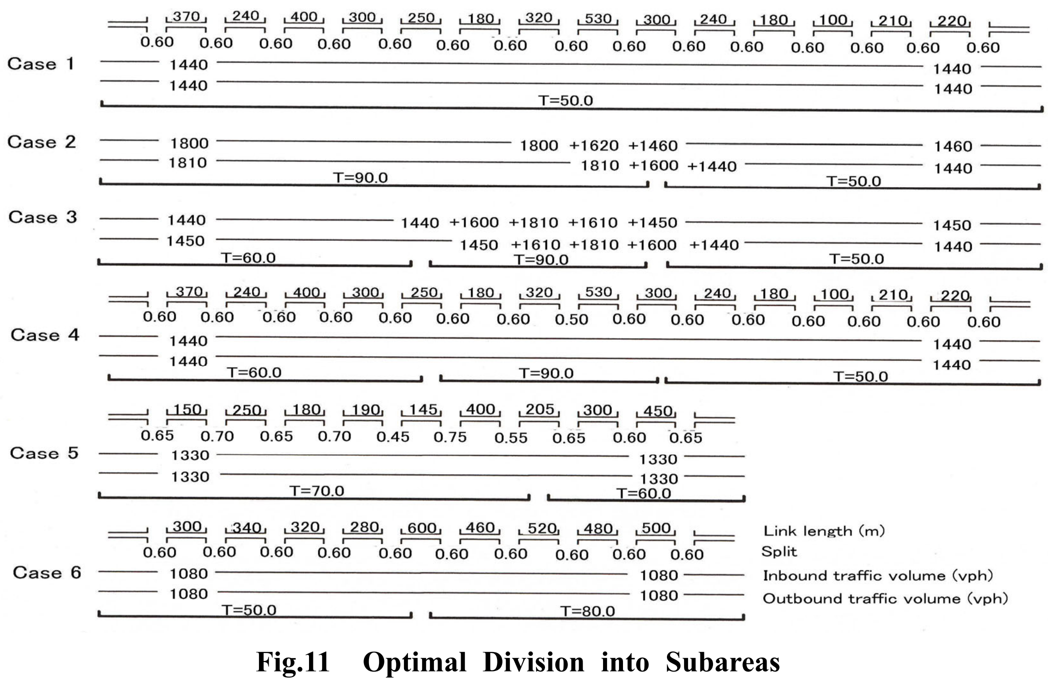

Fig.11 Optimal Division into Subareas and Optimal Cycle Length

Case 3 is also the same street as case 1. In this case, however, the traffic volume is heavy in the middle of the street and light at both sides of it. This may be supposed to be a street that passes through the center of a city. As a result of calculation, it is also found to be optimal to control the middle area with heavy traffic with a longer cycle length and the areas at both ends with a shorter cycle length.

Case 4 is also the same street and the same traffic volume as case 1. However, it includes a signal with a small split as a characteristic feature. This signal should be controlled with a longer cycle length because it may be a bottleneck on traffic capacity. As a result, it is found to be optimal to divide the street into 3 subareas.

In case 5, the arterial street consists of 10 signals, and the split of each intersection is different. A signal with a very small split is provided in the central part. As a result, it is found to be optimal to divide the street into two subareas and make the coordinated cycle length of the subarea including the signal with a split of 0.45 longer than that of the other subarea.

In case 6, though the traffic volume and split of each intersection is equal, link lengths are consciously arranged so as to raise the offset effect through coordinated control with a cycle length of 50s for the left 4 links of the street and with a cycle length of 80s for the right 4 links. As a result of calculation, it is found to be optimal to divide the street into two subareas as expected. With this, it may be said that it is necessary to make division in consideration of the link lengths and those arrangement as well as the traffic volume and split of each intersection.

From the above, as a rule, it may be better to control all signals with a coordinated cycle length without division into subareas. However, an area with signals with a small split and heavier traffic should be controlled with a longer cycle length than that of the other areas. In some cases where the split and traffic volume of each intersection are uniform, it may be better to divide the street into some subareas, though it depends on link lengths and those arrangements.

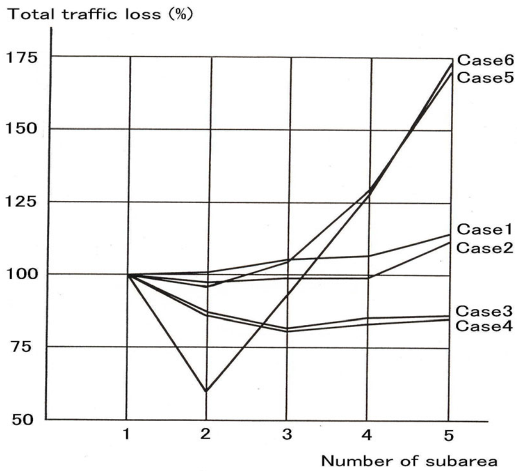

Fig.12 Relationship between the Number of Subarea and Total Traffic Loss

The relationship between the number of subareas and total traffic loss is as shown in **Fig.12**. The axis of ordinates is the total traffic loss represented by index with the total traffic loss as 100 when all signals are controlled with a common cycle length without division into subareas. As shown in the figure, the optimal control that minimizes the total traffic loss is not to be divided in case 1, to be divided into two subareas in cases 2, 5 and 6, and to be divided into 3 subareas in cases 3 and 4. This figure shows that as the number of division becomes far from the optimal number, the total traffic loss increases almost monotonously. When the street is optimally divided into subareas and each of them is controlled with the optimal cycle length, an improvement of about 20% in cases 3 and 4 and an improvement of about 40% in case 6 can be attained, respectively. Therefore, it seem to be important to make divisions into subareas and control them with the optimal cycle length though it depends on the conditions of the street.

6 SUMMARY AND CONCLUSION

This study proposed a method for optimally dividing the signal-coordinated arterial street into subareas of cycle length and described the features of optimal division showing some examples of calculation. A weighted sum of delay and stops was used as the performance index for optimization. Traffic conditions were assumed to be undersaturated and a platoon was represented by a single rectangular wave. Among the three parameters of cycle length, split and offset, the splits were treated as given.

This study can be summarized as follows:

(1) Optimal division into subareas and the optimal cycle length of each subarea can be obtained by the DP technique. The offsets of each subarea can also be optimized by DP.

(2) From some examples of calculation, the following has been made clear about optimal division into subareas. If there are no large differences in split, road and traffic conditions among intersections, division into subareas should not be made. If there are differences in these conditions, areas including signals with a small split or areas with heavier traffic should be controlled with a longer cycle length than that of the other areas. Nevertheless, division into subareas may be made desirably in some cases though it depends on link lengths and those arrangements.

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