

Detailed Evolution of Degree Distributions in Residual Graphs with Joint Degree Distributions

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SUMMARY Luby et al. derived evolution of degree distributions in residual graphs for irregular LDPC code ensembles. Evolution of degree distributions in residual graphs is important characteristic which is used for finite-length analysis of the expected block and bit error probability over the binary erasure channel. In this paper, we derive detailed evolution of degree distributions in residual graphs for irregular LDPC code ensembles with joint degree distributions.

key words: *detailedly represented irregular LDPC code ensembles, evolution of degree distributions in residual graphs, peeling algorithm, binary erasure channel*

1. Introduction

Gallager invented low-density parity-check (LDPC) codes [1] in 1963. LDPC codes are linear codes defined by sparse bipartite graphs. Luby et al. introduced irregular LDPC code ensembles [2] defined by some distributions which represent the degrees of nodes. We can obtain flexibility in balancing requirement that variable nodes want high degrees and check nodes want low degrees.

Luby et al. introduced the peeling algorithm (PA) [4], [7] for the binary erasure channel (BEC). PA is an iterative algorithm which is defined on Tanner graphs. As PA proceeds edges and nodes are removed, and the decoding is successful if the graph vanished. The residual graphs consist nodes and edges that are still unknown at each iteration. Evolution of degree distributions in residual graphs can be used for *finite-length scaling* of decoding performance of belief propagation (BP). Luby et al. [4] derived it for irregular LDPC code ensembles. Amraoui [5] showed that distributions of the number of check nodes of degree one in the residual graph convergences weakly to a Gaussian as block-length tends to infinity and that block and bit error probability of finite-length LDPC codes are derived by the average and the variance of the number of check node of degree one in the residual graph.

The check distributions of the optimized finite-length

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LDPC codes in [5] tend to have different degrees like $0.390753x^4 + 0.361589x^5 + 0.247658x^9$ while optimized infinite-length LDPC codes in [6] have concentrated degree distributions of check nodes like x^9 . It is known that irregular LDPC codes defined by joint degree distributions introduced in [3] performs well with such non-concentrated check degree distributions.

In this paper, as the first step to optimize finite-length irregular LDPC codes with joint degree distributions, we investigate evolution of residual graphs in PA. Furthermore, we give a graphical determination of successful decoding.

2. Preliminaries

2.1 Channel Model

In this paper, we consider the *binary erasure channel* (BEC). A channel input, denoted by X , is binary $X \in \{0, 1\}$. The corresponding output Y takes on values in the alphabet $\{0, 1, ?\}$, where ? indicating an erasure. Let ϵ be the erasure probability, i.e. $\epsilon = \Pr(Y = ?|X = x)$ and $1 - \epsilon = \Pr(Y = x|X = x)$.

2.2 Peeling Decoder

The peeling algorithm (PA) is an iterative decoding algorithm. It works on Tanner graphs of LDPC codes over BEC. BP and PA are completely equivalent in terms of their performance for BEC. Each check node has a one-bit memory cell. *Residual graphs* are defined as the graphs which consist nodes and edges that are still unknown at each iteration.

(1) Initialization

Variable nodes receive channel outputs: known messages '0' and '1' or an erasure '?'. The variable nodes with known received messages, send their value to check nodes by edges. They and all edges connecting to them are removed from the graph. Check nodes store the sum of the receiving values in their memory cells.

(2) Iteration

The decoder uniformly chooses a check node of degree one in the residual graph. The check node sends the value in its memory cell to the variable node connected to it. The variable node propagates this value to all check nodes connected to it and is removed with its edges. Each check node overwrites its memory cell with the sum of the receiving value

and the value of its memory cell.

(3) Decisions

If the decoder does not find any check node of degree one in the residual graph, then the decoding halts. If the residual graph is empty, decoding is succeeds, otherwise it fails and the variable nodes in the residual graph form a stopping set.

2.3 Detailedly Represented Irregular LDPC Code Ensembles (Detailedly Represented Ensembles)

In this paper, we use detailedly represented ensembles. Let b_i and d_j be a set of variable nodes of degree i and a set of check nodes of degree j , respectively. Let l_m and r_m be maximum degree of variable nodes and check nodes, respectively. Let B and D be

$$B := \{b_2, b_3, \dots, b_{l_m}\}, \quad D := \{d_2, d_3, \dots, d_{r_m}\}.$$

A function $\pi : B \times D \rightarrow [0, 1]$ is said to be a *joint degree distribution*, if $\sum_{b_i \in B} \sum_{d_j \in D} \pi(i, j) = 1$.

A detailedly represented ensemble $C(n, \pi)$ [3] is defined as an ensemble which consists of all bipartite graphs $G = (V \cup C, E)$ such that $n = |V|$ and $\pi(i, j) = |E_{i,j}|/|E|$ for all pair of $b_i \in B$, $d_j \in D$, where $E_{i,j}$ represents a set of edges connected to variable nodes of degree i and check nodes of degree j . Moreover we define $\lambda_i := \sum_j \pi(i, j)$, $\rho_j := \sum_i \pi(i, j)$, $\lambda_j(i) := \pi(i, j)/\rho_j$, $\rho_i(j) := \pi(i, j)/\lambda_i$.

3. Evolution of Degree Distributions in Residual Graphs for Detailedly Represented Ensembles

In this section, we derive expected degree distributions in residual graphs. We define *original degree* as the degree of nodes in the Tanner graph on which the code is defined. *Residual degree* of a node is defined as the degree of the node in the residual graph. Let t denote *time*, which starts from zero and increases by $\Delta t := 1/E$ for every decoding iteration, where E is the number of edges contained in the Tanner graph on which the code is defined. Let $L_k(t)$, $R_{s,\tilde{s}}(t)$ and $E_{k,s}(t)$ be the total number of edges connecting to variable nodes of residual degree k , connecting to check nodes of residual degree \tilde{s} and original degree s , and connecting to variable nodes of original degree k and check nodes of original degree s at time t , respectively. For a given residual graph, we define *variable degree distribution* l_k , *check degree distribution* $r_{s,\tilde{s}}$ and *joint degree distribution* $e_{k,s}$ as $l_k := L_k/E$, $r_{s,\tilde{s}} := R_{s,\tilde{s}}/E$ and $e_{k,s} := E_{k,s}/E$, respectively.

3.1 Differential Equations Description

From the iterations of PA, we can derive following differential equations.

Lemma 1: The differential equation for the joint degree distribution $e_{k,s}$, the variable degree distribution l_k and the check degree distribution $r_{s,\tilde{s}}$ in residual graph, are given as

$$\frac{de_{k,s}}{dt} = -h_s p_{k,s} - (k-1)q_{k,s} \sum_{j=2}^{r_m} h_j p_{k,j} \quad (1)$$

$$\frac{dl_k}{dt} = -k \sum_{j=2}^{r_m} h_j p_{k,j} \quad (2)$$

$$\frac{dr_{s,1}}{dt} = -h_s + \frac{(r_{s,2} - r_{s,1})}{\sum_{\tilde{s}'=1}^s r_{s,\tilde{s}'}} \sum_{i=2}^{l_m} (i-1)q_{i,s} \sum_{j=2}^{r_m} h_j p_{i,j} \quad (3)$$

$$\frac{dr_{s,\tilde{s}}}{dt} = \tilde{s} \frac{(r_{s,\tilde{s}+1} - r_{s,\tilde{s}})}{\sum_{\tilde{s}'=1}^s r_{s,\tilde{s}'}} \sum_{i=2}^{l_m} (i-1)q_{i,s} \sum_{j=2}^{r_m} h_j p_{i,j}, \quad (4)$$

where

$$h_j := \frac{r_{j,1}}{\sum_{j'} r_{j',1}}, \quad p_{i,j} := \frac{e_{i,j}}{\sum_{j'} e_{i,j'}}, \quad q_{i,j} := \frac{e_{i,j}}{\sum_{j'} e_{i,j'}}.$$

3.1.1 Proof of Eq. (1)

A check node of residual degree one is chosen uniformly and removed on each iteration. Let h_j be the probability that the chosen check node has original degree j . The probability h_j is given by

$$h_j = \frac{r_{j,1}}{\sum_{j'} r_{j',1}}. \quad (5)$$

Let $p_{i,j}$ be the probability that the edge connecting to the check node of original degree j connects to a variable node of original degree i at the other end. The probability $p_{i,j}$ is given by $p_{i,j} = e_{i,j}/\sum_{j'} e_{i,j'}$. Similarly, let $q_{i,j}$ be the probability that the edge connecting to the variable node of original degree i connects to a check node of original degree j at the other end. The probability $q_{i,j}$ is given by $q_{i,j} = e_{i,j}/\sum_{j'} e_{i,j'}$. Assume that we have a check node c of residual degree one and original degree j and a variable node v of original degree i . Let $v_{i,j}(u_2, \dots, u_{r_m})$ be the probability that i edges connecting to the variable node v connect to u_s check nodes of original degree s , for $s \in \{2, \dots, r_m\}$, including the check node c , where $\sum_{s=2}^{r_m} u_s = i$. In the blocklength limit, the probability $v_{i,j}(u_2, \dots, u_{r_m})$ is given by

$$v_{i,j}(u_2, \dots, u_{r_m}) = \binom{i-1}{u_2, \dots, u_j-1, \dots, u_{r_m}} \cdot q_{i,2}^{u_2} \cdots q_{i,j}^{u_j-1} \cdots q_{i,r_m}^{u_{r_m}}.$$

The probability generating function for $v_{i,j}(u_2, \dots, u_{r_m})$ is given by

$$\sum_{u_2, \dots, u_{r_m}} v_{i,j}(u_2, \dots, u_{r_m}) x_2^{u_2} \cdots x_{r_m}^{u_{r_m}} = x_j (q_{i,2} x_2 + \cdots + q_{i,r_m} x_{r_m})^{i-1}. \quad (6)$$

Let $\hat{f}_j^{(e_{k,s})}$ denote the expected change in the number of edges connecting to check nodes of original degree s and variable nodes of original degree k , conditioned on the fact that a check node of original degree j is removed. Taking the derivative of both sides of the Eq. (6) with respect to x_s and

$x_j = 1$ for all j , we can write for $j \neq s$

$$\begin{aligned} \hat{f}_j^{(e_{k,s})} &= - \sum_{u_2, \dots, u_{r_m}} p_{k,j} v_{k,j}(u_2, \dots, u_{r_m}) u_s \\ &= -(k-1) p_{k,j} q_{k,s}. \end{aligned} \tag{7}$$

Similarly, for $j = s$, we can write

$$\begin{aligned} \hat{f}_s^{(e_{k,s})} &= - \sum_{u_2, \dots, u_{r_m}} p_{k,s} v_{k,s}(u_2, \dots, u_{r_m}) u_s \\ &= -p_{k,s}(1 + (k-1)q_{k,s}). \end{aligned} \tag{8}$$

Let $\hat{f}^{(e_{k,s})}$ denote the expected change in the number of edges connecting to check nodes of original degree s and variable nodes of original degree k . From Eqs. (5), (7) and (8), we see that

$$\hat{f}^{(e_{k,s})} = \sum_{j=2}^{r_m} h_j \hat{f}_j^{(e_{k,s})} = -h_s p_{k,s} - (k-1) q_{k,s} \sum_{j=2}^{r_m} h_j p_{k,j}.$$

As $E \rightarrow \infty$, Δt tends to 0. Note that $\hat{f}^{(e_{k,s})} = E_{k,s}(t + \Delta t) - E_{k,s}(t) = \{e_{k,s}(t + \Delta t) - e_{k,s}(t)\} / \Delta t = \frac{de_{k,s}}{dt}$. Hence, we obtain Eq. (1).

3.1.2 Proof of Eq. (2)

Let $\hat{f}_j^{(l_k)}$ denote the expected change in the number of the edges connecting to variable nodes of residual degree k conditioned on the fact that a check node of original degree j is removed. We can write

$$\hat{f}_j^{(l_k)} = -k p_{k,j}. \tag{9}$$

Let $\hat{f}^{(l_k)}$ denote the expected change in the number of edges connecting to variable nodes of residual degree k . From Eqs. (5) and (9), we see that

$$\hat{f}^{(l_k)} = \sum_{j=2}^{r_m} h_j \hat{f}_j^{(l_k)} = -k \sum_{j=2}^{r_m} h_j p_{k,j}.$$

Similarly, note that $\hat{f}^{(l_k)} = \frac{dl_k}{dt}$. Hence we obtain Eq. (2).

3.1.3 Proof of Eqs. (3) and (4)

Let $q_{i,s,\tilde{s}}$ be the probability that the edge connecting to the variable node of original degree i has a check node of original degree s and residual degree \tilde{s} . The probability $q_{i,s,\tilde{s}}$ is given by $q_{i,s,\tilde{s}} = q_{i,s} r_{s,\tilde{s}} / \sum_{\tilde{s}} r_{s,\tilde{s}}$. Assume that we have a check node c of residual degree one and original degree j and a variable node v of original degree i . Let $w_{i,j}(u_{2,1}, \dots, u_{r_m,r_m})$ be the probability that i edges connecting to the variable node v connect to $u_{s,\tilde{s}}$ check nodes of original degree s and residual degree \tilde{s} , for $s \in \{2, \dots, r_m\}$ and $\tilde{s} \in \{1, \dots, s\}$, including the check node c , where $\sum_{s=2}^{r_m} \sum_{\tilde{s}=1}^s u_{s,\tilde{s}} = i$. In the blocklength limit, the probability $w_{i,j}(u_{2,1}, \dots, u_{r_m,r_m})$ is given by

$$\begin{aligned} w_{i,j}(u_{2,1}, \dots, u_{r_m,r_m}) &= \binom{i-1}{u_{2,1}, \dots, u_{j,1}-1, \dots, u_{r_m,r_m}} \\ &\quad \cdot q_{i,2,1}^{u_{2,1}} \cdots q_{i,j,1}^{u_{j,1}-1} \cdots q_{i,r_m,r_m}^{u_{r_m,r_m}}. \end{aligned}$$

From this equation, the probability generating function for $w_{i,j}(x_{2,1}, \dots, x_{r_m,r_m})$ is given by

$$\begin{aligned} \sum_{u_{2,1}, \dots, u_{r_m,r_m}} w_{i,j}(u_{2,1}, \dots, u_{r_m,r_m}) x_{2,1}^{u_{2,1}} \cdots x_{r_m,r_m}^{u_{r_m,r_m}} &= x_{j,1} (q_{i,2,1} x_{2,1} + \cdots + q_{i,r_m,r_m} x_{r_m,r_m})^{i-1}. \end{aligned} \tag{10}$$

We assume that the edge connecting to a check node of original degree s and residual degree \tilde{s} is removed. The decrease in the number of edges connecting to check nodes of original degree s and residual degree \tilde{s} is \tilde{s} . On the other hand, the increase in the number of edges connecting to check nodes of original degree s and residual degree $\tilde{s} - 1$ is $\tilde{s} - 1$. Let $\hat{f}_j^{(r_{s,\tilde{s}})}$ denote the expected change in the number of edges connecting to check nodes of original degree s and residual degree \tilde{s} , conditioned on the fact that a check node of original degree j is removed. Using Eq. (10), for $j \neq s$ or $\tilde{s} \neq 1$, we can write

$$\begin{aligned} \hat{f}_j^{(r_{s,\tilde{s}})} &= \tilde{s} \sum_{i=2}^{l_m} p_{i,j} \sum_{u_{2,1}, \dots, u_{r_m,r_m}} w_{i,j}(u_{2,1}, \dots, u_{r_m,r_m}) (u_{s,\tilde{s}+1} - u_{s,\tilde{s}}) \\ &= \tilde{s} \sum_{i=2}^{l_m} p_{i,j} (i-1) (q_{i,s,\tilde{s}+1} - q_{i,s,\tilde{s}}). \end{aligned} \tag{11}$$

Similarly, for $j = s$ and $\tilde{s} = 1$, we can write

$$\begin{aligned} \hat{f}_s^{(r_{s,1})} &= \sum_{i=2}^{l_m} p_{i,s} \sum_{u_{2,1}, \dots, u_{r_m,r_m}} w_{i,s}(u_{2,1}, \dots, u_{r_m,r_m}) (u_{s,2} - u_{s,1}) \\ &= -1 + \sum_{i=2}^{l_m} p_{i,s} (i-1) (q_{i,s,2} - q_{i,s,1}). \end{aligned} \tag{12}$$

Let $\hat{f}^{(r_{s,\tilde{s}})}$ denote the expected change in the number of edges connecting to check nodes of original degree s and residual degree \tilde{s} . From Eqs. (5), (11) and (12), for $\tilde{s} \geq 2$, we see that

$$\begin{aligned} \hat{f}^{(r_{s,\tilde{s}})} &= \sum_{j=2}^{r_m} h_j \hat{f}_j^{(r_{s,\tilde{s}})} \\ &= \tilde{s} \frac{(r_{s,\tilde{s}+1} - r_{s,\tilde{s}})}{\sum_{\tilde{s}} r_{s,\tilde{s}}} \sum_{i=2}^{l_m} (i-1) q_{i,s} \sum_{j=2}^{r_m} h_j p_{i,j}. \end{aligned}$$

Similarly, for $\tilde{s} = 1$, we see that

$$\begin{aligned} \hat{f}^{(r_{s,1})} &= \sum_{j=2}^{r_m} h_j \hat{f}_j^{(r_{s,1})} \\ &= -h_s + \frac{(r_{s,2} - r_{s,1})}{\sum_{\tilde{s}} r_{s,\tilde{s}}} \sum_{i=2}^{l_m} (i-1) q_{i,s} \sum_{j=2}^{r_m} h_j p_{i,j}. \end{aligned}$$

Note that $\hat{f}^{(r_{s,\bar{s}})} = \frac{dr_{s,\bar{s}}}{dt}$. Hence, we obtain Eqs. (3) and (4).

3.2 Evolution of Degree Distributions in Residual Graphs

Unfortunately, it is difficult to solve differential equations in Lemma 1. However, these differential equations are easily solved by parameters $\mathbf{y} = (y_2, \dots, y_{r_m})$. The parameters \mathbf{y} are defined such that

$$\frac{dy_j}{dt} = -\frac{y_j}{\sum_{i'=2}^{l_m} e_{i',j}} \frac{r_{j,1}}{\sum_{j'} r_{j',1}} \quad (13)$$

and $y_j = 1$ when $t = 0$.

Theorem 1: Using parameters $\mathbf{y} = (y_2, \dots, y_{r_m})$, the joint degree distribution $e_{k,s}$, the variable degree distribution l_k and the check degree distribution $r_{s,\bar{s}}$ in residual graph, are given as

$$r_{s,1} = \epsilon \sum_{i=2}^{l_m} \pi(i, s) \left(\sum_{j=2}^{r_m} \rho_i(j) y_j \right)^{i-1} \left[y_s - 1 + \left\{ 1 - \epsilon \sum_{i=2}^{l_m} \lambda_s(i) \left(\sum_{j=2}^{r_m} \rho_i(j) y_j \right)^{i-1} \right\}^{s-1} \right] \quad (14)$$

$$r_{s,\bar{s}} = \rho_s \left(\frac{s-1}{\bar{s}-1} \right) \left\{ \epsilon \sum_{i=2}^{l_m} \lambda_s(i) \left(\sum_{j=2}^{r_m} \rho_i(j) y_j \right)^{i-1} \right\}^{\bar{s}} \cdot \left\{ 1 - \epsilon \sum_{i=2}^{l_m} \lambda_s(i) \left(\sum_{j=2}^{r_m} \rho_i(j) y_j \right)^{i-1} \right\}^{s-\bar{s}} \quad (15)$$

$$l_k = \epsilon \lambda_k \left(\sum_{j=2}^{r_m} \rho_k(j) y_j \right)^k \quad (16)$$

$$e_{k,s} = \epsilon \pi(k, s) y_s \left(\sum_{j=2}^{r_m} \rho_k(j) y_j \right)^{k-1}, \quad (17)$$

for $k \in \{2, \dots, l_m\}$, $s \in \{2, \dots, r_m\}$ and $\bar{s} \in \{2, \dots, s\}$.

The proof of Theorem 1 is given in Appendix.

Example 1: Using Eqs. (14) and (17) for Eq. (13), we can write

$$\frac{dy_j}{dt} = -\frac{y_j - 1 + (1 - \epsilon \sum_i \lambda_j(i) Y_i^{i-1})^{j-1}}{\sum_{j'} r_{j',1}}, \quad (18)$$

where $\sum_j r_{j,1} = \epsilon \sum_j \sum_{i'} \pi(i', j) Y_{i'}^{i'-1} \{y_j - 1 + (1 - \epsilon \sum_i \lambda_j(i) Y_i^{i-1})^{j-1}\}$ and $Y_i = \sum_{j=2}^{r_m} \rho_i(j) y_j$. Solving Eq. (18) by numerical calculations, we have \mathbf{y} as a function of t . Using \mathbf{y} , from Theorem 1, we have degree distributions in residual graphs at time t . If decoding succeeds, the number of iteration of PA equals to the number of variable nodes in residual graph at $t = 0$, since each iteration of PA removes a variable node. The decoding halts at $t = t_{\max} := \epsilon \sum_i \frac{E \lambda_i}{i} \Delta t = \epsilon \sum_i \frac{\lambda_i}{i}$ for successful decoding. Figure 1 shows the evolution of the degree distributions $r_{5,1}(t)$ and $r_{10,1}(t)$ in residual graphs for $\pi(3, 5) = 0.12$,

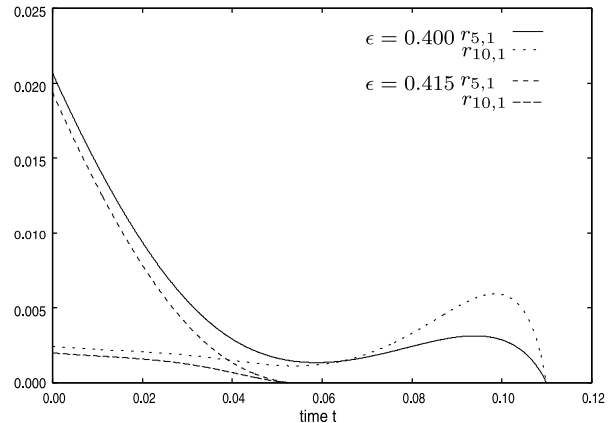


Fig. 1 Evolution of degree distributions $r_{5,1}$ and $r_{10,1}$ in residual graphs. $\pi(3, 5) = 0.12$, $\pi(3, 10) = 0.18$, $\pi(4, 5) = 0.28$, $\pi(4, 10) = 0.42$.

$\pi(3, 10) = 0.18$, $\pi(4, 5) = 0.28$, $\pi(4, 10) = 0.42$, $t_{\max} = 0.11$. Since the curves of $\epsilon = 0.400$ reach to t_{\max} in Fig. 1, we see that decoding succeeds. On the other hand, the curves of $\epsilon = 0.415$ touch the zero line at $t < t_{\max}$ and the decoding fails i.e. the residual graph has no check node of residual degree one.

4. Graphical Determination of Successful Decoding

In this section, we see $r_{s,1}$ in Eq. (14) as a function of \mathbf{y} . The following theorem shows a graphical determination of successful decoding from the contour of $\min_s r_{s,1}(\mathbf{y})$.

Theorem 2: For detailedly represented ensembles and $\epsilon \in (0, 1)$, a path from the point $(1, 1, \dots, 1)$ to $(0, 0, \dots, 0)$ exists in the set $\{\mathbf{y} \mid \min_s r_{s,1}(\epsilon, \mathbf{y}) > 0, \mathbf{y} \in [0, 1]^{|D|}\}$ if and only if the PA is successful.

The proof of Theorem 2 is given in Sects. 4.1 and 4.2. An example of Theorem 2 is given in the following.

Example 2 (Two Different Check Degrees): Let $y_m^{(\ell)}$ also represent the fraction of erased messages sent from the check nodes of degree m to variable nodes at the ℓ -th iteration of BP which is equivalent with PA in decoding results. (It is described in Sect. 4.1). Define $\mathbf{y}^{(\ell)} := (y_2^{(\ell)}, y_3^{(\ell)}, \dots, y_{r_m}^{(\ell)})$. If $\mathbf{y}^{(\ell)} = (0, 0, \dots, 0)$ for some ℓ , then BP decoding is successful. We define *path of density evolution* as the path made from the line segments bounded by the point $\mathbf{y}^{(\ell)}$ and $\mathbf{y}^{(\ell+1)}$ for all $\ell \geq 0$. The set $\{\mathbf{y} \mid \min\{r_{5,1}(\mathbf{y}), r_{10,1}(\mathbf{y})\} > 0, \mathbf{y} \in [0, 1]^2\}$ is the region inside of curves labeled 0.00 in Fig. 2 and Fig. 3. In Fig. 2, the point $(1, 1)$ and $(0, 0)$ are connected by the path of density evolution and decoding is successful. On the other hand, in Fig. 3, there does not exist any path from the point $(1, 1)$ and $(0, 0)$ in the set $\{\mathbf{y} \mid \min\{r_{5,1}(\mathbf{y}), r_{10,1}(\mathbf{y})\} > 0, \mathbf{y} \in [0, 1]^2\}$ and the decoding is not successful.

4.1 Monotonicity

For $s \in \{2, \dots, r_m\}$, functions $f_s(\epsilon, \mathbf{y})$ are defined as

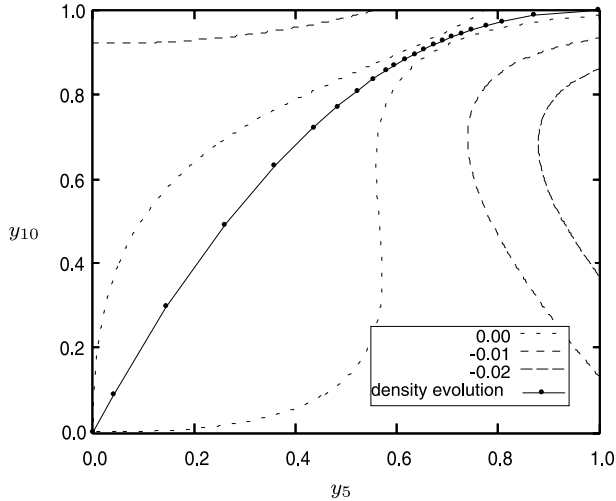


Fig. 2 The contour of $\min\{r_{5,1}, r_{10,1}\}$ and the path of density evolution (d.e.). $\pi(3,5) = 0.12$, $\pi(3,10) = 0.18$, $\pi(4,5) = 0.28$, $\pi(4,10) = 0.42$, $\epsilon = 0.40$.

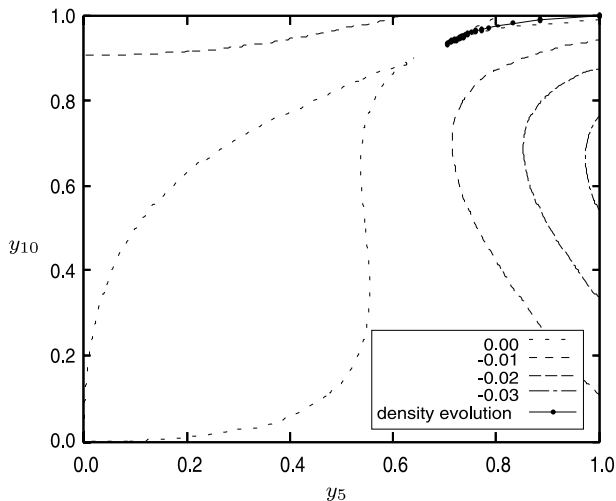


Fig. 3 The contour of $\min\{r_{5,1}, r_{10,1}\}$ and path of density evolution. $\pi(3,5) = 0.12$, $\pi(3,10) = 0.18$, $\pi(4,5) = 0.28$, $\pi(4,10) = 0.42$, $\epsilon = 0.415$.

$$f_s(\epsilon, \mathbf{y}) := 1 - \left\{ 1 - \epsilon \sum_{i=2}^{l_m} \lambda_s(i) \left(\sum_{j=2}^{r_m} \rho_i(j) y_j \right)^{i-1} \right\}^{s-1}. \quad (19)$$

These functions are called *density evolution equations*. We start with $\mathbf{y}^{(0)} = (1, \dots, 1)$ and apply the recursions $y_s^{(\ell+1)} = f_s(\epsilon, \mathbf{y}^{(\ell)})$. We denote $\mathbf{y}^{(\ell+1)} < \mathbf{y}^{(\ell)}$ if $y_s^{(\ell+1)} < y_s^{(\ell)}$ for all $s \in \{2, \dots, r_m\}$. A point \mathbf{p} is called a *fixed point* if $p_s = f_s(\epsilon, \mathbf{p})$ for all $s \in \{2, \dots, r_m\}$. To prove Theorem 2, we use the following lemmas.

Lemma 2: For $\epsilon \in (0, 1)$ and $\mathbf{y} \neq (0, \dots, 0)$, $f_s(\epsilon, \mathbf{y})$ is increasing in y_k for all $s, k \in \{2, \dots, r_m\}$.

Lemma 3: For $\epsilon \in (0, 1)$, the sequence $\{\mathbf{y}^{(\ell)}\}_{\ell \geq 0}$ converges to a fixed point and $\mathbf{y}^{(\ell+1)} < \mathbf{y}^{(\ell)}$ for all $\ell \geq 0$.

Proof of Lemma 2: For $k, s \in \{2, \dots, r_m\}$, $\epsilon \in (0, 1)$ and $\mathbf{y} \neq (0, \dots, 0)$, the partial derivative of Eq. (19) with respect to y_k is

$$\frac{\partial f_s(\epsilon, \mathbf{y})}{\partial y_k} = (s-1)\epsilon \sum_{i=2}^{l_m} \lambda_s(i)(i-1)\rho_i(k) \left(\sum_{j=2}^{r_m} \rho_i(j)y_j \right)^{i-2} \cdot \left\{ 1 - \epsilon \sum_{i=2}^{l_m} \lambda_s(i) \left(\sum_{j=2}^{r_m} \rho_i(j)y_j \right)^{i-1} \right\}^{s-2} > 0$$

and we conclude that $f_s(\epsilon, \mathbf{y})$ is increasing in y_k .

Proof of Lemma 3: Let $\epsilon \in (0, 1)$. It holds that $\mathbf{y}^{(1)} < \mathbf{y}^{(0)}$ since $y_s^{(1)} = f_s(\epsilon, \mathbf{y}^{(0)}) = 1 - (1 - \epsilon)^{s-1} < 1 = y_s^{(0)}$ for $s \in \{2, \dots, r_m\}$. From Lemma 2, if $\mathbf{y}^{(\ell+1)} < \mathbf{y}^{(\ell)}$ then $y_s^{(\ell+2)} = f_s(\epsilon, \mathbf{y}^{(\ell+1)}) < f_s(\epsilon, \mathbf{y}^{(\ell)}) = y_s^{(\ell+1)}$ for $s \in \{2, \dots, r_m\}$. This shows $\mathbf{y}^{(\ell+2)} < \mathbf{y}^{(\ell+1)}$. Hence, $\mathbf{y}^{(\ell+1)} < \mathbf{y}^{(\ell)}$ for all $\ell \geq 0$.

Let \mathbf{p} be a fixed point. It can be shown that from Lemma 2, if $\mathbf{p} \leq \mathbf{y}^{(\ell)}$ then $p_s = f_s(\epsilon, \mathbf{p}) \leq f_s(\epsilon, \mathbf{y}^{(\ell)}) = y_s^{(\ell+1)}$. This shows $\mathbf{p} \leq \mathbf{y}^{(\ell+1)}$. Therefore, the sequence $\{\mathbf{y}^{(\ell)}\}_{\ell \geq 0}$ converges. Let $\mathbf{y}^{(\infty)} := \lim_{\ell \rightarrow \infty} \mathbf{y}^{(\ell)}$. Since the sequence converges, $y_s^{(\infty)} = f_s(\epsilon, \mathbf{y}^{(\infty)})$ for all s . This shows $\mathbf{y}^{(\infty)}$ is a fixed point. Hence, the sequence $\{\mathbf{y}^{(\ell)}\}_{\ell \geq 0}$ converges to the fixed point.

4.2 Proof of Theorem 2

Define $\mathcal{R} := \{\mathbf{y} \mid \min_s r_{s,1}(\epsilon, \mathbf{y}) > 0, \mathbf{y} \in [0, 1]^{\text{pl}}\}$. From Eqs. (14) and (19), it holds for $\epsilon \in (0, 1)$

$$y_s > f_s(\epsilon, \mathbf{y}) \text{ iff } r_{s,1}(\epsilon, \mathbf{y}) > 0. \quad (20)$$

From Lemma 2 and Lemma 3, it can be shown $y_s^{(\ell+1)} = f_s(\epsilon, \mathbf{y}^{(\ell)}) > f_s(\epsilon, \mathbf{y}^{(\ell+1)})$ for all $s \in \{2, \dots, r_m\}$. From Eq. (20), $r_{s,1}(\epsilon, \mathbf{y}^{(\ell)}) > 0$ for $\ell \geq 1$. From Eq. (14), $r_{s,1}(\epsilon, \mathbf{y}^{(0)}) > 0$. Therefore, we obtain $\mathbf{y}^{(\ell)} \in \mathcal{R}$ for $\ell \geq 0$. Let $\theta \in (0, 1)$. Define $\mathbf{a}(\theta) := \theta \mathbf{y}^{(\ell)} + (1 - \theta) \mathbf{y}^{(\ell+1)}$. Since $\mathbf{y}^{(\ell+1)} < \mathbf{a}(\theta) < \mathbf{y}^{(\ell)}$, we obtain $f_s(\epsilon, \mathbf{a}(\theta)) < f_s(\epsilon, \mathbf{y}^{(\ell)}) = y_s^{(\ell+1)}$ for all $s \in \{2, \dots, r_m\}$ by Lemma 2. This shows $f_s(\epsilon, \mathbf{a}(\theta)) < a_s(\theta)$ for all s . From Eq. (20), $\mathbf{a}(\theta) \in \mathcal{R}$. Define $\mathcal{L}^{(\ell)} := \{\theta \mathbf{y}^{(\ell)} + (1 - \theta) \mathbf{y}^{(\ell+1)} \mid \theta \in [0, 1]\}$. We conclude that $\mathcal{L}^{(\ell)} \subset \mathcal{R}$ for all $\ell \geq 0$. If decoding is successful, then $\mathbf{y}^{(\infty)} = (0, 0, \dots, 0)$. There exists a path from the point $(1, \dots, 1)$ to $(0, \dots, 0)$ by line segments $\mathcal{L}^{(\ell)}$. Since $\mathcal{L}^{(\ell)} \subset \mathcal{R}$, this path is in the set \mathcal{R} .

If decoding is not successful, then some fixed points, represented by $\mathbf{p} \in (0, 1]^{\text{pl}}$, exist. Define $\mathcal{P}_s := \{\mathbf{y} \mid y_i \in [p_i, 1], \forall i \in \{2, \dots, r_m\} / \{s\}, y_s = p_s\}$ for $s \in \{2, \dots, r_m\}$. Let $\mathbf{b}_s \in \mathcal{P}_s$. Since $\mathbf{p} \leq \mathbf{b}_s$, it holds that $f_s(\epsilon, \mathbf{p}) \leq f_s(\epsilon, \mathbf{b}_s)$ by Lemma 2. Since $p_s - f_s(\epsilon, \mathbf{p}) \geq p_s - f_s(\epsilon, \mathbf{b}_s)$, we obtain $0 = r_{s,1}(\epsilon, \mathbf{p}) \geq r_{s,1}(\epsilon, \mathbf{b}_s)$ from Eq.(20). Therefore, $\mathcal{P}_s \cap \mathcal{R} = \emptyset$. Every path from the point $(1, \dots, 1)$ to $(0, \dots, 0)$ includes a point in a set \mathcal{P}_s . Hence, there does not exist any path from the point $(1, \dots, 1)$ to $(0, \dots, 0)$ in the set \mathcal{R} .

5. Conclusion and Future Work

In this paper, we derive detailed evolution of degree distributions in residual graphs using joint degree distribution. From this results, we can derive the average of the number of check nodes of degree one in residual graph. Moreover, we give a graphical determination of successful decoding.

As a future work, we will derive the variance of the number of check nodes of degree one in residual graphs, and will show finite-length scaling [5] for detailedly represented ensembles. Moreover we will optimize finite-length LDPC codes for detailedly represented ensembles.

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Appendix: Proof of Theorem1

For $j \in \{2, \dots, r_m\}$ and $k \in \{2, \dots, l_m\}$, define the variables $y_{j,k}$ such that

$$\frac{\rho_k(j)}{\sum_{j=2}^{r_m} \rho_k(j) y_{j,k}} dy_{j,k} := -\frac{h_j p_{k,j}}{l_k} dt, \quad (\text{A.1})$$

where $y_{j,k} = 1$ when $t = 0$ and they are called initial conditions. We define $\mathbf{y}_k := (y_{2,k}, \dots, y_{r_m,k})$.

A.1 Proof of Eqs. (16) and (17)

Using Eq. (A.1) for Eq. (2), we can write

$$\begin{aligned} dl_k &= -k \sum_{j=2}^{r_m} h_j p_{k,j} dt \\ dl_k(\mathbf{y}_k) &= k \frac{\sum_{j=2}^{r_m} \rho_k(j') dy_{j',k}}{\sum_{j=2}^{r_m} \rho_k(j) y_{j,k}} l_k(\mathbf{y}_k). \end{aligned} \quad (\text{A.2})$$

We define the substitution $Y_k = \sum_{j=2}^{r_m} \rho_k(j) y_{j,k}$ which gives $dY_k = \sum_{j=2}^{r_m} \rho_k(j') dy_{j',k}$. These transform Eq. (A.2) to

$$dl_k(\mathbf{y}_k) = k \frac{dY_k}{Y_k} l_k(\mathbf{y}_k).$$

The solution is given by

$$l_k(\mathbf{y}_k) = C (Y_k)^k = C \left(\sum_{j=2}^{r_m} \rho_k(j) y_{j,k} \right)^k$$

for a constant C to be determined from the initial condition. From initial conditions, we see that $l_k(t = 0) = C = \epsilon \lambda_k$, since the erasure probability of each variable node is ϵ . Hence, we obtain

$$l_k(\mathbf{y}_k) = \epsilon \lambda_k \left(\sum_{j=2}^{r_m} \rho_k(j) y_{j,k} \right)^k. \quad (\text{A.3})$$

Using Eqs. (A.1) and (A.3) for Eq. (1), we can write

$$\begin{aligned} de_{k,s} &= -h_s p_{k,s} dt - (k-1) q_{k,s} \sum_{j=2}^{r_m} h_j p_{k,j} dt \\ &= \epsilon \pi(k, s) \left(\sum_{j=2}^{r_m} \rho_k(j) y_{j,k} \right)^{k-1} dy_{s,k} \\ &\quad + (k-1) e_{k,s}(\mathbf{y}_k) \frac{\sum_{j=2}^{r_m} \rho_k(j') dy_{j',k}}{\sum_{j=2}^{r_m} \rho_k(j) y_{j,k}}, \end{aligned}$$

where we use $q_{k,s} = e_{k,s}/l_k$. We can make this total differential equation exact, as following,

$$\begin{aligned} 0 = df &= -\frac{de_{k,s}}{\left(\sum_{j=2}^{r_m} \rho_k(j) y_{j,k} \right)^{k-1}} + \epsilon \pi(k, s) dy_{s,k} \\ &\quad + (k-1) e_{k,s} \frac{\sum_{j=2}^{r_m} \rho_k(j') dy_{j',k}}{\left(\sum_{j=2}^{r_m} \rho_k(j) y_{j,k} \right)^k}. \end{aligned} \quad (\text{A.4})$$

We assume that the variables $y_{s,k}$ ($s \in \{2, \dots, r_m\}$, $k \in \{2, \dots, l_m\}$) are constants and the solution is given by

$$f = -\frac{e_{k,s}}{\left(\sum_{j=2}^{r_m} \rho_k(j) y_{j,k} \right)^{k-1}} + C(\mathbf{y}_k), \quad (\text{A.5})$$

for the function $C(\mathbf{y}_k)$ to be determined from Eq.(A.4). To determine the function $C(\mathbf{y}_k)$, the partial derivative of Eq. (A.5) with respect to $y_{j',k}$ is

$$\frac{\partial f}{\partial y_{j',k}} = (k-1) \frac{\rho_k(j') e_{k,s}}{\left(\sum_{j=2}^{r_m} \rho_k(j) y_{j,k} \right)^k} + \frac{\partial C(\mathbf{y}_k)}{\partial y_{j',k}}.$$

From Eq. (A.4), we can write

$$\frac{\partial C(\mathbf{y}_k)}{\partial y_{j',k}} = 0 \quad (j' \neq s), \quad \frac{\partial C(\mathbf{y}_k)}{\partial y_{s,k}} = \epsilon \pi(k, s).$$

Therefore, we see that $C(\mathbf{y}) = \epsilon \pi(k, s) y_{s,k} + C$, where C is a constant. Since f is a constant, let $C' := f - C$. From Eq. (A.5), we see that

$$\begin{aligned} C' &= -\frac{e_{k,s}}{\left(\sum_{j=2}^{r_m} \rho_k(j) y_{j,k} \right)^{k-1}} + \epsilon \pi(k, s) y_{s,k} \\ e_{k,s}(\mathbf{y}_k) &= (\epsilon \pi(k, s) y_{s,k} - C') \left(\sum_{j=2}^{r_m} \rho_k(j) y_{j,k} \right)^{k-1}. \end{aligned}$$

From the initial condition $e_{k,s}(t=0) = \epsilon\pi(k,s)$, we see that $C' = 0$. Hence, we obtain

$$e_{k,s}(\mathbf{y}_k) = \epsilon\pi(k,s)y_{s,k} \left(\sum_{j=2}^{r_m} \rho_k(j)y_{j,k} \right)^{k-1}. \quad (\text{A} \cdot 6)$$

Using Eqs. (A.3) and (A.6) for Eq. (A.1), we obtain for $j \in \{2, \dots, r_m\}, k \in \{2, \dots, l_m\}$

$$\frac{dy_{j,k}}{y_{j,k}} = -\frac{h_j}{\sum_{i'=2}^{l_m} e_{i',j}} dt.$$

We see that $y_{j,k'} = y_{j,k}$ for $k' \neq k$. Define $y_j := y_{j,k}$ and $\mathbf{y} := \mathbf{y}_k$. Hence, we obtain Eqs. (16) and (17) from Eqs. (A.3) and (A.6). Moreover, this differential equation becomes Eq. (13).

A.2 Proof of Eqs. (14) and (15)

Note that $\sum_{k=2}^{l_m} e_{k,s} = \sum_{\bar{s}=1}^s r_{s,\bar{s}}$. Using Eqs. (A.1) and (A.6) for Eq. (4), we obtain

$$\begin{aligned} dr_{s,\bar{s}} &= \bar{s} \frac{r_{s,\bar{s}+1} - r_{s,\bar{s}}}{\sum_i e_{i,s}} \sum_{i=2}^{l_m} (i-1) \frac{e_{i,s}}{l_i} \sum_{j'=2}^{r_m} h_{j'} p_{i,j'} dt \\ &= \frac{\tilde{s}(r_{s,\bar{s}}(\mathbf{y}) - r_{s,\bar{s}+1}(\mathbf{y}))}{\sum_{i=2}^{l_m} \lambda_s(i) \left(\sum_{j=2}^{r_m} \rho_i(j)y_j \right)^{i-1}} \sum_{i=2}^{l_m} (i-1) \lambda_s(i) \\ &\quad \cdot \left(\sum_{j=2}^{r_m} \rho_i(j)y_j \right)^{i-2} \sum_{j'=2}^{r_m} \rho_i(j') dy_{j'} \end{aligned} \quad (\text{A} \cdot 7)$$

To solve Eq. (A.7), we define a substitution $H(\mathbf{y}) := \sum_{i=2}^{l_m} \lambda_s(i) \left(\sum_{j=2}^{r_m} \rho_i(j)y_j \right)^{i-1}$, which gives

$$dH(\mathbf{y}) = \sum_{i=2}^{l_m} (i-1) \lambda_s(i) \left(\sum_{j=2}^{r_m} \rho_i(j)y_j \right)^{i-2} \sum_{j'=2}^{r_m} \rho_i(j') dy_{j'}.$$

Using this substitutions, Eq. (A.7) is written as

$$dr_{s,\bar{s}}(H) = \tilde{s}(r_{s,\bar{s}}(H) - r_{s,\bar{s}+1}(H)) \frac{dH}{H}.$$

This equation is a first order linear differential equation. The solution is given by

$$r_{s,\bar{s}}(H) = H^{\tilde{s}} \left\{ -\tilde{s} \int r_{s,\bar{s}+1}(H) H^{-\tilde{s}} \frac{dH}{H} + C_{s,\bar{s}} \right\}$$

for some constants $C_{s,\bar{s}}$ to be determined from the initial conditions. These equations can be solved recursively, starting with $r_{s,s}$. We see that

$$r_{s,\bar{s}}(H) = \sum_{i \geq \bar{s}} (-1)^{i+\bar{s}} \binom{i-1}{\bar{s}-1} C_{s,i} H^i. \quad (\text{A} \cdot 8)$$

This equation can be solved recursively, starting with $C_{s,s}$. Using initial conditions for Eq. (A.8), we see that

$$C_{s,i} = \sum_{j \geq i} \binom{j-1}{i-1} r_{s,j}(t=0). \quad (\text{A} \cdot 9)$$

Note that $H(t=0) = 1$. Let R_s be the number of check nodes of original degree s . We see that

$$R_{s,j}(t=0) = jR_s \binom{m}{j} \epsilon^j (1-\epsilon)^{s-j}.$$

Let the both sides be multiplied by $1/E$. We obtain

$$r_{s,j}(t=0) = \rho_s \binom{s-1}{j-1} \epsilon^j (1-\epsilon)^{s-j}$$

where $\rho_s = \sum_i \pi(i,s)$. From this equation and Eq. (A.9), we can write

$$C_{s,i} = \rho_s \binom{s-1}{i-1} \epsilon^i.$$

(Use the identity $\binom{j-1}{i-1} \binom{s-1}{j-1} = \binom{s-1}{i-1} \binom{s-i}{j-i}$). From this equation and Eq. (A.8), we can write

$$\begin{aligned} r_{s,\bar{s}}(\mathbf{y}) &= \rho_s \binom{s-1}{\bar{s}-1} (\epsilon H(\mathbf{y}))^{\bar{s}} (1 - \epsilon H(\mathbf{y}))^{s-\bar{s}} \\ &= \rho_s \binom{s-1}{\bar{s}-1} \left\{ \epsilon \sum_{i=2}^{l_m} \lambda_s(i) \left(\sum_{j=2}^{r_m} \rho_i(j)y_j \right)^{i-1} \right\}^{\bar{s}} \\ &\quad \cdot \left\{ 1 - \epsilon \sum_{i=2}^{l_m} \lambda_s(i) \left(\sum_{j=2}^{r_m} \rho_i(j)y_j \right)^{i-1} \right\}^{s-\bar{s}}. \end{aligned}$$

Hence, we obtain Eq. (15).

Note that $\sum_{k=2}^{l_m} e_{k,s}(\mathbf{y}) = \sum_{\bar{s}=1}^s r_{s,\bar{s}}(\mathbf{y})$. Using Eqs. (15) and (17), we can write

$$\begin{aligned} r_{s,1}(\mathbf{y}) &= \sum_{k=2}^{l_m} e_{k,s}(\mathbf{y}) - \sum_{\bar{s}=2}^s r_{s,\bar{s}}(\mathbf{y}) \\ &= \epsilon \sum_{i=2}^{l_m} \pi(i,s) \left(\sum_{j=2}^{r_m} \rho_i(j)y_j \right)^{i-1} \left[y_s - 1 \right. \\ &\quad \left. + \left\{ 1 - \epsilon \sum_{i=2}^{l_m} \lambda_s(i) \left(\sum_{j=2}^{r_m} \rho_i(j)y_j \right)^{i-1} \right\}^{s-1} \right]. \end{aligned}$$

Hence, we obtain Eq. (14).



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