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TE Volume Modes in Anisotropic Single-Negative Slab with Negative Component in Permeability Tensor

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SUMMARY Existence of backward TE volume modes which are to be identified as Magnetostatic Wave (MSW) in anisotropic single-negative slab with partly negative permeability tensor component have already been revealed by present authors. In this paper, detailed modal analysis has been carried out for this kind of TE volume modes to find out their novel and peculiar properties. From these numerical results, it has been clarified that dispersion curve of the lowest order mode for thicker slab has a frequency of turning point below which both forward and backward waves can be simultaneously observed and also there is a critical slab thickness for each order of TE volume modes to exist.

key words: *volume mode, single-negative, tensor material parameters, metamaterial*

1. Introduction

Metamaterial with negative material parameters has attracted a great deal of attention due to their eternal possibility of a new vista in the field of electromagnetic (EM) applications [1]-[5]. Ordinary material in nature is double-positive media with all positive material parameters, however, it has been reported that the appropriately constructed metamaterials only with nonmagnetic and non-ionized media show the negative permittivity and/or permeability [5]. The materials with negative parameters can be broadly divided into two categories; one is double-negative media with both permittivity and permeability negative, and the other is single-negative media with either permittivity or permeability negative which is the intermediate state between the double-positive and the double-negative media. Applying these metamaterials with negative parameters to the novel devices, their modal properties and the peculiar propagation behaviors of the wave in those materials must be revealed in advance.

Based on the modal analysis for single-negative slab, the present authors have revealed so far that only TE surface waves can exist on the isotropic single-negative slab with scalar negative permeability and also only TM surface modes on the slab with scalar negative permittivity [6]. In addition, it has already been presented that backward TE volume modes which are to be identified as the magnetostatic wave can be observed in the anisotropic single-negative slab whose permeability tensor has partly negative component [7]-[9].

In this paper, the detailed modal analysis has been achieved for the anisotropic single-negative slab with the permeability tensor whose x-component is negative and dispersive, in order to pursue the novel and peculiar properties of these TE volume modes.

At first, the detailed inspection on the dispersion characteristics of the TE volume modes would be carried out for the anisotropic single-negative slab with different thickness. Then, we will estimate the eigenvalues for each order TE volume modes as a function of the slab thickness. From these results, the peculiar modal characteristics of the TE volume modes in the anisotropic single-negative slab would be presented.

2. Basic Structure and Eigenvalue Equation

We analyze TE volume modes in the anisotropic single-negative slab as shown in Fig.1. The region II whose thickness is set as T is single-negative media and their tensor permittivity and permeability are assumed to be expressed as

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}, \ \hat{\mu} = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \quad (1)$$

where their components are allowed to be negative. The regions I and III are chosen as the free-space with the

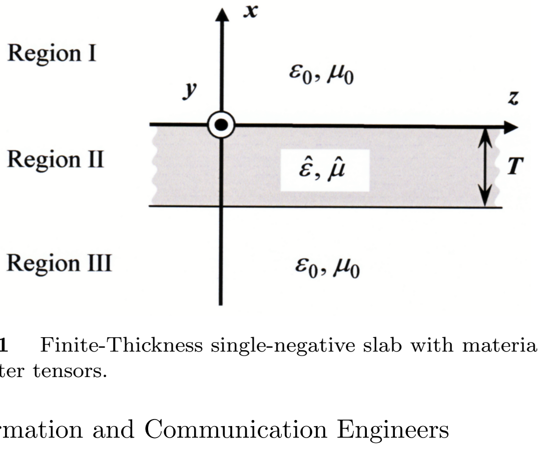

Fig. 1 Finite-Thickness single-negative slab with material parameter tensors.

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material parameters ε_0 and μ_0.

By solving the Maxwell 's equation in each region and applying the boundary conditions for tangential electric and magnetic fields, E_y and H_z, at $x = 0$ and $x = -T$ surfaces, the following eigenvalue equation for TE volume modes can be obtained.

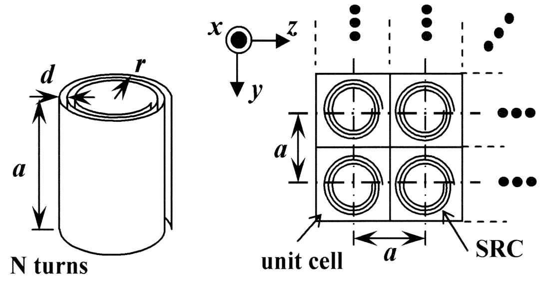
$$\left(\mu_0^2\delta_1^2 - \mu_z^2\delta_0^2\right)\sin\delta_1 T = 2\mu_0\mu_z\delta_0\delta_1\cos\delta_1 T \qquad (2)$$

with

$$\delta_0 = \sqrt{\beta^2 - \omega^2\varepsilon_0\mu_0}, \quad \delta_1 = \sqrt{\frac{\mu_z}{\mu_x}(\omega^2\varepsilon_y\mu_x - \beta^2)},$$

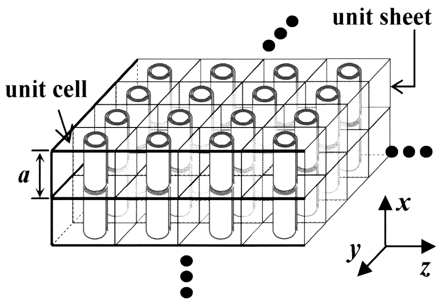
where ω is the radial frequency and β is the propagation constant, respectively.

Next, we define the positive and negative sign of the material tensor components in region II used for the numerical analysis. It has already been presented that TE volume modes can exist in the anisotropic single-negative slab which has all positive components in permittivity tensor but the component of the permeability tensor toward the normal to the slab surface, that is the x-direction in Fig.1, is negative [7]-[9]. Therefore, we shall take this sign combination for the material tensor components used for the analysis. The metamaterial having such permeability tensor would be realized by the stacked substrate sheets in which the periodically arrayed Swiss-Roll Capacitors (SRCs) are embedded as shown in Fig.2. It has been reported in the Ref.[5] that the material parameters of this structure can be estimated by using the concept of the effective permittivity and permeability which is valid for the periodic structures. According to this concept, if the whole structure can be regarded as the assembly of cubic unit cells whose dimensions are much smaller than the wavelength of the corresponding electromagnetic wave, the contents of the unit cell will define the effective response of the system as a whole structure, that is, the obtained material parameters for the unit cell would be equivalent to the effective material parameters of the whole structure. Therefore, as the unit cell of the structure shown in Fig.2, we have assumed the small cubic cell of dimensions a.



(a) Swiss-Roll Capacitor (SRC)

(b) Arranging sequence of SRCs in yz plane



(c) Stacking sequence of unit sheets

Fig. 2 Structure of metamaterial with negative permeability in x-direction by using arrayed SRCs [5], [8], where r = 0.2mm, a = 0.5mm, d = 0.01mm and N = 3.

Taking the shape of SRC into our considerations, the x-components of the permeability and permittivity tensors would exhibit a negative and dispersive permeability $\mu_x = \mu_r(\omega)\mu_0$, and a positive and non-dispersive permittivity $\varepsilon_x = \varepsilon_r\varepsilon_0$, where the relative permeability and permittivity of this structure can be expressed by using the structural parameters of SRCs as

$$\mu_r(\omega) = 1 - \frac{\pi r^2/a^2}{1 - \left\{dc_0^2/2\pi^2\omega^2(N-1)r^3\right\}}, \qquad (3)$$

and

$$\varepsilon_r = \left(1 - \frac{\pi r^2}{a^2}\right)^{-1} \qquad (4)$$

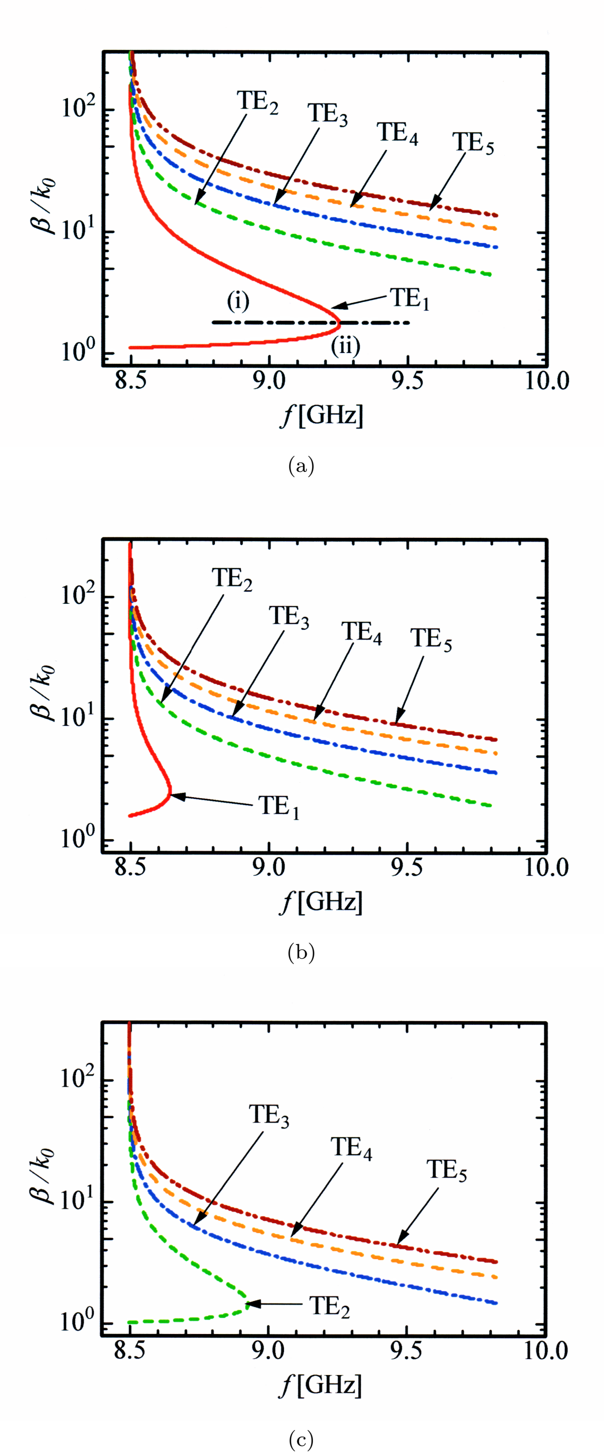
where the resistance of SRC is assumed to be 0.0 Ω/m^2 [5]. In Eqs.(3) and (4), c_0 presents the velocity of the light and the other dimensions and structural parameters used for the analysis are shown in Fig.2. Under these structural parameters, the relative permeability in the x-direction, $\mu_r(\omega)$, becomes smaller than -1.0 over 8.50GHz to 9.81GHz frequency range. The remaining components of the material tensor are set as $\mu_y = \mu_z = \mu_0$ and $\varepsilon_y = \varepsilon_z = \varepsilon_0$ [5], [8].

3. Modal Properties for TE Volume Modes

3.1 Dispersion Characteristics

By solving the eigenvalue equation (2) numerically, we obtained the dispersion characteristics of TE volume modes for three different thicknesses of the slab as shown in Figs.3.

Figure 3(a) presents the dispersion characteristics for the TE volume modes with slab thickness T=5.0mm. From this result, the propagation constants except for TE_1 mode go down toward the cutoff frequency at 9.81GHz. This nature tells us that the obtained TE modes excepting the first order mode are the backward waves. However, the dispersion curve of TE_1 mode shows a turning point around 9.25GHz below which there are two propagation constants at the same operating frequency [8], [9]. Thus the dispersion



(a)

(b)

(c)

Fig. 3 Dispersion characteristics of the TE volume modes in the anisotropic single-negative slab with thickness (a) T=5.0mm, (b) T=10.0mm, and (c) T=20.0mm, respectively.

curves of this mode can be divided into two categories (i) and (ii). The propagation constant on the branch (i) is the eigenvalue for the backward wave and the other on the branch (ii) is that for the forward wave. Incidentally, from another numerical result, it has been confirmed that the magnetic energy stored in the obtained TE$_1$ volume mode is extremely larger than the electric counterpart and also the dispersion curve of the TE$_1$ volume mode with the large propagation constant β/k_0 on branch (i) coincide with the results obtained by the magnetostatic approximation for the same structure, whereas their difference becomes larger when the frequency is closed to that of the turning point. Judging from these results, only a part of the obtained backward TE$_1$ volume mode with large β/k_0 would be identified as the magnetostatic wave but the TE$_1$ mode obtained around the frequency of turning point seems to be the magnetic wave [7] which defined as the wave with the larger magnetic energy than the electric energy.

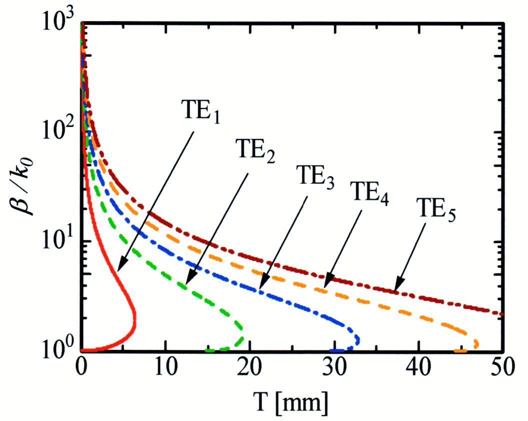


Fig. 4 The eigenvalues for TE volume modes as a function of the slab thickness, where the operating frequency is kept at 9.0GHz.

Next, Fig. 3(b) shows the results for the slab thickness T=10.0mm. In this case, the dispersion curve of TE$_1$ mode also shows a turning point at the lower frequency than that of T=5.0mm.

Furthermore, Fig.3(c) illustrates the dispersion curves of the TE volume modes for the much thicker slab thickness T=20.0mm. The noteworthy fact in this result is that the overall tendency of the dispersion curves is almost the same as those in Figs.3 (a) and (b) excepting for the disappearance of the dispersion curve of TE$_1$ mode from the analytical frequency range. In other words, it seems that the eigenvalues for the TE$_1$ volume mode are swept out from the analytical frequency range where the x-component of relative permeability tensor is less than -1.0.

3.2 Dependence on Slab Thickness

In order to clarify the relationship between the mode existence and the slab thickness in detail, we explore the eigenvalues of the TE volume modes as a function of the slab thickness with the operating frequency kept at 9.0GHz. The result is shown in Fig.4. From this result, it seems that there is the critical slab thickness for each order of the TE volume modes to exist. In other words, as the slab region becomes thicker and reaches the critical thickness, the lower order TE volume modes are disappeared in turn. For example, as the slab thickness is thinner than 6.4mm with the fixed operating frequency 9.5GHz, the eigenvalues for all TE volume modes can exist. However, if the slab thickness becomes thicker than 6.4mm, the eigenvalue for the first order TE_1 mode can not be observed.

In addition to this point, when the slab thickness is thinner than 6.4mm, there are two eigenvalues for TE_1 mode but the other higher order modes have only one eigenvalue, respectively. The same phenomenon for the TE_2 mode can be observed at the slab thickness around 19.1mm. These facts would be a cause of the appearance of the turning point on the dispersion curves and the disappearance of the TE_1 mode in Fig.3(c). This phenomenon is peculiarly different from those of the volume modes in the ordinary slab waveguide with the double-positive materials.

4. Conclusions

In this paper, we have presented the peculiar modal properties for TE volume modes in anisotropic single-negative slab whose permeability tensor has negative and dispersive x-component.

According to the careful inspection on the dispersion characteristic of these TE volume modes for different thickness of slab, it has been brought to light that the dispersion curve of the lowest order mode for the thicker slab has a frequency of turning point below which there are simultaneously two eigenvalues for forward and backward waves. In addition, we have indicated that not only obtained higher order TE volume modes but also a part of the lowest order backward TE volume mode with large β/k_0 on the thicker slab would be identified as the magnetostatic wave in spite of the fact that the metamaterial assumed as the slab region in this paper does not contain a magnetic material and there are no induced external magnetic field.

Furthermore, by inspecting the dependence of the propagation constants on the slab thickness more in detail, it has been clarified that there is the critical slab thickness for each order of the TE volume modes to exist and the critical thickness of higher order mode is thicker than those of lower order modes.

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