On the Correspondence of Dual Orbits Related to Some Representation, II

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(Received September 27, 2013)

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The purpose of this paper is to investigate the dual orbits correspondence of the non-regular simple prehomogeneous representation $(GL(1)^2 \times SL(6), \Lambda_3 + \Lambda_1, V(20) \oplus V(6))$ (see Proposition 2) .

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Preliminaries. In the following, we denote by G the group $GL(1)^2 \times SL(6)$ and by ρ the representation $\Lambda_3 + \Lambda_1$ of $SL(6)$ with scalar multiplications.

We define an element u_i of $\mathbb{C}^6$ by $u_i = {}^t(0, \cdots, 0, \overset{i}{\check{1}}, 0, \cdots, 0)$ for $1 \leq i \leq 6$. The representation space of ρ is identified with

$$V = \{\tilde{x} = (x_1, x_2); x_1 = \sum_{1 \leq i < j < k \leq 6} x_{ijk}\ u_i \wedge u_j \wedge u_k \in \overset{3}{\wedge}\mathbb{C}^6, x_2 = \sum_{l=1}^{6} y_l u_l \in \mathbb{C}^6\}.$$

Then the action ρ is given by

$$\rho(\tilde{g})\tilde{x} = (\alpha \rho_3(g) x_1, \beta g x_2)$$

for $\tilde{g} = (\alpha, \beta; g) \in G = GL(1)^2 \times SL(6)$ and $\tilde{x} = (x_1, x_2) \in V$, where $\rho_3(g) x_1 = \sum_{1 \leq i < j < k \leq 6} x_{ijk}(g u_i) \wedge (g u_j) \wedge (g u_k)$.

Proposition 1. The triplet (G, ρ, V) has fifteen orbits $\rho(G)\tilde{x}_i$ $(1 \leqq i \leqq 15)$ and the representative points $\tilde{x}_i$ $(1 \leqq i \leqq 15)$ are given as follows:

Representative point	Codimension
(1) $\tilde{x}_1 = (u_1 \wedge u_2 \wedge u_3 + u_4 \wedge u_5 \wedge u_6, u_1 + u_4)$	0
(2) $\tilde{x}_2 = (u_1 \wedge u_2 \wedge u_3 + u_4 \wedge u_5 \wedge u_6, u_1)$	3
(3) $\tilde{x}_3 = (u_1 \wedge u_2 \wedge u_3 + u_4 \wedge u_5 \wedge u_6, 0)$	6
(4) $\tilde{x}_4 = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5 + u_2 \wedge u_4 \wedge u_6, u_6)$	1
(5) $\tilde{x}_5 = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5 + u_2 \wedge u_4 \wedge u_6, u_1)$	4
(6) $\tilde{x}_6 = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5 + u_2 \wedge u_4 \wedge u_6, 0)$	7
(7) $\tilde{x}_7 = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5, u_6)$	5
(8) $\tilde{x}_8 = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5, u_2)$	6
(9) $\tilde{x}_9 = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5, u_1)$	10
(10) $\tilde{x}_{10} = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5, 0)$	11
(11) $\tilde{x}_{11} = (u_1 \wedge u_2 \wedge u_3, u_6)$	10
(12) $\tilde{x}_{12} = (u_1 \wedge u_2 \wedge u_3, u_1)$	13
(13) $\tilde{x}_{13} = (u_1 \wedge u_2 \wedge u_3, 0)$	16
(14) $\tilde{x}_{14} = (0, u_1)$	20
(15) $\tilde{x}_{15} = (0, 0)$	26

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In this section, we use the notations in [2]. Let Λ be the conormal bundle of an orbit S in V and Λ^* that of an orbit S^* in V^*. When $\Lambda = \Lambda^*$, we say that S and S^* are the dual orbits of each other. Since G is reductive, we have $(G, \rho^*, V^*) \cong (G, \rho, V)$ and hence the dual space V^* has also fifteen $\rho(G)$-orbits. We identify V and V^* as usual.

Proposition 2. The dual orbits correspondence of $(GL(1)^2 \times SL(6), \Lambda_3 + \Lambda_1)$ is given as follows:

Representative point	dual orbit	Representative point	dual orbit
$\tilde{x}_1$	$\tilde{x}_{15}$	$\tilde{x}_9$	$\tilde{x}_8$
$\tilde{x}_2$	$\tilde{x}_{12}$	$\tilde{x}_{10}$	$\tilde{x}_7$
$\tilde{x}_3$	$\tilde{x}_{14}$	$\tilde{x}_{11}$	$\tilde{x}_6$
$\tilde{x}_4$	$\tilde{x}_{13}$	$\tilde{x}_{12}$	$\tilde{x}_2$
$\tilde{x}_5$	$\tilde{x}_5$	$\tilde{x}_{13}$	$\tilde{x}_4$
$\tilde{x}_6$	$\tilde{x}_{11}$	$\tilde{x}_{14}$	$\tilde{x}_3$
$\tilde{x}_7$	$\tilde{x}_{10}$	$\tilde{x}_{15}$	$\tilde{x}_1$
$\tilde{x}_8$	$\tilde{x}_9$		

Put

$$\mathfrak{G} = \{(\alpha, \beta; A = (a_{ij})) : \alpha, \beta \in \mathbb{C}, A \in \mathfrak{gl}(6), \sum_{i=1}^{6} a_{ii} = 0\}.$$

We set $e_1 = u_1 \wedge u_2 \wedge u_3, e_2 = u_1 \wedge u_2 \wedge u_4, e_3 = u_1 \wedge u_2 \wedge u_5, e_4 = u_1 \wedge u_2 \wedge u_6, e_5 = u_1 \wedge u_3 \wedge u_4, e_6 = u_1 \wedge u_3 \wedge u_5, e_7 = u_1 \wedge u_3 \wedge u_6, e_8 = u_1 \wedge u_4 \wedge u_5, e_9 = u_1 \wedge u_4 \wedge u_6, e_{10} = u_1 \wedge u_5 \wedge u_6, e_{11} = u_2 \wedge u_3 \wedge u_4, e_{12} = u_2 \wedge u_3 \wedge u_5, e_{13} = u_2 \wedge u_3 \wedge u_6, e_{14} = u_2 \wedge u_4 \wedge u_5, e_{15} = u_2 \wedge u_4 \wedge u_6, e_{16} = u_2 \wedge u_5 \wedge u_6, e_{17} = u_3 \wedge u_4 \wedge u_5, e_{18} = u_3 \wedge u_4 \wedge u_6, e_{19} = u_3 \wedge u_5 \wedge u_6$ and $e_{20} = u_4 \wedge u_5 \wedge u_6$.

For $A = \begin{pmatrix} a_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_2 & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_3 & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_4 & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_5 & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_6 \end{pmatrix} \in \mathfrak{gl}(6)$ with $\sum_{i=1}^{6} a_i = 0$, we have the following equation:

$$d\rho_3(A)(e_1, e_2, e_3, \cdots, e_{20}) = (e_1, e_2, e_3, \cdots, e_{20}) \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix},$$

where

$$B_{11} = \begin{pmatrix} A_{123} & a_{34} & a_{35} & a_{36} & -a_{24} & -a_{25} & -a_{26} & 0 & 0 & 0 \\ a_{43} & A_{124} & a_{45} & a_{46} & a_{23} & 0 & 0 & -a_{25} & -a_{26} & 0 \\ a_{53} & a_{54} & A_{125} & a_{56} & 0 & a_{23} & 0 & a_{24} & 0 & -a_{26} \\ a_{63} & a_{64} & a_{65} & A_{126} & 0 & 0 & a_{23} & 0 & a_{24} & a_{25} \\ -a_{42} & a_{32} & 0 & 0 & A_{134} & a_{45} & a_{46} & -a_{35} & -a_{36} & 0 \\ -a_{52} & 0 & a_{32} & 0 & a_{54} & A_{135} & a_{56} & a_{34} & 0 & -a_{36} \\ -a_{62} & 0 & 0 & a_{32} & a_{64} & a_{65} & A_{136} & 0 & a_{34} & a_{35} \\ 0 & -a_{52} & a_{42} & 0 & -a_{53} & a_{43} & 0 & A_{145} & a_{56} & -a_{46} \\ 0 & -a_{62} & 0 & a_{42} & -a_{63} & 0 & a_{43} & a_{65} & A_{146} & a_{45} \\ 0 & 0 & -a_{62} & a_{52} & 0 & -a_{63} & a_{53} & -a_{64} & a_{54} & A_{156} \end{pmatrix},$$

$$B_{21} = \begin{pmatrix} a_{41} & -a_{31} & 0 & 0 & a_{21} & 0 & 0 & 0 & 0 & 0 \\ a_{51} & 0 & -a_{31} & 0 & 0 & a_{21} & 0 & 0 & 0 & 0 \\ a_{61} & 0 & 0 & -a_{31} & 0 & 0 & a_{21} & 0 & 0 & 0 \\ 0 & a_{51} & -a_{41} & 0 & 0 & 0 & 0 & a_{21} & 0 & 0 \\ 0 & a_{61} & 0 & -a_{41} & 0 & 0 & 0 & 0 & a_{21} & 0 \\ 0 & 0 & a_{61} & -a_{51} & 0 & 0 & 0 & 0 & 0 & a_{21} \\ 0 & 0 & 0 & 0 & a_{51} & -a_{41} & 0 & a_{31} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{61} & 0 & -a_{41} & 0 & a_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{61} & -a_{51} & 0 & 0 & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{61} & -a_{51} & a_{41} \end{pmatrix},$$

$$B_{12} =$$
$$\begin{pmatrix}
a_{14} & a_{15} & a_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-a_{13} & 0 & 0 & a_{15} & a_{16} & 0 & 0 & 0 & 0 & 0 \\
0 & -a_{13} & 0 & -a_{14} & 0 & a_{16} & 0 & 0 & 0 & 0 \\
0 & 0 & -a_{13} & 0 & -a_{14} & -a_{15} & 0 & 0 & 0 & 0 \\
a_{12} & 0 & 0 & 0 & 0 & 0 & a_{15} & a_{16} & 0 & 0 \\
0 & a_{12} & 0 & 0 & 0 & 0 & -a_{14} & 0 & a_{16} & 0 \\
0 & 0 & a_{12} & 0 & 0 & 0 & 0 & -a_{14} & -a_{15} & 0 \\
0 & 0 & 0 & a_{12} & 0 & 0 & a_{13} & 0 & 0 & a_{16} \\
0 & 0 & 0 & 0 & a_{12} & 0 & 0 & a_{13} & 0 & -a_{15} \\
0 & 0 & 0 & 0 & 0 & a_{12} & 0 & 0 & a_{13} & a_{14}
\end{pmatrix},$$

$$B_{22} =$$
$$\begin{pmatrix}
A_{234} & a_{45} & a_{46} & -a_{35} & -a_{36} & 0 & a_{25} & a_{26} & 0 & 0 \\
a_{54} & A_{235} & a_{56} & a_{34} & 0 & -a_{36} & -a_{24} & 0 & a_{26} & 0 \\
a_{64} & a_{65} & A_{236} & 0 & a_{34} & a_{35} & 0 & -a_{24} & -a_{25} & 0 \\
-a_{53} & a_{43} & 0 & A_{245} & a_{56} & -a_{46} & a_{23} & 0 & 0 & a_{26} \\
-a_{63} & 0 & a_{43} & a_{65} & A_{246} & a_{45} & 0 & a_{23} & 0 & -a_{25} \\
0 & -a_{63} & a_{53} & -a_{64} & a_{54} & A_{256} & 0 & 0 & a_{23} & a_{24} \\
a_{52} & -a_{42} & 0 & a_{32} & 0 & 0 & A_{345} & a_{56} & -a_{46} & a_{36} \\
a_{62} & 0 & -a_{42} & 0 & a_{32} & 0 & a_{65} & A_{346} & a_{45} & -a_{35} \\
0 & a_{62} & -a_{52} & 0 & 0 & a_{32} & -a_{64} & a_{54} & A_{356} & a_{34} \\
0 & 0 & 0 & a_{62} & -a_{52} & a_{42} & a_{63} & -a_{53} & a_{43} & A_{456}
\end{pmatrix}$$

and $A_{ijk} = a_i + a_j + a_j,\ 1 \leq i < j < k \leq 6.$

(1) The case of $\tilde{x}_1 = (u_1 \wedge u_2 \wedge u_3 + u_4 \wedge u_5 \wedge u_6, u_1 + u_4)$.

$$\mathfrak{G}_{\tilde{x}_1} = \{(0, a_2 + a_3;$$
$$\begin{pmatrix}
-a_2 - a_3 & a_{12} & a_{13} & 0 & 0 & 0 \\
0 & a_2 & a_{23} & 0 & 0 & 0 \\
0 & a_{32} & a_3 & 0 & 0 & 0 \\
0 & 0 & 0 & -a_2 - a_3 & a_{45} & a_{46} \\
0 & 0 & 0 & 0 & a_5 & a_{56} \\
0 & 0 & 0 & 0 & a_{65} & a_2 + a_3 - a_5
\end{pmatrix}) \in \mathfrak{G}\}.$$

$V^*_{\tilde{x}_1} = \{\ (0,0)\ \}$. $\tilde{y}_0 = (0,0) \in \rho^*(G)\tilde{x}_{15}$.

(2) The case of $\tilde{x}_2 = (u_1 \wedge u_2 \wedge u_3 + u_4 \wedge u_5 \wedge u_6, u_1)$.

$$\mathfrak{G}_{\tilde{x}_2} = \{\tilde{A} = (0, a_2+a_3; \begin{pmatrix} -a_2-a_3 & a_{12} & a_{13} & 0 & 0 & 0 \\ 0 & a_2 & a_{23} & 0 & 0 & 0 \\ 0 & a_{32} & a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & a_{45} & a_{46} \\ 0 & 0 & 0 & a_{54} & a_5 & a_{56} \\ 0 & 0 & 0 & a_{65} & a_{65} & -a_4-a_5 \end{pmatrix}) \in \mathfrak{G}\}.$$

$V^*_{\tilde{x}_2} = \mathbb{C}\langle v_1, v_2, v_3 \rangle$, where $v_1 = (u_2 \wedge u_3 \wedge u_4, -u_4)$, $v_2 = (u_2 \wedge u_3 \wedge u_5, -u_5)$ and $v_3 = (u_2 \wedge u_3 \wedge u_6, -u_6)$.

$$d\rho_{\tilde{x}_2}(\tilde{A})(v_1, v_2, v_3) = (v_1, v_2, v_3) \begin{pmatrix} A_1 & -a_{54} & -a_{64} \\ -a_{45} & A_2 & -a_{65} \\ -a_{46} & -a_{56} & A_3 \end{pmatrix},$$

where $A_1 = -a_2 - a_3 - a_4$, $A_2 = -a_2 - a_3 - a_5$ and $A_3 = -a_2 - a_3 + a_4 + a_5$.
In this case, we have $(G_{x_2}, \rho_{x_2}, V^*_{x_2}) \cong (GL(3), \Lambda_1, V(3))$.
$\tilde{y}_0 = v_1 \in \rho^*(G)\tilde{x}_{12}$.

(3) The case of $\tilde{x}_3 = (u_1 \wedge u_2 \wedge u_3 + u_4 \wedge u_5 \wedge u_6, 0)$.

$$\mathfrak{G}_{\tilde{x}_3} = \{\tilde{A} = (0, \beta; A = \begin{pmatrix} a_1 & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{21} & a_2 & a_{23} & 0 & 0 & 0 \\ a_{31} & a_{32} & -a_1-a_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & a_{45} & a_{46} \\ 0 & 0 & 0 & a_{54} & a_5 & a_{56} \\ 0 & 0 & 0 & a_{65} & a_{65} & -a_4-a_5 \end{pmatrix}) \in \mathfrak{G}\}.$$

$V^*_{\tilde{x}_3} = \mathbb{C}\langle v_1, v_2, v_3, v_4, v_5, v_6 \rangle$, where $v_1 = (0, u_1)$, $v_2 = (0, u_2)$, $v_3 = (0, u_3)$, $v_4 = (0, u_4)$, $v_5 = (0, u_5)$ and $v_6 = (0, u_6)$.
$d\rho_{\tilde{x}_3}(\tilde{A})(0, y) = (0, -{}^tAy - \beta y)$ for $(0, y) \in V^*_{\tilde{x}_3}$.
$\tilde{y}_0 = v_1 + v_4 = (0, e_1 + e_4) \in \rho^*(G)\tilde{x}_{14}$.

(4) The case of $\tilde{x}_4 = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5 + u_2 \wedge u_4 \wedge u_6, u_6)$.

$$\mathfrak{G}_{\tilde{x}_4} = \{\tilde{A} = (a_1 + a_3 + a_5, -3a_1 - 2a_3 - 2a_5;$$
$$\begin{pmatrix} a_1 & a_{12} & a_{13} & a_{14} & a_{15} & 0 \\ 0 & a_2 & a_{23} & a_{24} & a_{25} & 0 \\ 0 & 0 & a_3 & 0 & a_{35} & 0 \\ 0 & -a_{35} & a_{25} & a_4 & a_{45} & 0 \\ 0 & 0 & -a_{24} & 0 & a_5 & 0 \\ 0 & 0 & a_{14} & 0 & -a_{12} & a_6 \end{pmatrix}) \in \mathfrak{G} : \begin{array}{l} a_2 = -2a_1 - 2a_3 - a_5, \\ a_4 = -2a_1 - a_3 - 2a_5, \\ a_6 = 3a_1 + 2a_3 + 2a_5 \end{array} \}.$$

$V^*_{\tilde{x}_4} = \mathbb{C}\langle v_1 \rangle$, where $v_1 = (u_3 \wedge u_5 \wedge u_6, 0)$.
$d\rho_{\tilde{x}_4}(\tilde{A})(v_1) = -4(a_1 + a_3 + a_5)v_1$.
$\tilde{y}_0 = v_1 = (u_3 \wedge u_5 \wedge u_6, 0) \in \rho^*(G)\tilde{x}_{13}$ and $\tilde{y}_1 = (0,0) \in \rho^*(G)\tilde{x}_{15}$.

(5) The case of $\tilde{x}_5 = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5 + u_2 \wedge u_4 \wedge u_6, u_1)$.

$$\mathfrak{G}_{\tilde{x}_5} = \{\tilde{A} = (a_1 + a_3 + a_5, -a_1; \begin{pmatrix} a_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_2 & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & 0 & a_3 & 0 & a_{35} & 0 \\ 0 & -a_{35} & a_{25} - a_{16} & a_4 & a_{45} & a_{46} \\ 0 & 0 & -a_{24} & 0 & a_5 & 0 \\ 0 & 0 & a_{14} & 0 & -a_{12} & a_6 \end{pmatrix}) \in \mathfrak{G} :$$
$$a_2 = -2a_1 - 2a_3 - a_5, a_4 = -2a_1 - a_3 - 2a_5, a_6 = 3a_1 + 2a_3 + 2a_5 \}.$$

$V^*_{\tilde{x}_5} = \mathbb{C}\langle v_1, v_2, v_3, v_4 \rangle$, where $v_1 = (u_1 \wedge u_3 \wedge u_5 - u_2 \wedge u_3 \wedge u_6 - u_4 \wedge u_5 \wedge u_6, 2u_6)$, $v_2 = (u_2 \wedge u_3 \wedge u_5, -u_5)$, $v_3 = (u_3 \wedge u_4 \wedge u_5, -u_3)$ and $v_4 = (u_3 \wedge u_5 \wedge u_6, 0)$.

$$d\rho_{\tilde{x}_5}(\tilde{A})(v_1, v_2, v_3, v_4) = (v_1, v_2, v_3, v_4) \begin{pmatrix} A_1 & 0 & 0 & 0 \\ -2a_{12} & A_2 & -a_{35} & 0 \\ 2a_{14} & a_{24} & A_3 & 0 \\ -2a_{16} & -a_{26} & a_{46} & A_4 \end{pmatrix},$$

where $A_1 = -2(a_1 + a_3 + a_5), A_2 = a_1 - a_5, A_3 = a_1 - a_3$ and $A_4 = -4(a_1 + a_3 + a_5)$.
$\tilde{y}_0 = v_1 \in \rho^*(G)\tilde{x}_5$ and $\tilde{y}_1 = v_2 \in \rho^*(G)\tilde{x}_{12}$.

(6) The case of $\tilde{x}_6 = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5 + u_2 \wedge u_4 \wedge u_6, 0)$.

$$\mathfrak{G}_{\tilde{x}_6} = \{\tilde{A} = (a_1 + a_3 + a_5, \beta; \begin{pmatrix} a_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_2 & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & 0 & a_3 & 0 & a_{35} & a_{36} \\ a_{36} & -a_{35} & a_{25} - a_{16} & a_4 & a_{45} & a_{46} \\ 0 & 0 & -a_{24} & 0 & a_5 & -a_{21} \\ 0 & 0 & a_{14} & 0 & -a_{12} & a_6 \end{pmatrix}) \in \mathfrak{G} :$$
$$a_2 = -2a_1 - 2a_3 - a_5, a_4 = -2a_1 - a_3 - 2a_5, a_6 = 3a_1 + 2a_3 + 2a_5 \}.$$

$V^*_{\tilde{x}_6} = \mathbb{C}\langle v_1, v_2, v_3, v_4, v_5, v_6, v_7 \rangle$, where $v_1 = (u_3 \wedge u_5 \wedge u_6, 0)$, $v_2 = (0, u_1)$, $v_3 = (0, u_2)$, $v_4 = (0, u_3)$, $v_5 = (0, u_4)$, $v_6 = (0, u_5)$ and $v_7 = (0, u_6)$.

$d\rho_{\tilde{x}_6}(\tilde{A})(x_{19}u_3 \wedge u_5 \wedge u_6, y) = (-4(a_1+a_3+a_5)x_{19}u_3 \wedge u_5 \wedge u_6, -{}^tAy-\beta y)$ for $(x_{19}u_3 \wedge u_5 \wedge u_6, y) \in V^*_{\tilde{x}_6}$.

$\tilde{y}_0 = v_1 + v_2 = (u_3 \wedge u_5 \wedge u_6, u_1) \in \rho^*(G)\tilde{x}_{11}$ and $\tilde{y}_1 = v_2 \in \rho^*(G)\tilde{x}_{14}$.

(7) The case of $\tilde{x}_7 = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5, u_6)$.

$$\mathfrak{G}_{\tilde{x}_7} = \{\tilde{A} = (\alpha, -a_6; \begin{pmatrix} a_1 & a_{12} & a_{13} & a_{14} & a_{15} & 0 \\ 0 & a_2 & a_{23} & a_{24} & a_{25} & 0 \\ 0 & a_{32} & a_3 & a_{34} & a_{35} & 0 \\ 0 & -a_{35} & a_{25} & a_4 & a_{45} & 0 \\ 0 & a_{34} & -a_{24} & a_{54} & a_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_6 \end{pmatrix}) \in \mathfrak{G} : \begin{array}{l} a_3 = -a_1 - a_2 - \alpha, \\ a_5 = -a_1 - a_4 - \alpha, \\ a_6 = a_1 + 2\alpha \end{array} \}.$$

$V^*_{\tilde{x}_7} = \mathbb{C}\langle v_1, v_2, v_3, v_4, v_5 \rangle$, where $v_1 = (u_2 \wedge u_3 \wedge u_6 - u_4 \wedge u_5 \wedge u_6, 0)$, $v_2 = (u_2 \wedge u_4 \wedge u_6, 0)$, $v_3 = (u_2 \wedge u_5 \wedge u_6, 0)$, $v_4 = (u_3 \wedge u_4 \wedge u_6, 0)$ and $v_5 = (u_3 \wedge u_5 \wedge u_6, 0)$.

$$d\rho_{\tilde{x}_7}(\tilde{A})(v_1, v_2, v_3, v_4, v_5) = (v_1, v_2, v_3, v_4, v_5) \begin{pmatrix} -2\alpha & -a_{25} & a_{24} & -a_{35} & a_{34} \\ -2a_{34} & A_2 & -a_{54} & -a_{32} & 0 \\ -2a_{35} & -a_{45} & A_3 & 0 & -a_{32} \\ 2a_{24} & -a_{23} & 0 & A_4 & -a_{54} \\ 2a_{25} & 0 & -a_{23} & -a_{45} & A_5 \end{pmatrix},$$

where $A_2 = -a_1 - a_2 - a_4 - 3\alpha$, $A_3 = -a_2 + a_4 - 2\alpha$, $A_4 = a_2 - a_4 - 2\alpha$ and $A_5 = a_1 + a_2 + a_4 - \alpha$.

$\tilde{y}_0 = v_1 \in \rho^*(G)\tilde{x}_{10}$ and $\tilde{y}_1 = v_2 \in \rho^*(G)\tilde{x}_{13}$.

(8) The case of $\tilde{x}_8 = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5, u_2)$.

$$\mathfrak{G}_{\tilde{x}_8} = \{\tilde{A} = (\alpha, -a_2; \begin{pmatrix} a_1 & 0 & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_2 & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & 0 & a_3 & 0 & 0 & a_{36} \\ 0 & 0 & a_{25} & a_4 & a_{45} & a_{46} \\ 0 & 0 & -a_{24} & a_{54} & a_5 & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_6 \end{pmatrix}) \in \mathfrak{G} : \begin{array}{l} a_3 = -a_1 - a_2 - \alpha, \\ a_5 = -a_1 - a_4 - \alpha, \\ a_6 = a_1 + 2\alpha \end{array} \}.$$

$V^*_{\tilde{x}_8} = \mathbb{C}\langle v_1, v_2, v_3, v_4, v_5, v_6 \rangle$, where $v_1 = (u_1 \wedge u_3 \wedge u_6, u_6)$, $v_2 = (u_2 \wedge u_3 \wedge u_6 - u_4 \wedge u_5 \wedge u_6, 0)$, $v_3 = (u_2 \wedge u_4 \wedge u_6, 0)$, $v_4 = (u_2 \wedge u_5 \wedge u_6, 0)$,

$v_5 = (u_3 \wedge u_4 \wedge u_6, 0)$ and $v_6 = (u_3 \wedge u_5 \wedge u_6, 0)$.

$$d\rho_{\tilde{x}_8}(\tilde{A})(v_1, v_2, v_3, v_4, v_5, v_6) = (v_1, v_2, v_3, v_4, v_5, v_6) \begin{pmatrix} A_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2\alpha & -a_{25} & a_{24} & 0 & 0 \\ 0 & 0 & A_3 & -a_{54} & 0 & 0 \\ 0 & 0 & -a_{45} & A_4 & 0 & 0 \\ a_{14} & 2a_{24} & -a_{23} & 0 & A_5 & -a_{54} \\ a_{15} & 2a_{25} & 0 & -a_{23} & -a_{45} & A_6 \end{pmatrix},$$

where $A_1 = -a_1 + a_2 - 2\alpha, A_3 = -a_1 - a_2 - a_4 - 3\alpha, A_4 = -a_2 + a_4 - 2\alpha$, $A_5 = a_2 - a_4 - 2\alpha$ and $A_6 = a_1 + a_2 + a_4 - \alpha$.

$\tilde{y}_0 = v_1 + v_4 \in \rho^*(G)\tilde{x}_9$ and $\tilde{y}_1 = v_3 + v_6 \in \rho^*(G)\tilde{x}_{10}$.

(9) The case of $\tilde{x}_9 = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5, u_1)$.

$$\mathfrak{G}_{\tilde{x}_9} = \{\tilde{A} = (\alpha, -a_1; \begin{pmatrix} a_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_2 & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & a_{32} & a_3 & a_{34} & a_{35} & a_{36} \\ 0 & -a_{35} & a_{25} & a_4 & a_{45} & a_{46} \\ 0 & a_{34} & -a_{24} & a_{54} & a_5 & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_6 \end{pmatrix}) \in \mathfrak{G} : \begin{array}{l} a_3 = -a_1 - a_2 - \alpha, \\ a_5 = -a_1 - a_4 - \alpha, \\ a_6 = a_1 + 2\alpha \end{array} \}.$$

$V^*_{\tilde{x}_9} = \mathbb{C}\langle v_1, v_2, v_3, \cdots, v_{10} \rangle$, where $v_1 = (u_2 \wedge u_3 \wedge u_4, -u_4)$, $v_2 = (u_2 \wedge u_3 \wedge u_5, -u_5)$, $v_3 = (u_2 \wedge u_3 \wedge u_6, -u_6)$, $v_4 = (u_2 \wedge u_4 \wedge u_5, -u_2)$, $v_5 = (u_2 \wedge u_4 \wedge u_6, 0)$, $v_6 = (u_2 \wedge u_5 \wedge u_6, 0)$, $v_7 = (u_3 \wedge u_4 \wedge u_5, -u_3)$, $v_8 = (u_3 \wedge u_4 \wedge u_6, 0)$, $v_9 = (u_3 \wedge u_5 \wedge u_6, 0)$ and $v_{10} = (u_4 \wedge u_5 \wedge u_6, -u_6)$.

$$d\rho_{\tilde{x}_9}(\tilde{A})(v_1, v_2, v_3, \cdots, v_{10}) = (v_1, v_2, v_3, \cdots, v_{10})B,$$

$$B = \begin{pmatrix} A_1 & -a_{54} & 0 & -a_{24} & 0 & 0 & -a_{34} & 0 & 0 & 0 \\ -a_{45} & A_2 & 0 & -a_{25} & 0 & 0 & -a_{35} & 0 & 0 & 0 \\ -a_{46} & -a_{56} & A_3 & 0 & -a_{25} & a_{24} & 0 & -a_{35} & a_{34} & 0 \\ a_{35} & -a_{34} & 0 & A_4 & 0 & 0 & -a_{32} & 0 & 0 & 0 \\ a_{36} & 0 & -a_{34} & -a_{56} & A_5 & -a_{54} & 0 & -a_{32} & 0 & a_{34} \\ 0 & a_{36} & -a_{35} & a_{46} & -a_{45} & A_6 & 0 & 0 & -a_{32} & a_{35} \\ -a_{25} & a_{24} & 0 & -a_{23} & 0 & 0 & A_7 & 0 & 0 & 0 \\ -a_{26} & 0 & a_{24} & 0 & -a_{23} & 0 & -a_{56} & A_8 & -a_{54} & -a_{24} \\ 0 & -a_{26} & a_{25} & 0 & 0 & -a_{23} & a_{46} & -a_{45} & A_9 & -a_{25} \\ 0 & 0 & 0 & -a_{26} & a_{25} & -a_{24} & -a_{36} & a_{35} & -a_{34} & A_{10} \end{pmatrix},$$

where $A_1 = a_1 - a_4, A_2 = a_1 - a_5, A_3 = a_1 - a_6, A_4 = a_1 - a_2, A_5 = -a_1 - a_2 - a_4 - 3\alpha, A_6 = -a_2 + a_4 - 2\alpha, A_7 = 2a_1 + a_2 + \alpha, A_8 = a_2 - a_4 - 2\alpha, A_9 = a_1 + a_2 + a_4 - \alpha$ and $A_{10} = -2\alpha$.

$\tilde{y}_0 = v_1 + v_6 \in \rho^*(G)\tilde{x}_8$.

(10) The case of $\tilde{x}_{10} = (u_1 \wedge u_2 \wedge u_3 + u_1 \wedge u_4 \wedge u_5, 0)$.

$$\mathfrak{G}_{\tilde{x}_{10}} = \{\tilde{A} = (\alpha, \beta;$$

$$A = \begin{pmatrix} a_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_2 & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & a_{32} & a_3 & a_{34} & a_{35} & a_{36} \\ 0 & -a_{35} & a_{25} & a_4 & a_{45} & a_{46} \\ 0 & a_{34} & -a_{24} & a_{54} & a_5 & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_6 \end{pmatrix}) \in \mathfrak{G} : \begin{array}{l} a_3 = -a_1 - a_2 - \alpha, \\ a_5 = -a_1 - a_4 - \alpha, \\ a_6 = a_1 + 2\alpha \end{array} \}.$$

$V^*_{\tilde{x}_{10}} = \mathbb{C}\langle v_1, v_2, v_3, \cdots, v_{11} \rangle$, where $v_1 = (u_2 \wedge u_3 \wedge u_6 - u_4 \wedge u_5 \wedge u_6, 0)$, $v_2 = (u_2 \wedge u_4 \wedge u_6, 0)$, $v_3 = (u_2 \wedge u_5 \wedge u_6, 0)$, $v_4 = (u_3 \wedge u_4 \wedge u_6, 0)$, $v_5 = (u_3 \wedge u_5 \wedge u_6, 0)$, $v_6 = (0, u_1)$, $v_7 = (0, u_2)$, $v_8 = (0, u_3)$, $v_9 = (0, u_4)$, $v_{10} = (0, u_5)$ and $v_{11} = (0, u_6)$.

$$d\rho_{\tilde{x}_{10}}(\tilde{A})(v_1, v_2, v_3, \cdots, v_{11}) = (v_1, v_2, v_3, \cdots, v_{11}) \begin{pmatrix} B & 0 \\ 0 & -{}^tA - \beta I_6 \end{pmatrix},$$

$$B = \begin{pmatrix} -2\alpha & -a_{25} & a_{24} & -a_{35} & a_{34} \\ -2a_{34} & A_2 & -a_{54} & -a_{32} & 0 \\ -2a_{35} & -a_{45} & A_3 & 0 & -a_{32} \\ 2a_{24} & -a_{23} & 0 & A_4 & -a_{54} \\ 2a_{25} & 0 & -a_{23} & -a_{45} & A_5 \end{pmatrix},$$

where $A_2 = -a_1 - a_2 - a_4 - 3\alpha, A_3 = -a_2 + a_4 - 2\alpha, A_4 = a_2 - a_4 - 2\alpha$ and $A_5 = a_1 + a_2 + a_4 - \alpha$.

$\tilde{y}_0 = v_1 + v_6 \in \rho^*(G)\tilde{x}_7$ and $\tilde{y}_1 = v_2 + v_6 \in \rho^*(G)\tilde{x}_{11}$.

(11) The case of $\tilde{x}_{11} = (u_1 \wedge u_2 \wedge u_3, u_6)$.

$$\mathfrak{G}_{\tilde{x}_{11}} = \{\tilde{A} = (-a_1 - a_2 - a_3, a_6;$$

$$\begin{pmatrix} a_1 & a_{12} & a_{13} & a_{14} & a_{15} & 0 \\ a_{21} & a_2 & a_{23} & a_{24} & a_{25} & 0 \\ a_{31} & a_{32} & a_3 & a_{34} & a_{35} & 0 \\ 0 & 0 & 0 & a_4 & a_{45} & 0 \\ 0 & 0 & 0 & a_{54} & a_5 & 0 \\ 0 & 0 & 0 & a_{64} & a_{65} & a_6 \end{pmatrix}) \in \mathfrak{G} : a_6 = -\sum_{i=1}^{5} a_i\}.$$

$V^*_{\tilde{x}_{11}} = \mathbb{C}\langle v_1, v_2, v_3, \cdots, v_{10} \rangle$, where $v_1 = (u_1 \wedge u_4 \wedge u_5, 0)$, $v_2 = (u_1 \wedge u_4 \wedge u_6, 0)$, $v_3 = (u_1 \wedge u_5 \wedge u_6, 0)$, $v_4 = (u_2 \wedge u_4 \wedge u_5, 0)$, $v_5 = (u_2 \wedge u_4 \wedge u_6, 0)$, $v_6 = (u_2 \wedge u_5 \wedge u_6, 0)$, $v_7 = (u_3 \wedge u_4 \wedge u_5, 0)$, $v_8 = (u_3 \wedge u_4 \wedge u_6, 0)$, $v_9 = (u_3 \wedge u_5 \wedge u_6, 0)$ and $v_{10} = (u_4 \wedge u_5 \wedge u_6, 0)$.

$$d\rho_{\tilde{x}_{11}}(\tilde{A})(v_1, v_2, v_3, \cdots, v_{10}) = (v_1, v_2, v_3, \cdots, v_{10})B,$$

$$B = \begin{pmatrix} A_1 & -a_{65} & a_{64} & -a_{21} & 0 & 0 & -a_{31} & 0 & 0 & 0 \\ 0 & A_2 & -a_{54} & 0 & -a_{21} & 0 & 0 & -a_{31} & 0 & 0 \\ 0 & -a_{45} & A_3 & 0 & 0 & -a_{21} & 0 & 0 & -a_{31} & 0 \\ -a_{12} & 0 & 0 & A_4 & -a_{65} & a_{64} & -a_{32} & 0 & 0 & 0 \\ 0 & -a_{12} & 0 & 0 & A_5 & -a_{54} & 0 & -a_{32} & 0 & 0 \\ 0 & 0 & -a_{12} & 0 & -a_{45} & A_6 & 0 & 0 & -a_{32} & 0 \\ -a_{13} & 0 & 0 & -a_{23} & 0 & 0 & A_7 & -a_{65} & a_{64} & 0 \\ 0 & -a_{13} & 0 & 0 & -a_{23} & 0 & 0 & A_8 & -a_{54} & 0 \\ 0 & 0 & -a_{13} & 0 & 0 & -a_{23} & 0 & -a_{45} & A_9 & 0 \\ 0 & a_{15} & -a_{14} & 0 & a_{25} & -a_{24} & 0 & a_{35} & -a_{34} & A_{10} \end{pmatrix},$$

where $A_1 = a_2 + a_3 - a_4 - a_5, A_2 = a_1 + 2a_2 + 2a_3 + a_5, A_3 = a_1 + 2a_2 + 2a_3 + a_4, A_4 = a_1 + a_3 - a_4 - a_5, A_5 = 2a_1 + a_2 + 2a_3 + a_5, A_6 = 2a_1 + a_2 + 2a_3 + a_4, A_7 = a_1 + a_2 - a_4 - a_5, A_8 = 2a_1 + 2a_2 + a_3 + a_5, A_9 = 2a_1 + 2a_2 + a_3 + a_4$ and $A_{10} = 2a_1 + 2a_2 + 2a_3$.

$\tilde{y}_0 = v_1 + v_5 + v_9 \in \rho^*(G)\tilde{x}_6$.

(12) The case of $\tilde{x}_{12} = (u_1 \wedge u_2 \wedge u_3, u_1)$.

$$\mathfrak{G}_{\tilde{x}_{12}} = \{\tilde{A} = (-a_1 - a_2 - a_3, -a_1; \begin{pmatrix} a_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_2 & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & a_{32} & a_3 & a_{34} & a_{35} & a_{36} \\ 0 & 0 & 0 & a_4 & a_{45} & a_{46} \\ 0 & 0 & 0 & a_{54} & a_5 & a_{56} \\ 0 & 0 & 0 & a_{64} & a_{65} & a_6 \end{pmatrix}) \in \mathfrak{G} : a_6 = -\sum_{i=1}^{5} a_i\}.$$

$V^*_{\tilde{x}_{12}} = \mathbb{C}\langle v_1, v_2, v_3, \cdots, v_{13} \rangle$, where $v_1 = (u_1 \wedge u_4 \wedge u_5, 0)$, $v_2 = (u_1 \wedge u_4 \wedge u_6, 0)$, $v_3 = (u_1 \wedge u_5 \wedge u_6, 0)$, $v_4 = (u_2 \wedge u_3 \wedge u_4, -u_4)$, $v_5 = (u_2 \wedge u_3 \wedge u_5, -u_5)$, $v_6 = (u_2 \wedge u_3 \wedge u_6, -u_6)$, $v_7 = (u_2 \wedge u_4 \wedge u_5, 0)$, $v_8 = (u_2 \wedge u_4 \wedge u_6, 0)$, $v_9 = (u_2 \wedge u_5 \wedge u_6, 0)$, $v_{10} = (u_3 \wedge u_4 \wedge u_5, 0)$, $v_{11} = (u_3 \wedge u_4 \wedge u_6, 0)$, $v_{12} = (u_3 \wedge u_5 \wedge u_6, 0)$ and $v_{13} = (u_4 \wedge u_5 \wedge u_6, 0)$.

$$d\rho_{\tilde{x}_{12}}(\tilde{A})(v_1, v_2, v_3, \cdots, v_{13}) = (v_1, v_2, v_3, \cdots, v_{13})B,$$

$$B =$$

$$\begin{pmatrix} A_1 & -a_{65} & a_{64} & 0 & 0 & 0 & -a_{21} & 0 & 0 & -a_{31} & 0 & 0 & 0 \\ -a_{56} & A_2 & -a_{54} & 0 & 0 & 0 & 0 & -a_{21} & 0 & 0 & -a_{31} & 0 & 0 \\ a_{46} & -a_{45} & A_3 & 0 & 0 & 0 & 0 & 0 & -a_{21} & 0 & 0 & -a_{31} & 0 \\ 0 & 0 & 0 & A_4 & -a_{54} & -a_{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_{45} & A_5 & -a_{65} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_{46} & -a_{56} & A_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -a_{12} & 0 & 0 & a_{35} & -a_{34} & 0 & A_7 & -a_{65} & a_{64} & -a_{32} & 0 & 0 & 0 \\ 0 & -a_{12} & 0 & a_{36} & 0 & -a_{34} & -a_{56} & A_8 & -a_{54} & 0 & -a_{32} & 0 & 0 \\ 0 & 0 & -a_{12} & 0 & a_{36} & -a_{35} & a_{46} & -a_{45} & A_9 & 0 & 0 & -a_{32} & 0 \\ -a_{13} & 0 & 0 & -a_{25} & a_{24} & 0 & -a_{23} & 0 & 0 & A_{10} & -a_{65} & a_{64} & 0 \\ 0 & -a_{13} & 0 & -a_{26} & 0 & a_{24} & 0 & -a_{23} & 0 & -a_{56} & A_{11} & -a_{54} & 0 \\ 0 & 0 & -a_{13} & 0 & -a_{26} & a_{25} & 0 & 0 & -a_{23} & a_{46} & -a_{45} & A_{12} & 0 \\ -a_{16} & a_{15} & -a_{14} & 0 & 0 & 0 & -a_{26} & a_{25} & -a_{24} & -a_{36} & a_{35} & -a_{34} & A_{13} \end{pmatrix},$$

where $A_1 = a_2 + a_3 - a_4 - a_5, A_2 = a_1 + 2a_2 + 2a_3 + a_5, A_3 = a_1 + 2a_2 + 2a_3 + a_4, A_4 = a_1 - a_4, A_5 = a_1 - a_5, A_6 = a_1 - a_6, A_7 = a_1 + a_3 - a_4 - a_5, A_8 = 2a_1 + a_2 + 2a_3 + a_5, A_9 = 2a_1 + a_2 + 2a_3 + a_4, A_{10} = a_1 + a_2 - a_4 - a_5, A_{11} = 2a_1 + 2a_2 + a_3 + a_5, A_{12} = 2a_1 + 2a_2 + a_3 + a_4$ and $A_{13} = 2a_1 + 2a_2 + 2a_3$.

$\tilde{y}_0 = v_3 + v_4 \in \rho^*(G)\tilde{x}_2$.

(13) The case of $\tilde{x}_{13} = (u_1 \wedge u_2 \wedge u_3, 0)$.

$$\mathfrak{G}_{\tilde{x}_{13}} = \{\tilde{A} = (-a_1 - a_2 - a_3, \beta;$$

$$A = \begin{pmatrix} a_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_2 & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_3 & a_{34} & a_{35} & a_{36} \\ 0 & 0 & 0 & a_4 & a_{45} & a_{46} \\ 0 & 0 & 0 & a_{54} & a_5 & a_{56} \\ 0 & 0 & 0 & a_{64} & a_{65} & a_6 \end{pmatrix}) \in \mathfrak{G} : a_6 = -\sum_{i=1}^{5} a_i\}.$$

$V^*_{\tilde{x}_{13}} = \mathbb{C}\langle v_1, v_2, v_3, \cdots, v_{16} \rangle$, where $v_1 = (u_1 \wedge u_4 \wedge u_5, 0)$, $v_2 = (u_1 \wedge u_4 \wedge u_6, 0)$, $v_3 = (u_1 \wedge u_5 \wedge u_6, 0)$, $v_4 = (u_2 \wedge u_4 \wedge u_5, 0)$, $v_5 = (u_2 \wedge u_4 \wedge u_6, 0)$, $v_6 = (u_2 \wedge u_5 \wedge u_6, 0)$, $v_7 = (u_3 \wedge u_4 \wedge u_5, 0)$, $v_8 = (u_3 \wedge u_4 \wedge u_6, 0)$, $v_9 = (u_3 \wedge u_5 \wedge u_6, 0)$, $v_{10} = (u_4 \wedge u_5 \wedge u_6, 0)$, $v_{11} = (0, u_1)$, $v_{12} = (0, u_2)$, $v_{13} = (0, u_3)$, $v_{14} = (0, u_4)$, $v_{15} = (0, u_5)$ and $v_{16} = (0, u_6)$.

$$d\rho_{\tilde{x}_{13}}(\tilde{A})(v_1, v_2, v_3, \cdots, v_{16}) = (v_1, v_2, v_3, \cdots, v_{16}) \begin{pmatrix} B & 0 \\ 0 & -{}^tA - \beta I_6 \end{pmatrix},$$

$$B=\begin{pmatrix} A_1 & -a_{65} & a_{64} & -a_{21} & 0 & 0 & -a_{31} & 0 & 0 & 0 \\ -a_{56} & A_2 & -a_{54} & 0 & -a_{21} & 0 & 0 & -a_{31} & 0 & 0 \\ a_{46} & -a_{45} & A_3 & 0 & 0 & -a_{21} & 0 & 0 & -a_{31} & 0 \\ -a_{12} & 0 & 0 & A_4 & -a_{65} & a_{64} & -a_{32} & 0 & 0 & 0 \\ 0 & -a_{12} & 0 & -a_{56} & A_5 & -a_{54} & 0 & -a_{32} & 0 & 0 \\ 0 & 0 & -a_{12} & a_{46} & -a_{45} & A_6 & 0 & 0 & -a_{32} & 0 \\ -a_{13} & 0 & 0 & -a_{23} & 0 & 0 & A_7 & -a_{65} & a_{64} & 0 \\ 0 & -a_{13} & 0 & 0 & -a_{23} & 0 & -a_{56} & A_8 & -a_{54} & 0 \\ 0 & 0 & -a_{13} & 0 & 0 & -a_{23} & a_{46} & -a_{45} & A_9 & 0 \\ -a_{16} & a_{15} & -a_{14} & -a_{26} & a_{25} & -a_{24} & -a_{36} & a_{35} & -a_{34} & A_{10} \end{pmatrix},$$

where $A_1 = a_2 + a_3 - a_4 - a_5, A_2 = a_1 + 2a_2 + 2a_3 + a_5, A_3 = a_1 + 2a_2 + 2a_3 + a_4, A_4 = a_1 + a_3 - a_4 - a_5, A_5 = 2a_1 + a_2 + 2a_3 + a_5, A_6 = 2a_1 + a_2 + 2a_3 + a_4, A_7 = a_3 - a_6, A_8 = 2a_1 + 2a_2 + a_3 + a_5, A_9 = 2a_1 + 2a_2 + a_3 + a_4$ and $A_{10} = 2a_1 + 2a_2 + 2a_3$.

. $\tilde{y}_0 = v_1 + v_5 + v_9 + v_{11} \in \rho^*(G)\tilde{x}_4$.

(14) The case of $\tilde{x}_{14} = (0, u_1)$.

$$\mathfrak{G}_{\tilde{x}_{14}} = \{\tilde{A} = (\alpha, -a_1;\ A = \begin{pmatrix} a_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_2 & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & a_{32} & a_3 & a_{34} & a_{35} & a_{36} \\ 0 & a_{42} & a_{43} & a_4 & a_{45} & a_{46} \\ 0 & a_{52} & a_{53} & a_{54} & a_5 & a_{56} \\ 0 & a_{62} & a_{63} & a_{64} & a_{65} & a_6 \end{pmatrix}) \in \mathfrak{G} : a_6 = -\sum_{i=1}^{5} a_i\}.$$

$V^*_{\tilde{x}_{14}} = \mathbb{C}\langle v_1, v_2, v_3, \cdots, v_{20} \rangle$ where $v_i = (e_i, 0), 1 \leqq i \leqq 20$.
$d\rho_{\tilde{x}_{14}}(\tilde{A})(v_1, v_2, \cdots, v_{20}) = (v_1, v_2, \cdots, v_{20})\,(-\,{}^t d\rho_3(A) - \alpha I_{20})$.
$\tilde{y}_0 = v_1 + v_{10} + v_{20} \in \rho^*(G)\tilde{x}_3$.

(15) The case of $\tilde{x}_{15} = (0, 0)$.
In this case, we have $(G_{x_{15}}, \rho_{x_{15}}, V^*_{x_{15}}) \cong (G, \rho, V)$. $\tilde{y}_0 = \tilde{x}_1,\ \tilde{y}_1 = \tilde{x}_2$.

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