Artificial Life and Robotics manuscript No.
(will be inserted by the editor)

The effect of the internal parameters on association performance of a chaotic neural network

Shun Watanabe · Takashi Kuremoto · Kunikazu Kobayashi · Masanao Obayashi

Received: date / Accepted: date

Abstract A chaotic neural network proposed (CNN) by Aihara et al. is able to recollect stored patterns dynamically. But there are difficult cases such as its long time processing of association, and difficult to recall a specific stored pattern during the dynamical associations. We have proposed to find the optimal parameters using meta-heuristics methods to improve association performance, for example, the shorter recalling time and higher recollection rates of stored patterns in our previous works. However, the relationship between the different values of parameters of chaotic neurons and the association performance of CNN was not investigated clearly. In this paper, we propose a method to analyze the spatiotemporal changes of internal states in CNN and, by the method, analyze how the change of values of internal parameters of chaotic neurons affects the characteristics of chaotic neurons when multiple patterns are stored in the CNN. Quantile-Quantile plot (Q-Q plot), least square approximation (LSA), hierarchical clustering (HC), and Hilbert transform (HT) are used to investigate the similarity of internal states of chaotic neurons, and to classify the neurons. Simulation results showed that how different values of an internal parameter yielded different behaviors of chaotic neurons and it suggests the optimal parameter which generates higher association performance may concern with the stored patterns of the CNN.

Keywords Chaotic neural network · Quantile-Quantile plot · Least squares approximation · Hierarchical clustering · Hilbert transform

This work was presented in part at the 18th International Symposium on Artificial Life and Robotics, Daejeon, Korea, January 30-February 1, 2013.

S. Watanabe · T. Kuremoto (✉) · M. Obayashi
Graduate School of Science and Engineering, Yamaguchi University, Tokiwadai 2-16-1, Ube-shi, Yamaguchi 755-8611, Japan
Tel.: +81-836-859005, Fax: +81-836-859016
E-mail: wu@yamaguchi-u.ac.jp

K. Kobayashi
Aichi Prefectural University

1 Introduction

The chaotic neural network proposed (CNN) by Aihara et al. is well-known as a recurrent neural network which is able to dynamically recollect stored patterns [1,2]. And CNN has been widely applied to optimization [3], parallel distribution processing [4,5], robotics [6], and so on. In our previous works, we have proposed some meta-heuristic methods (e.g. genetic algorithm, particle swarm optimization) to determine the optimal parameters of chaotic neurons to realize higher performance of association memory of CNN [7,8]. However, the causality between the determined parameters and the association ability was not investigated. In this paper, we propose the method to extract the features of the spatiotemporal changes of internal states in CNN, analyze them and intend to analyze how the varied internal states of chaotic neurons are yielded by the different parameters of neuron dynamics. The spatiotemporal changes of the internal states of each chaotic neuron are observed, and the comparison of these time series data is given by kinds of methods such as Quantile-Quantile plot (Q-Q plot) [9], least squares approximation (LSA), hierarchical clustering (HC) [10]. And Hilbert transform (HT) [11]. Here, Q-Q plot, LSA, and HC are used to show the similarity between the dynamical changes of the internal states of chaotic neurons of CNN, meanwhile HT is used to show the synchronization of neurons during association process. Associative simulation showed that how change of an internal parameter affected the characteristics of neurons and the optimal parameter may concern with the stored patterns.

2 Chaotic neural network

Aihara et al.'s chaotic neural network (CNN) is a kind of interconnected recurrent artificial neural network which neurons perform chaotic output [1,2]. The dynamics of a chaotic neuron i of CNN is defined as follows:

$$\eta_i(t+1) = k_f \eta_i(t) + \sum_{j=0}^{N} w_{ij} x_j(t) \tag{1}$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha x_i(t) + a \tag{2}$$

$$y_i(t+1) = \eta_i(t+1) + \zeta_i(t+1) \tag{3}$$

$$x_i(t+1) = \frac{1}{1 + e^{-y_i(t+1)/\varepsilon}} \tag{4}$$

where N is number of chaotic neurons in network, $\eta_i(t)$ and $\zeta_i(t)$ are internal state for the feedback inputs and refractoriness, k_f and k_r are the decay parameters for the feedback input and the refractoriness, w_{ij} is a synaptic weight from the jth neuron to the ith neuron, α is a refractory scaling parameter, a is a threshold, $y_i(t)$ is the internal state, $x_i(t)$ is the chaotic neuron output, ε is a steepness parameter. In this paper, the internal states of the chaotic neurons are limited to $y_i(t), i = 0, 1, \ldots, N-1$ and the internal parameter k_r is investigated in detail.

Hebb learning rule [12] is used to store patterns in CNN, that is, the synaptic weight between two arbitrary neurons is modified as follow:

$$\Delta w_{ij} = \begin{cases} \beta \sum_{m=0}^{M-1} \chi_{mi} \chi_{mj} & \text{if } i \neq j \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where $\beta = 1/M$ is a learning coefficient, χ_{mi} is the ith bipolar value of the mth pattern.

3 Feature extraction

In this paper, Quantile-Quantile plot (Q-Q plot), least squares approximation (LSA), hierarchical clustering (HC), and Hilbert transform (HT) are used for analyzing the characteristics of the dynamical association in CNN. The dynamics that is in equilibrium, cycle or chaos but not divergence moves in bounded and then is able to represent by state distribution. So the results of the comparison between the internal state distributions in CNN represent as the characteristics of the relationship between the state distributions of neurons. And the characteristics of the relationship between the temporal changes of neurons are represented by the synchronization between neurons. This synchronization is able to give that the mutual change relative to fluctuation is happening at the same time, at a delay or at different timing. Firstly, Q-Q plot is used to compare the distribution types of each chaotic neuron internal states in CNN. Secondly, the curve plotted by Q-Q plot is approximated to a function by LSA. Finally, the approximation errors and approximated parameters are clustered by HC, and the clustered data as the similarity between neurons is shown. Because the synchronization depends on time, cannot be represented with the internal state distribution of each chaotic neuron, so HT is used to observe the synchronization between internal states of chaotic neuron.

3.1 Quantile-Quantile plot

Quantile-Quantile plot (Q-Q plot) is a graphical method to represent the similarity and characteristics between the distributions of two data sets [9]. When the elements of two data sets are sorted in ascending order, the elements of one data set are plotted on X-dimension in order, and the elements of another data set are plotted on Y-dimension either, then the correlation and the similarity of the two data sets are easily to be observed. If the plotted graph has linearity, it means that their data distributions were a certain similarity. On other hand, if the graph has nonlinearity, it means that their data are not similar. In this paper, similarities of all observed internal states of chaotic neurons in CNN are investigated using Q-Q plot, and the linearities are calculated by least squares approximation described in the next subsection.

3.2 Least squares approximation

Least squares approximation (LSA) is a method to find an approximate fitting function using samples of data set. In this paper, linear least square as a LSA method is used to quantitatively measure the plot graph (the sorted two data sets) in Q-Q plot mentioned in 3.1 section. A polynomial equation of the first degree (linear function) as an approximation function is used to measure the similarity (linearity) between the distributions of two data in Q-Q plot. Using the linear function given by LSA, we can measure the similarity of two data distributions by squares errors.

For example, as the quantitative values of the characteristics in Q-Q plot, the coefficient, the intercept and the squares error of approximation linear function are 1, 0 and 0 in the case those two distributions are equal. However, the coefficient and the intercept get to different values in the cases that the mutual scale of the distributions is different and that the mutual mean of distributions is different, respectively. The squares error is given a value of more than zero in the case that mutual distributions are different.

3.3 Hierarchical clustering

Hierarchical clustering (HC) is a clustering method that iteratively merges clusters in higher similarity [10]. And the method creates the clusters that have a tree-like structure. In this paper, the nearest neighbor method is used in HC, and Euclidean distance, as the degree of similarity of the internal state, is used to classify the chaotic neurons. Shorter Euclidean distance represents higher similarity between two clusters. The quantitative values calculated by LSA in Q-Q plot between each chaotic neuron and all are used in HC. So the similarities between the internal state distributions of all neurons are able to be represented in the tree-like structure of HC. And HC is able to give the clusters on voluntary hierarchy (at voluntary distance) by limiting maximum distance (setting the lower limit similarity). This means that the clusters and their clustered neurons at the quantitative value of similarity in HC are represented as the characteristics.

3.4 Hilbert transform

Hilbert transform (HT) is a transform technique to calculate the complex signal from an observed time series signal [11]. Here, the complex signal is transformed from the internal state time series signal of each chaotic neuron in CNN. And the synchronization between neurons is observed with the complex signals. The method of HT is as follows:

$$\text{FT}: y_i(t) \to y_i(\omega) \tag{6}$$

$$y_i(\omega) \equiv \text{Re}\left[y_i(\omega)\right] + I\text{Im}\left[y_i(\omega)\right] \tag{7}$$

$$y'_i(\omega) \equiv \text{Im}\left[y_i(\omega)\right] - I\text{Re}\left[y_i(\omega)\right] \tag{8}$$

$$\text{FT}^{-1}: y'_i(\omega) \to y_{iH}(t) \tag{9}$$

where $y_i(t)$ is an observed signal, FT is Fourier transform, I is the imaginary unit, FT^{-1} is inverse Fourier transform, $y_{iH}(t)$ is the signal of HT.

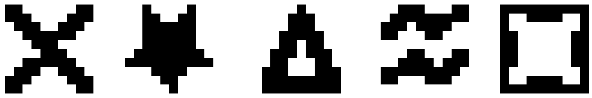
The phase difference $\phi_i(t) - \phi_j(t)$ of two signals $(y_i(t), y_j(t))$ is calculated by following equation:

$$\phi_i(t) - \phi_j(t) = \tan^{-1} \frac{y_{iH}(t) y_j(t) - y_i(t) y_{jH}(t)}{y_i(t) y_j(t) + y_{iH}(t) y_{jH}(t)} \tag{10}$$

This phase difference means the relationship between the fluctuations of two signals. For example, the phase difference gets to zero always in the case that two signals are same equilibrium, cycle or chaos and their temporal changes are happening at the same time (however, mutual scale does not affect the phase difference). And, the phase difference is not zero always in the case that two signals are same cycle and one side is delayed for a time interval. However, the phase difference changes frequently in the case that the mutual fluctuations of two signals change at different timing.

4 Computational simulations

The features of the internal states in CNN were observed when the value of the internal parameter k_r of CNN was different. Other parameters of the associative



"cross" "star" "triangle" "wave" "initial"

Fig. 1 "cross", "star", "triangle" and "wave" are stored patterns in CNN, and "initial" is an initial pattern of output in CNN, where both width and height of respective pattern are 10, black cell of respective pattern represents $\chi_{mi} = -1$, on other hand, white cell represents $\chi_{mi} = +1$

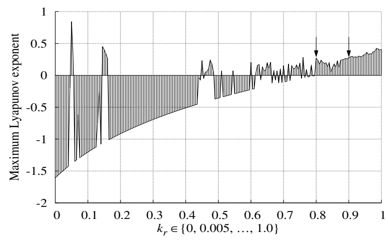
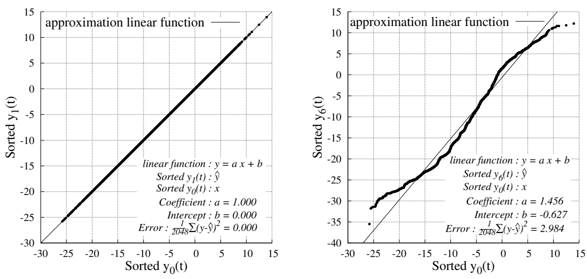


Fig. 2 Maximum Lyapunov exponents with different values of k_r

Table 1 Recalling times of stored patterns of each CNN and it's total times, where the stored pattern is regarded as recalling when Euclidean distance between output pattern and a stored pattern (as black cell is -1 and white cell is $+1$) is less than 1.

k_r	"cross"	"star"	"triangle"	"wave"	Total
0.9	6	2	8	14	30
0.8	0	7	12	0	19



(a) (b)

Fig. 3 The approximation linear function and the quantitative values (coefficient, intercept and squares error) by LSA in Q-Q plot with different chaotic neuron internal states.

model were set as $N = 100$ (10×10 network), $\zeta_i(0) = 0, \eta_i(0) = 0, k_f = 0.2, \alpha = 10, a = 2, \varepsilon = 0.015, \beta = 0.25$. The stored patterns in CNN were "cross", "star", "triangle", and "wave" as shown in Fig. 1 and they have been used usually as the stored patterns in CNN [2]. "initial" in Fig. 1 means the initial output state of CNN in recollection process. The internal states of chaotic neurons in CNN were recorded from $t = 2,048$ to $t = 4,095$ during a recalling.

The different values of k_r are 0.9 and 0.8 those are value used in [2] and simple value. And the maximum Lyapunov exponent of CNN with $k_r = 0.9$ was 0.290, and CNN with $k_r = 0.8$ was 0.245 from Fig. 2. The dynamical attractor of which the maximum Lyapunov exponent is positive is chaos (strange attractor). So both the CNN with $k_r = 0.9$ and the CNN with $k_r = 0.8$

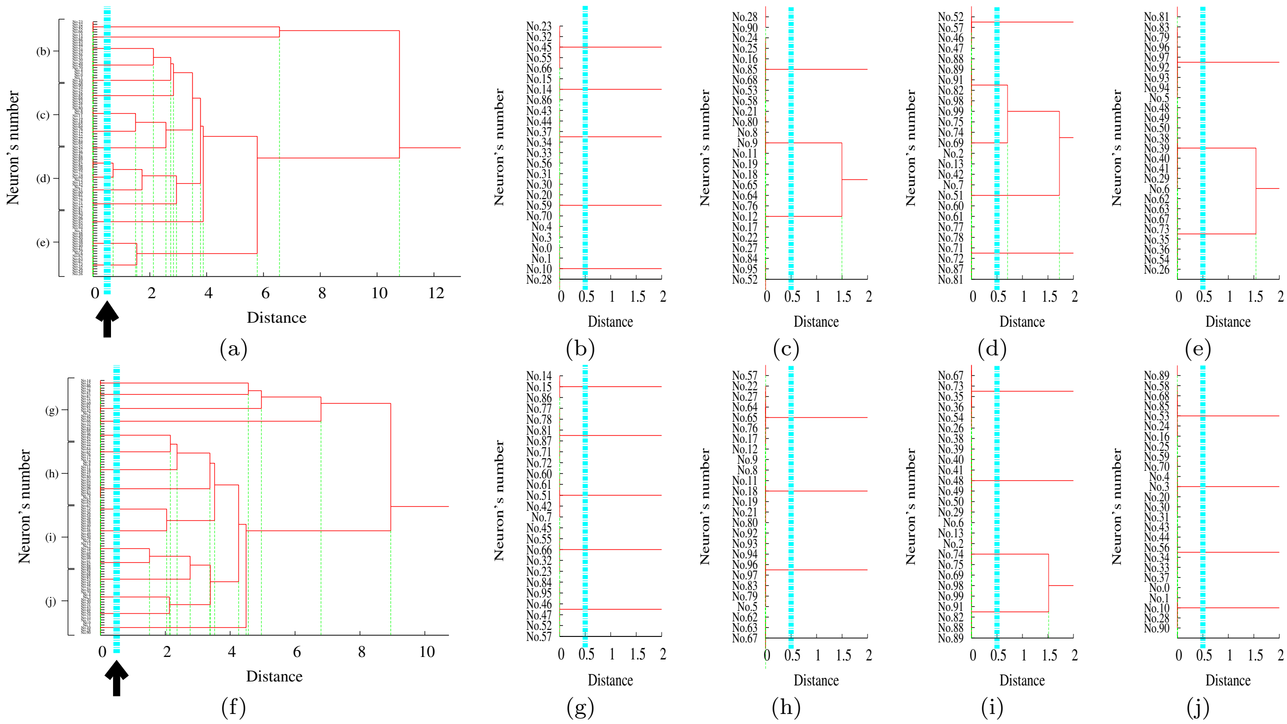


Fig. 4 The results of hierarchical clustering, horizontal axes are distance between clusters (similarity between neurons), vertical axes are neuron's number, arrow and broken line point at 0.5 distance and present clusters there: (a) is a whole image of hierarchical clustering in CNN with $k_r = 0.9$, (b), (c), (d), and (e) are parts of (a); (f) is in CNN with $k_r = 0.8$, (g), (h), (i), and (j) are parts of (f)

suggest that they are generating chaotic associative process. Table 1 shows the recalling times of the different stored patterns. The CNN with $k_r = 0.9$ had a higher recalling rates.

Fig. 3 shows examples of Q-Q plots and quantitations by LSA. The plotted graph has linearity in Fig. 3 (a) and then the quantitative values (coefficient, intercept and squares error) of approximation linear function by LSA are 1, 0 and 0 because the distributions of 0th neuron internal state and one of 1th neuron are more similar. On the other hand, because the internal state distributions of 0th neuron and 6th neuron are not similar, the graph in Fig. 3 (b) is nonlinear and then the quantitative values get to different values from what mutual distributions are similar.

The HC results of the case $k_r = 0.9$ (which was the value used in the original paper [2]) and the case $k_r = 0.8$ (which was an optional value used in this simulation) are shown in Fig. 4.

In Fig. 4, 100 neurons are arranged on the vertical axes which orders are according to their clusters. When the distances between the clusters (horizontal axes) is at the point with arrows in Fig. 4 (a) and Fig. 4 (f), the numbers of clusters were both 16, but neurons in those clusters were different when the parameter $k_r = 0.9$ and $k_r = 0.8$. Neurons in 16 clusters yielded by the different parameters are listed in Table 2.

Table 2 In the results of clustered neurons which internal states were similar during recollection using HC, cluster at 0.5 distance and neurons that belong to it's cluster.

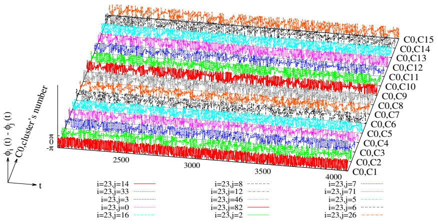
Cluster's No.	Neuron's No. ($k_r = 0.9$)	Neuron's No. ($k_r = 0.8$)
C0	23, 32, 45, 55, 66	14, 15, 86
C1	14, 15, 86	71, 72, 77, 78, 81, 87
C2	33, 34, 37, 43, 44, 56	7, 42, 51, 60, 61
C3	3, 4, 20, 30, 31, 59, 70	23, 32, 45, 55, 66
C4	0, 1, 10, 28, 90	46, 47, 52, 57, 84, 95
C5	16, 24, 25 53, 58, 68, 85	12, 17, 22, 27, 64, 65, 76
C6	8, 9, 11, 18, 19, 21, 80	8, 9, 11, 18, 19, 21, 80
C7	12, 17, 22, 27, 64, 65, 76	5, 79, 83, 92, 93, 94, 96, 97
C8	46, 47, 52, 57, 84, 95	26, 35, 36, 54, 62, 63, 67, 73
C9	82, 88, 89, 91, 98, 99	6, 29, 38, 39, 40, 41, 48, 49, 50
C10	2, 13, 69, 74, 75	2, 13, 69, 74, 75
C11	7, 42, 51, 60, 61	82, 88, 89, 91, 98, 99
C12	71, 72, 77, 78, 81, 87	16, 24, 25 53, 58, 68, 85
C13	5, 79, 83, 92, 93, 94, 96, 97	3, 4, 20, 30, 31, 59, 70
C14	6, 29, 38, 39, 40, 41, 48, 49, 50	33, 34, 37, 43, 44, 56
C15	26, 35, 36, 54, 62, 63, 67, 73	0, 1, 10, 28, 90

The phases between the neurons in the same cluster showed no any difference ($\phi_i(t) - \phi_j(t) \simeq 0, i, j \in \bigcup_{k=0}^{15} Ck$). Table 3 shows sample means and sample standard deviations as some examples of the phase comparison using neuron No. 23 in C0 via neuron No. 32, 45, 55, and 66 (the first row in Table 2 where $k_r = 0.9$), and No. 14 via No. 15, and 86 (the first row in Table 2 where $k_r = 0.8$). So the value of parameter k_r did not affect the synchronization of the neurons in the same cluster.

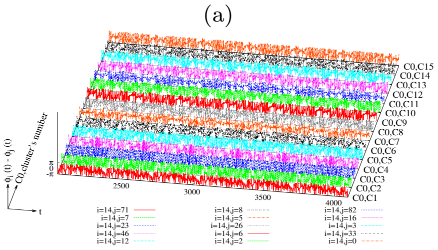
Fig. 5 shows the phase differences between the cluster C0 and other clusters (C1-C15) at each time. In Fig. 5, the horizontal axis indicates the time of recollection, the vertical axis is the phase difference, and the depth

Table 3 The means and the standard deviations of phase differences between the neurons of cluster C0, where sample means is $\overline{\phi_i - \phi_j} = \frac{1}{4095-2048+1}\sum_{t=2048}^{4095}(\phi_i(t) - \phi_j(t))$, sample standard deviations $s_{ij} = \sqrt{\frac{1}{4095-2048+1}}\sqrt{\sum_{t=2048}^{4095}[(\phi_i(t) - \phi_j(t)) - (\overline{\phi_i - \phi_j})]^2}$, t is recollection time.

kr	Combination of neuron no.	$\overline{\phi_i - \phi_j}$ (s_{ij})
0.9	$i = 23, j = 32$	$1.629 \cdot 10^{-18}$ $(2.001 \cdot 10^{-16})$
	$i = 23, j = 45$	$1.567 \cdot 10^{-17}$ $(1.314 \cdot 10^{-15})$
	$i = 23, j = 55$	$1.919 \cdot 10^{-17}$ $(1.292 \cdot 10^{-15})$
	$i = 23, j = 66$	$-5.407 \cdot 10^{-17}$ $(2.594 \cdot 10^{-15})$
0.8	$i = 14, j = 15$	$0.000 \cdot 10^{0}$ $(0.000 \cdot 10^{0})$
	$i = 14, j = 86$	$-9.780 \cdot 10^{-18}$ $(3.230 \cdot 10^{-16})$



(a)



(b)

Fig. 5 The phase differences between the neurons of different clusters, where three axes are recollection time t, phase difference $(\phi_i(t) - \phi_j(t))$, and combination with C0, respectively, recollection time t is from $t = 2048$ to $t = 4095$: (a) $k_r = 0.9$, $i = 23$ of C0, j is $14, 33, 3, 0, 16, 8, 12, 46, 82, 2, 7, 71, 5, 6$, or 26 of 16 clusters from C0 to C15. (b) $k_r = 0.8$, $i = 14$ of C0, j is $71, 7, 23, 46, 12, 8, 5, 26, 6, 2, 82, 16, 3, 33$, or 0 of 16 clusters from C0 to C15

axis corresponds to different clusters (from C1 to C15). And Fig. 6 shows the means and the standard deviations of the phase differences between clusters from $t = 2,048$ to $t = 4,095$ in CNN with $k_r = 0.9$ (shown in Fig. 4 (a)-(e)). So we can find that the change of the phase difference happened between the neurons of different clusters in CNN with $k_r = 0.9$ because the standard deviations of the phase differences were not zero. And though the standard deviations in CNN with $k_r = 0.8$ are not shown in this paper, they were not zero likewise.

Furthermore, to investigate the relationship between the stored patterns and the neuron spatial positions, we listed the output of neurons in 16 clusters, and it was interested that the number of clusters (i.e. 16) equaled to the number of the combination of stored patterns (see Table 4). For example, neuron No. 23, 32, 45, 55, and 66 in C0 when $k_r = 0.9$ and in C3 when $k_r = 0.8$, output "-1" when stored pattern "cross" was recalled.

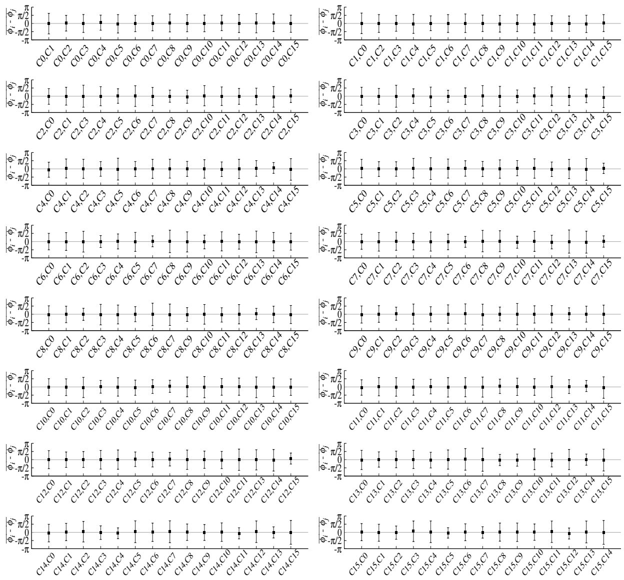


Fig. 6 The means and the standard deviations of the phase differences between clusters from $t = 2,048$ to $t = 4,095$ in CNN with $k_r = 0.9$

Table 4 The causality between elements combinations of stored patterns and clustered neurons.

Neuron's output corresponding to the stored patterns in Fig.1	Neuron's No.	Cluster's No. ($k_r = 0.9$)	Cluster's No. ($k_r = 0.8$)
-1, -1, -1, -1	26, 35, 36, 54, 62, 63, 67, 73	C15	C8
-1, +1, -1, -1	71, 72, 77, 78, 81, 87	C12	C1
-1, -1, -1, +1	33, 34, 37, 43, 44, 56	C2	C14
-1, -1, +1, -1	12, 17, 22, 27, 64, 65, 76	C7	C5
+1, -1, -1, -1	16, 24, 25 53, 58, 68, 85	C5	C12
-1, +1, -1, +1	82, 88, 89, 91, 98, 99	C9	C11
-1, +1, +1, -1	8, 9, 11, 18, 19, 21, 80	C6	C6
-1, -1, +1, +1	23, 32, 45, 55, 66	C0	C3
+1, +1, -1, -1	14, 15, 86	C1	C0
+1, -1, -1, +1	46, 47, 52, 57, 84, 95	C8	C4
+1, -1, +1, -1	2, 13, 69, 74, 75	C10	C10
-1, +1, +1, +1	0, 1, 10, 28, 90	C4	C15
+1, +1, -1, +1	5, 79, 83, 92, 93, 94, 96, 97	C13	C7
+1, +1, +1, -1	3, 4, 20, 30, 31, 59, 70	C3	C13
+1, -1, +1, +1	7, 42, 51, 60, 61	C11	C2
+1, +1, +1, +1	6, 29, 38, 39, 40, 41, 48, 49, 50	C14	C9

"+1", "+1", "-1" were output corresponding to the recalling of "star", "triangle" and "wave", respectively.

On the other hand, in 2 cases of CNN with $k_r = 0.5$ and CNN with $k_r = 0.795$ as non-chaotic associative process, Table 5 shows clusters at 0.5 distance and neurons that belong to their cluster on HC in CNN with $k_r = 0.5$ and CNN with $k_r = 0.795$ and Fig. 7 (a) and (b) show the means and the standard deviations of the phase differences between clusters in respective k_r. 0.5 and 0.795 are selected as the values that maximum lyapunov exponents are negative and one side is

Table 5 In the results of clustered neurons which internal states were similar during recollection using HC, cluster at 0.5 distance and neurons that belong to it's cluster.

Cluster's No.	Neuron's No. ($k_r = 0.5$)	Neuron's No. ($k_r = 0.795$)
C0	2, 13, 14, 15, 69, 74, 75, 86	2, 13, 69, 74, 75
C1	82, 88, 89, 91, 98, 99	8, 9, 11, 18, 19, 21, 80
C2	71, 72, 77, 78, 81, 87	82, 88, 89, 91, 98, 99
C3	46, 47, 52, 57, 84, 95	71, 72, 77, 81, 87
C4	33, 34, 37, 43, 44, 56	46, 47, 52, 57, 84, 95
C5	26, 35, 36, 54, 62, 63, 67, 73	33, 34, 37, 43, 44, 56
C6	23, 32, 45, 55, 66	26, 35, 36, 54, 62, 63, 67, 73
C7	16, 24, 25, 53, 58, 68, 85	23 ,32, 45, 55, 66
C8	12, 17, 22, 27, 64, 65, 76	16, 24, 25, 53, 58, 68, 85
C9	8, 9, 11, 18, 19, 21, 80	14, 15, 86
C10	7, 42, 51, 60, 61	12, 17, 22, 27, 64, 65, 76
C11	6, 29, 38, 39, 40, 41, 48, 49, 50	7, 42, 51, 60, 61
C12	5, 79, 83, 92, 93, 94, 96, 97	6, 29, 38, 39, 40, 41, 48, 49, 50
C13	3, 4, 20, 30, 31, 59, 70	5, 79, 83, 92, 93, 94, 96, 97
C14	0, 1, 10, 28, 90	3, 4, 20, 30, 31, 59, 70
C15		0, 1, 10, 28, 90

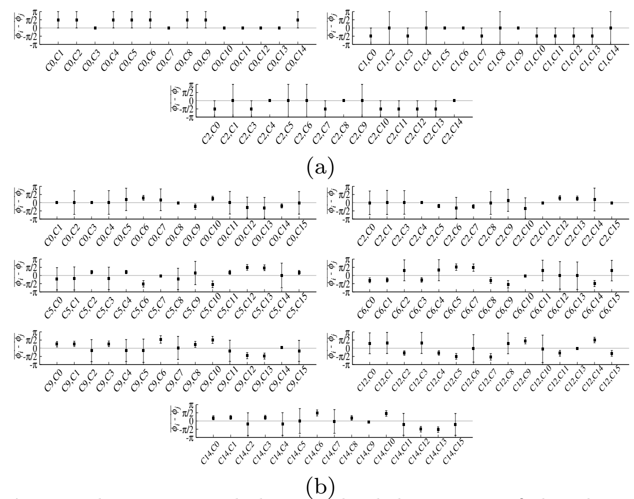


Fig. 7 The means and the standard deviations of the phase differences between clusters from $t = 2,048$ to $t = 4,095$; (a) CNN with $k_r = 0.5$; (b) CNN with $k_r = 0.795$

nearly zero from Fig. 2. The number of clusters in CNN with $k_r = 0.5$ of Table 5 is not different from Table 2 but, in CNN with $k_r = 0.795$ of Table 5, the number of clusters and their neurons are equal to Table 2. These mean that the internal state distributions in CNN with $k_r = 0.5$ have the different relationship from $k_r = 0.8$ and $k_r = 0.9$ and, in CNN with $k_r = 0.795$, have the similarly relationship from $k_r = 0.8$ and $k_r = 0.9$. However, Fig. 7 (a) and (b) show the synchronization between clusters because the standard deviations of the phase differences between some clusters (e.g. C0, C3, C7, C10, C11, C12 and C13; C1, C5, C6 and C9; C2, C4, C8 and C14 in $k_r = 0.5$, C0, C1, C3 and C8; C2, C4, C11 and C15; C5 and C7; C6 and C10; C12 and C13 in $k_r = 0.795$) are zero (synchronous). So, as non-chaotic association, CNN with $k_r = 0.5$ and CNN with $k_r = 0.795$ have the different characteristics from $k_r = 0.8$ and $k_r = 0.9$ as chaotic association.

Namely, the proposed analysis method is able to give the differences and the characteristics between the internal states in CNN with $k_r = 0.8$ or $k_r = 0.9$ as chaotic associative process and the internal states in CNN with $k_r = 0.5$ or $k_r = 0.795$ as non-chaotic associative process. And then the method gives that the neurons in chaotic associative process have the synchronization in the clusters relative to the elements combinations of stored patterns and the synchronization between clusters.

5 Conclusion

In this paper, we proposed the method to analyze the spatiotemporal changes of internal states in CNN and investigated the effect of the internal parameter of CNN on the association process by the proposed analysis method. The spatiotemporal change of the internal states of neurons was used to show the differences yielded by the change of the value of the internal parameter: the decay parameter for the refractoriness of chaotic neurons. The chaotic neurons were clustered according to their similarities of internal states, and the characterization of synchronization seemed affected by the parameter values. So these results show the validity of the proposed analysis method. The method is able to expect to support our previous works which suggested the importance of the optimal parameter for the dynamic association model CNNs.

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