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PAPER

Optimal Polyphase Asymmetric ZCZ Sequence Sets Including Uncorrelated Sequences

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Abstract **The present paper proposes a new method for constructing polyphase asymmetric zero-correlation zone (A-ZCZ) sequence sets using discrete Fourier transform (DFT) matrices and orthogonal codes. The proposed method can generate A-ZCZ sequence sets that cannot be obtained using known methods. The newly obtained A-ZCZ sequence sets include optimal ZCZ sequence sets. In addition, two arbitrary sequences that belong to different sequence subsets become uncorrelated sequences. The proposed method is expected to be useful for reducing or avoiding inter-cell interference from adjacent cells in approximately synchronized code-division multiple-access (AS-CDMA) systems.**

Keywords: spread spectrum communication, AS-CDMA, asymmetric ZCZ sequence sets, optimal ZCZ sequence sets, spreading sequences

1. Introduction

Code-division multiple-access (CDMA) has been widely applied in digital cellular systems. In CDMA systems, channel separation is provided by the correlation properties of pseudo-random codes referred to as spreading sequences. Therefore, spreading sequences with good autocorrelation and cross-correlation properties play an important part in CDMA systems.

In recent years, approximately synchronized CDMA (AS-CDMA) systems have attracted a great deal of attention because co-channel interference within a cell does not exist in some types of AS-CDMA systems [1]. In such AS-CDMA systems, zero-correlation zone (ZCZ) sequence sets are used as spreading sequences in order to realize this advantage [1]–[15]. Generally, a ZCZ sequence set is characterized by the sequence period, the number of sequences, the ZCZ length, and the number of phases of the sequence elements. The ZCZ length is restricted by a mathematical upper bound [6], [16], and ZCZ sequence sets that satisfy this mathematical upper bound are referred to as optimal ZCZ sequence sets. A number of studies have evaluated optimal ZCZ sequence sets [1], [6], [11], [14], [15]. On the other hand, quasi-optimal ZCZ sequence sets have a ZCZ length that is equal to one less than the mathematical upper bound. Quasi-optimal ZCZ sequence sets have also been investigated extensively [8], [13]. These methods can generate quasi-optimal ZCZ sequence sets under the conditions whereby no optimal ZCZ sequence sets exist.

Recently, new ZCZ sequence sets that are composed of several sequence subsets have been proposed [17]–[28]. These new ZCZ sequence sets have the property whereby the ZCZ length between sequences that belong to different sequence subsets is larger than the ZCZ length between sequences that belong to the same sequence subset. In order to emphasize this asymmetric property, we herein refer to these types of ZCZ sequence sets as asymmetric ZCZ (A-ZCZ) sequence sets. In addition, in order to distinguish between the different types of ZCZ lengths, we refer to these lengths as the zero-cross-correlation zone (ZCCZ) length between different sequence subsets and the ZCZ length in each sequence subset, respectively. Although some types of AS-CDMA systems can avoid co-channel interference in each cell, it is difficult to avoid inter-cell interference from adjacent cells in such AS-CDMA systems. However, if sequence subsets of an A-ZCZ sequence set are assigned to adjacent cells, the asymmetric property can be useful in reducing or avoiding inter-cell interference because of the larger ZCCZ length between different sequence subsets. Tang *et al.* proposed several types of binary A-ZCZ sequence sets [18], [20], and Hayashi *et al.* proposed several types of binary and ternary A-ZCZ sequence sets [17], [22], [23], [25], [28]. In addition, Hayashi *et al.* proposed a method for constructing A-ZCZ sequence sets that can be regarded as optimal ZCZ sequence sets [26]. Zhang *et al.* proposed complementary A-ZCZ sequence sets [21]. Moreover, we previously proposed a method for constructing polyphase A-ZCZ sequence sets that can be regarded as quasi-optimal ZCZ sequence sets [19], [24].

In the present paper, we propose a new method for constructing polyphase A-ZCZ sequence sets using discrete Fourier transform (DFT) matrices and orthogonal codes. The proposed method can generate A-ZCZ sequence sets that cannot be obtained using known methods. The newly obtained A-ZCZ sequence sets include optimal ZCZ sequence sets. In addition, the cross-correlation function between two arbitrary sequences that belong to different sequence subsets is zero at all shifts, i.e., two arbitrary sequences that belong to different sequence subsets become uncorrelated sequences. No A-ZCZ sequence sets that can be regarded as optimal ZCZ sequence sets and include uncorrelated sequences have been reported in previous studies. The proposed method is expected to be more useful for reducing or avoiding inter-cell interferences from adjacent cells than known A-ZCZ sequence sets.

2. Preliminaries

In this section, we define ZCZ sequence sets, A-ZCZ sequence sets, and related terms.

2.1 ZCZ Sequence Sets

In this subsection, we explain ZCZ sequence sets.

Let Z be a sequence set with M sequences of period P. Then, Z can be represented as

$$\begin{aligned} Z &= \{Z_m \mid 0 \le m \le M-1\} \\ Z_m &= \left(z_0^{(m)}, z_1^{(m)}, \cdots, z_p^{(m)}, \cdots, z_{P-1}^{(m)}\right) \end{aligned} \quad (1)$$

where Z_m and $z_p^{(m)}$ represent a sequence and a sequence element, respectively. Let $R_{Z_{m_0},Z_{m_1}}(\tau)$ be the periodic correlation function between Z_{m_0} and Z_{m_1}. For the sake of simplicity, the modulo operator is represented as %, i.e.,

$$x\%P \stackrel{\text{def}}{=} x \bmod P \quad (2)$$

Then, $R_{Z_{m_0},Z_{m_1}}(\tau)$ is defined as

$$R_{Z_{m_0},Z_{m_1}}(\tau) \stackrel{\text{def}}{=} \sum_{p=0}^{P-1} z_p^{(m_0)} z_{(p+\tau)\%P}^{(m_1)*} \quad (3)$$

where the symbol $*$ denotes complex conjugation. When Z_{m_0} does not correspond to Z_{m_1}, $R_{Z_{m_0},Z_{m_1}}(\tau)$ is referred to as the periodic cross-correlation function between Z_{m_0} and Z_{m_1}. On the other hand, when Z_{m_0} corresponds to Z_{m_1}, $R_{Z_{m_0},Z_{m_1}}(\tau) = R_{Z_{m_0},Z_{m_0}}(\tau)$ is referred to as the periodic autocorrelation function of Z_{m_0}. Note that $R_{Z_{m_0},Z_{m_1}}(\tau) = R_{Z_{m_0},Z_{m_1}}(\tau + kP)$, where k is an integer. If all of the sequences in Z satisfy the following conditions, then Z is referred to as a ZCZ sequence set:

$$\begin{aligned} &\forall m_0, 1 \le |\tau| \le L \\ &\qquad R_{Z_{m_0},Z_{m_0}}(\tau) = 0 \quad (4) \\ &\forall m_0 \ne m_1, |\tau| \le L \\ &\qquad R_{Z_{m_0},Z_{m_1}}(\tau) = 0 \quad (5) \end{aligned}$$

The integer $L\,(< P)$ is referred to as a ZCZ length. The ZCZ sequence set is represented as $Z_{cz}(P, M, L)$ in order to illustrate the sequence period, the number of sequences, and the ZCZ length. The ZCZ length is restricted by the following mathematical upper bound [6], [16]:

$$L \le \frac{P}{M} - 1 \quad (6)$$

If a ZCZ sequence set satisfies the following condition, the ZCZ sequence set is referred to as an optimal ZCZ sequence set:

$$L = \frac{P}{M} - 1 \quad (7)$$

Although L is always an integer, P/M is not always an integer. In the case in which P/M is not an integer, (7) can be generalized as follows [29]:

$$L = \left\lfloor \frac{P}{M} \right\rfloor - 1 \quad (8)$$

where $\lfloor P/M \rfloor$ is the floor function of P/M, i.e., $\lfloor P/M \rfloor$ is the maximum integer that does not exceed P/M.

2.2 A-ZCZ Sequence Sets

In this subsection, we explain A-ZCZ sequence sets.

Let A be a set with N sequence sets containing M sequences of period P. Then, A can be represented as

$$\begin{aligned} A &= \{A_n \mid 0 \le n \le N-1\} \\ A_n &= \left\{A_m^{(n)} \mid 0 \le m \le M-1\right\} \\ A_m^{(n)} &= \left(a_0^{(n,m)}, a_1^{(n,m)}, \cdots,\right. \\ &\qquad \left. a_p^{(n,m)}, \cdots, a_{P-1}^{(n,m)}\right) \end{aligned} \quad (9)$$

where A_n, $A_m^{(n)}$, and $a_p^{(n,m)}$ represent a sequence set, a sequence, and a sequence element, respectively. In the present paper, each A_n is referred to as a sequence subset. If all of the sequences in A satisfy the following conditions, then A is referred to as an A-ZCZ sequence set:

$$\begin{aligned} &\forall n_0, \forall m_0, 1 \le |\tau| \le \tilde{L} \\ &\quad R_{A_{m_0}^{(n_0)}, A_{m_0}^{(n_0)}}(\tau) = 0 \end{aligned} \tag{10}$$

$$\begin{aligned} &\forall n_0, \forall m_0 \ne m_1, |\tau| \le \tilde{L} \\ &\quad R_{A_{m_0}^{(n_0)}, A_{m_1}^{(n_0)}}(\tau) = 0 \end{aligned} \tag{11}$$

$$\begin{aligned} &\forall n_0 \ne n_1, \forall m_0, \forall m_1, |\tau| \le \Lambda \\ &\quad R_{A_{m_0}^{(n_0)}, A_{m_1}^{(n_1)}}(\tau) = 0 \end{aligned} \tag{12}$$

The integer $\tilde{L} \,(< P)$ is referred to as a ZCZ length in each sequence subset, and the integer $\Lambda \,(< P)$ is referred to as a ZCCZ length between different sequence subsets. The A-ZCZ sequence set is represented as $A_{zcz}\left(P, \{N, M\}, \left\{\tilde{L}, \Lambda\right\}\right)$ in order to illustrate the sequence period, the number of sequence subsets, the number of sequences in each sequence subset, the ZCZ length in each sequence subset, and the ZCCZ length between different sequence subsets. Note that an A-ZCZ sequence set can be regarded as a ZCZ sequence set. In this case, L is determined by the following formula:

$$L = \min\left\{\tilde{L}, \Lambda\right\} \tag{13}$$

3. New A-ZCZ Sequence Sets

In this section, we propose a new method for constructing polyphase A-ZCZ sequence sets based on DFT matrices and orthogonal codes. In addition, we present examples and a proof.

3.1 Sequence Generation

In this subsection, a new method for constructing polyphase A-ZCZ sequence sets is proposed.

Let F be the P-dimensional DFT matrix, i.e., F can be represented as

$$\begin{aligned} &F = [f_{i,j}] \\ &f_{i,j} = \frac{1}{\sqrt{P}} \exp\left(\frac{-2\pi\sqrt{-1}}{P} ij\right) \end{aligned} \tag{14}$$

where $f_{i,j}$ represents the (i,j)-th element of F, and $0 \le i, j \le P-1$. Let $R_{f_{i_0}, f_{i_1}}(\tau)$ be the periodic correlation function between the i_0-th row and the i_1-th row of the DFT matrix. The DFT matrix has the following correlation property:

$$\begin{aligned} &\forall i_0 \ne i_1, \forall \tau \\ &\quad R_{f_{i_0}, f_{i_1}}(\tau) = 0 \end{aligned} \tag{15}$$

Two sequences, the periodic cross-correlation function of which gives 0 at all shifts, are referred to as uncorrelated sequences. Therefore, two arbitrary rows in the DFT matrix are uncorrelated sequences. Two integers, M and N, are defined as follows:

$$\left\lfloor \frac{P}{M} \right\rfloor = N, M > 1, N > 1 \tag{16}$$

Let O be a set of M orthogonal codes of length M, i.e., O can be represented as

$$\begin{aligned} &O = \{O_m \mid 0 \le m \le M-1\} \\ &O_m = \left(o_0^{(m)}, o_1^{(m)}, \cdots, o_p^{(m)}, \cdots, o_{M-1}^{(m)}\right) \end{aligned} \tag{17}$$

$$\begin{aligned} &\forall m_0 \ne m_1 \\ &\quad R_{O_{m_0}, O_{m_1}}(0) = \sum_{p=0}^{M-1} o_p^{(m_0)} o_p^{(m_1)*} = 0 \end{aligned} \tag{18}$$

where O_m and $o_p^{(m)}$ represent a sequence and a sequence element, respectively. Using F and O, an A-ZCZ sequence set A is obtained by the following formula:

$$\begin{aligned} &A = \{A_n \mid 0 \le n \le N-1\} \\ &A_n = \left\{A_m^{(n)} \mid 0 \le m \le M-1\right\} \\ &A_m^{(n)} = \left(a_0^{(n,m)}, a_1^{(n,m)}, \cdots, a_p^{(n,m)}, \cdots, a_{PM-1}^{(n,m)}\right) \\ &a_p^{(n,m)} = f_{nM+p\%M, \lfloor p/M \rfloor} \cdot o_{p\%M}^{(m)} \end{aligned} \tag{19}$$

Then, A can be represented as $A_{zcz}(PM, \{N, M\}, \{M-1, PM-1\})$. Since the sequence period is PM and the ZCCZ length between different sequence subsets is $PM-1$, two arbitrary sequences that belong to different sequence subsets are uncorrelated sequences. This A-ZCZ sequence set can be regarded as a ZCZ sequence set. Since this A-ZCZ sequence set includes N sequence subsets and each sequence subset includes M sequences, the total number of sequences is NM. In addition, based on (13), it is clear that the ZCZ length is $M-1$. Therefore, this A-ZCZ sequence set is $Z_{cz}(PM, NM, M-1)$. If $P = NM$ is satisfied, $PM/NM = P/N$ corresponds with M. In this case, this A-ZCZ sequence set is an optimal ZCZ sequence set from the viewpoint of (7). Now, we consider the case in which at least one of M and N is not a divisor of P. If $\lfloor P/N \rfloor = M$ is satisfied, this A-ZCZ sequence set becomes an optimal ZCZ sequence set from the viewpoint of (8). Note that no A-ZCZ sequence sets that can be regarded as optimal ZCZ sequence sets and include uncorrelated sequences have been reported.

Thus, the proposed method can generate A-ZCZ sequence sets that have the following properties:

- Two arbitrary sequences that belong to different sequence subsets are uncorrelated sequences.

- If $P = NM$ is satisfied, the A-ZCZ sequence sets can be regarded as optimal ZCZ sequence sets in terms of (7).
- If at least one of M and N is not a divisor of P and $\lfloor P/N \rfloor = M$ is satisfied, the A-ZCZ sequence sets can be regarded as optimal ZCZ sequence sets in terms of (8).
- If $\lfloor P/N \rfloor = M$ is not satisfied, the A-ZCZ sequence sets are not optimal ZCZ sequence sets.

3.2 Examples

In this subsection, we present simple examples of the proposed method.

Example 1 First, we present an example that satisfies (7). Suppose that $P = 8$, then the eight-dimensional DFT matrix is represented as

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 0 & 6 & 4 & 2 & 0 & 6 & 4 & 2 \\ 0 & 5 & 2 & 7 & 4 & 1 & 6 & 3 \\ 0 & 4 & 0 & 4 & 0 & 4 & 0 & 4 \\ 0 & 3 & 6 & 1 & 4 & 7 & 2 & 5 \\ 0 & 2 & 4 & 6 & 0 & 2 & 4 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$

where each element represents a power of $\exp(2\pi\sqrt{-1}/8)$. Let $M = 4$ and $N = 2$. In addition, suppose that

$$O_0 = (0000)$$
$$O_1 = (0123)$$
$$O_2 = (0202)$$
$$O_3 = (0321)$$

where each element represents a power of $\exp(2\pi\sqrt{-1}/4)$. Then, $O = \{O_m \mid 0 \le m \le 3\}$ is a set of four quadriphase orthogonal codes of length 4, and an eight-phase A-ZCZ sequence set is obtained from (19). Each sequence of this A-ZCZ sequence set is given in Table 1. The periodic autocorrelation function of $A_{m_0}^{(n_0)}$ is given by

$$R_{A_{m_0}^{(n_0)}, A_{m_0}^{(n_0)}}(\tau) = (32, 0, 0, 0, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, 0, 0, 0)$$

where each x_i represents a complex number. Similarly, the periodic cross-correlation function between $A_{m_0}^{(n_0)}$ and $A_{m_1}^{(n_0)}$ $(m_0 \neq m_1)$ is given by

$$R_{A_{m_0}^{(n_0)}, A_{m_1}^{(n_0)}}(\tau) = (0, 0, 0, 0, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, 0, 0, 0)$$

Table 1 Sequence elements of Example 1

$A_0^{(0)}$	00000765064205270404036102460123
$A_1^{(0)}$	02460123000007650642052704040361
$A_2^{(0)}$	04040361024601230000076506420527
$A_3^{(0)}$	06420527040403610246012300000765
$A_0^{(1)}$	00004321064241630404472502464567
$A_1^{(1)}$	02464567000043210642416304044725
$A_2^{(1)}$	04044725024645670000432106424163
$A_3^{(1)}$	06424163040447250246456700004321

On the other hand, the periodic cross-correlation function between $A_{m_0}^{(n_0)}$ and $A_{m_1}^{(n_1)}$ $(n_0 \neq n_1)$ is given by

$$R_{A_{m_0}^{(n_0)}, A_{m_1}^{(n_1)}}(\tau) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$

Since the ZCZ length in each sequence subset is 3 and the ZCCZ length between different sequence subsets is 31, this A-ZCZ sequence set is $A_{zcz}(32, \{2, 4\}, \{3, 31\})$. Note that two arbitrary sequences that belong to different sequence subsets are uncorrelated sequences because the sequence period is 32 and the ZCCZ length between different sequence subsets is 31. From the viewpoint of (7), this A-ZCZ sequence set is also an optimal ZCZ sequence set that is represented as $Z(32, 8, 3)$.

Example 2 Next, we give another example that satisfies (8). Suppose that $P = 5$, then the five-dimensional DFT matrix is represented as

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 \\ 0 & 3 & 1 & 4 & 2 \\ 0 & 2 & 4 & 1 & 3 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

where each element represents a power of $\exp(2\pi\sqrt{-1}/5)$. Let $M = 2$ and $N = 2$. In addition, suppose that

$$O_0 = (00)$$
$$O_1 = (01)$$

where each element represents a power of $\exp(2\pi\sqrt{-1}/2)$. Then, $O = \{O_m \mid 0 \le m \le 1\}$ is a set of two binary orthogonal codes of length 2, and a ten-phase A-ZCZ sequence set is obtained from (19). Each sequence of this A-ZCZ sequence set is given in Table 2. Each element in Table 2 represents a power of $\exp(2\pi\sqrt{-1}/10)$. The periodic autocorrelation function of $A_{m_0}^{(n_0)}$ is given by

$$R_{A_{m_0}^{(n_0)}, A_{m_0}^{(n_0)}}(\tau) = (10, 0, x_2, x_3, x_4, x_5, x_6, x_7, x_8, 0)$$

Table 2 Sequence elements of Example 2

$A_0^{(0)}$	0008060402
$A_1^{(0)}$	0503010907
$A_0^{(1)}$	0064288246
$A_1^{(2)}$	0569238741

Similarly, the periodic cross-correlation function between $A_{m_0}^{(n_0)}$ and $A_{m_1}^{(n_0)}$ ($m_0 \neq m_1$) is given by

$$R_{A_{m_0}^{(n_0)},A_{m_1}^{(n_0)}}(\tau) = (0,0,x_2,x_3,x_4,x_5,x_6,x_7,x_8,0)$$

On the other hand, the periodic cross-correlation function between $A_{m_0}^{(n_0)}$ and $A_{m_1}^{(n_1)}$ ($n_0 \neq n_1$) is given by

$$R_{A_{m_0}^{(n_0)},A_{m_1}^{(n_1)}}(\tau) = (0,0,0,0,0,0,0,0,0,0)$$

Since the ZCZ length in each sequence subset is 1 and the ZCCZ length between different sequence subsets is 9, this A-ZCZ sequence set is $A_{zcz}(10,\{2,2\},\{1,9\})$. Note that two arbitrary sequences that belong to different sequence subsets are uncorrelated sequences because the sequence period is 10 and the ZCCZ length between different sequence subsets is 9. From the viewpoint of (8), this A-ZCZ sequence set is also an optimal ZCZ sequence set that is represented as $Z(10,4,1)$.

3.3 Proof

In this subsection, we prove the proposed method. Integers, p, p_0, p_1, τ, τ_0, and τ_1, are defined as

$$p = p_1 M + p_0 \tag{20}$$
$$\tau = \tau_1 M + \tau_0 \tag{21}$$
$$0 \leq p_0, \tau_0 \leq M-1 \tag{22}$$
$$0 \leq p_1, \tau_1 \leq P-1 \tag{23}$$
$$0 \leq p, \tau \leq PM-1 \tag{24}$$

In addition, an integer, ϵ, is defined as follows:

$$\text{For } p_0+\tau_0 < M$$
$$\epsilon = 0 \tag{25}$$
$$\text{For } p_0+\tau_0 \geq M$$
$$\epsilon = 1 \tag{26}$$

Then, we have

$$(p_0+\tau_0)\,\%M = p_0+\tau_0-\epsilon M \tag{27}$$
$$\lfloor (p_0+\tau_0)/M \rfloor = \epsilon \tag{28}$$

From (19), the periodic correlation function between $A_{m_0}^{(n_0)}$ and $A_{m_1}^{(n_1)}$ can be represented as

$$\begin{aligned}
&R_{A_{m_0}^{(n_0)},A_{m_1}^{(n_1)}}(\tau) \\
&= \sum_{p=0}^{PM-1} a_p^{(n_0,m_0)} \cdot a_{(p+\tau)\%PM}^{(n_1,m_1)*} \\
&= \sum_{p_0=0}^{M-1} o_{p_0}^{(m_0)} \cdot o_{(p_0+\tau_0)\%M}^{(m_1)*} \\
&\quad \cdot \sum_{p_1=0}^{P-1} f_{n_0 M+p_0,p_1} \\
&\quad \cdot f^*_{n_1 M+(p_0+\tau_0)\%M,(p_1+\tau_1+\lfloor (p_0+\tau_0)/M \rfloor)\%P} \\
&= \sum_{p_0=0}^{M-1} o_{p_0}^{(m_0)} \cdot o_{p_0+\tau_0-\epsilon M}^{(m_1)*} \\
&\quad \cdot \sum_{p_1=0}^{P-1} f_{n_0 M+p_0,p_1} \\
&\quad \cdot f^*_{n_1 M+p_0+\tau_0-\epsilon M,(p_1+\tau_1+\epsilon)\%P} \\
&= \sum_{p_0=0}^{M-1} o_{p_0}^{(m_0)} \cdot o_{p_0+\tau_0-\epsilon M}^{(m_1)*} \\
&\quad \cdot R_{f_{n_0 M+p_0},f_{n_1 M+p_0+\tau_0-\epsilon M}}(\tau_1+\epsilon)
\end{aligned} \tag{29}$$

Now, we consider the following two cases.

[A] Suppose that the two sequences $A_{m_0}^{(n_0)}$ and $A_{m_1}^{(n_1)}$ are included in the same sequence subset, namely, $n_0 = n_1$. In addition, suppose that $1 \leq \tau \leq M-1$. Then, from (21), (22), and (23),

$$1 \leq \tau_0 \leq M-1 \tag{30}$$
$$\tau_1 = 0 \tag{31}$$

In this case, we have

$$\tau_0 \neq \epsilon M \tag{32}$$

Therefore, the following condition is satisfied:

$$n_0 M + p_0 \neq n_1 M + p_0 + \tau_0 - \epsilon M \tag{33}$$

Note that $n_0 = n_1$. From (15), we have

$$R_{f_{n_0 M+p_0},f_{n_1 M+p_0+\tau_0-\epsilon M}}(\tau_1+\epsilon) = 0 \tag{34}$$

Based on (29), this means that

$$\begin{aligned}&\forall n_0, \forall m_0, \forall m_1, 1 \leq \tau \leq M-1 \\ &R_{A_{m_0}^{(n_0)},A_{m_1}^{(n_0)}}(\tau) = 0\end{aligned} \tag{35}$$

If m_0 and m_1 are exchanged, we have

$$\begin{aligned}&\forall n_0, \forall m_0, \forall m_1, 1 \leq \tau \leq M-1 \\ &R_{A_{m_1}^{(n_0)},A_{m_0}^{(n_0)}}(\tau) = 0\end{aligned} \tag{36}$$

The periodic correlation function has the following symmetric property [30]:

$$R_{A_{m_0}^{(n_0)},A_{m_1}^{(n_0)}}(-\tau) = R^*_{A_{m_1}^{(n_0)},A_{m_0}^{(n_0)}}(\tau) \tag{37}$$

From (36) and (37), we have

$$\forall n_0, \forall m_0, \forall m_1, -M+1 \le \tau \le -1$$
$$R_{A_{m_0}^{(n_0)}, A_{m_1}^{(n_0)}}(\tau) = 0 \tag{38}$$

When $\tau = 0$, we have $\tau_0 = \tau_1 = 0$ and $\epsilon = 0$ from (25). Therefore, (29) becomes

$$\begin{aligned} &R_{A_{m_0}^{(n_0)}, A_{m_1}^{(n_0)}}(\tau) \\ &\quad = R_{f_{n_0 M+p_0}, f_{n_0 M+p_0}}(0) \\ &\quad \cdot \sum_{p_0=0}^{M-1} o_{p_0}^{(m_0)} \cdot o_{p_0}^{(m_1)*} \end{aligned} \tag{39}$$

Since O is a set of orthogonal codes, (39) means that

$$\forall n_0, \forall m_0 \ne m_1$$
$$R_{A_{m_0}^{(n_0)}, A_{m_1}^{(n_0)}}(0) = 0 \tag{40}$$

[B] Suppose that the two sequences $A_{m_0}^{(n_0)}$ and $A_{m_1}^{(n_1)}$ are included in different sequence subsets, namely, $n_0 \ne n_1$. Since $\epsilon = 0$ when $\tau_0 = 0$, we have

$$(n_0 - n_1 + \epsilon) M \ne \tau_0 \tag{41}$$

Note that $n_0 \ne n_1$ and $0 \le \tau_0 \le M-1$. Therefore, the following condition is satisfied:

$$n_0 M + p_0 \ne n_1 M + p_0 + \tau_0 - \epsilon M \tag{42}$$

From (15), we have

$$R_{f_{n_0 M+p_0}, f_{n_1 M+p_0+\tau_0-\epsilon M}}(\tau_1 + \epsilon) = 0 \tag{43}$$

Based on (29), this means that

$$\forall n_0 \ne n_1, \forall m_0, \forall m_1, \forall \tau$$
$$R_{A_{m_0}^{(n_0)}, A_{m_1}^{(n_1)}}(\tau) = 0 \tag{44}$$

From (35), (38), (40), and (44),

$$\forall n_0, \forall m_0, 1 \le |\tau| \le M-1$$
$$R_{A_{m_0}^{(n_0)}, A_{m_0}^{(n_0)}}(\tau) = 0 \tag{45}$$
$$\forall n_0, \forall m_0 \ne m_1, |\tau| \le M-1$$
$$R_{A_{m_0}^{(n_0)}, A_{m_1}^{(n_0)}}(\tau) = 0 \tag{46}$$
$$\forall n_0 \ne n_1, \forall m_0, \forall m_1, \forall \tau$$
$$R_{A_{m_0}^{(n_0)}, A_{m_1}^{(n_1)}}(\tau) = 0 \tag{47}$$

Thus, the proposed method has been proven.

4. Conclusion

We have proposed a new method for constructing polyphase A-ZCZ sequence sets using DFT matrices and orthogonal codes. The proposed method can generate A-ZCZ sequence sets that cannot be obtained using known methods. The proposed method was obtained by improving our previously proposed methods [19], [24]. Although our previous methods use perfect sequences and orthogonal codes in order to construct A-ZCZ sequence sets, the proposed method uses DFT matrices instead of perfect sequences. The A-ZCZ sequence sets obtained by our previous methods can be regarded as quasi-optimal ZCZ sequence sets. On the other hand, the newly obtained A-ZCZ sequence sets include optimal ZCZ sequence sets. In addition, in our previous methods, two arbitrary sequences that belong to different sequence subsets are not uncorrelated sequences. On the other hand, in the proposed method, two arbitrary sequences that belong to different sequence subsets become uncorrelated sequences. These properties mean that the ZCZ length in each sequence subset and the ZCCZ length between different sequence subsets of the newly obtained A-ZCZ sequence sets are larger than those of the A-ZCZ sequence sets obtained by our previous methods. Moreover, no A-ZCZ sequence sets that can be regarded as optimal ZCZ sequence sets and include uncorrelated sequences have been reported by other researchers. Therefore, the proposed method is expected to be more useful for reducing or avoiding inter-cell interference from adjacent cells than known A-ZCZ sequence sets.

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