IEICE TRANSACTIONS

on Electronics

VOL.E76-C
NO.8
AUGUST 1993

The Institute of Electronics, Information and Communication Engineers
Kikai-Shinko-Kaikan Bldg., 5-8, Shibakoen 3chome, Minato-ku, Tokyo, 105 *Japan*

PAPER

Application of Beam Propagation Method to Discontinuities of Weakly Guiding Structures

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SUMMARY The beam propagation method (*BPM*) is a powerful and manageable method for the analysis of wave propagation along weakly guiding optical waveguides. However, the effects of reflected waves are not considered in the original *BPM*. In this paper, we propose two simple modifications of the *BPM* to make it relevant in characterizing waveguide discontinuities at which a significant amount of reflection is expected to be observed. Validity of the present modifications is confirmed by the numerical results for the slab waveguide discontinuities and the butt-joints between different slab waveguides which either support the dominant mode or higher order modes.
key words: *beam propagation method, reflected wave, waveguide discontinuity, overlap integral*

1. Introduction

In the design consideration of optical systems it is of fundamental importance to understand the behavior of optical waves propagating along various kinds of waveguides. The beam propagation method (*BPM*) proposed by Feit and Fleck [1] has been successfully applied to the analysis of a wide variety of weakly guiding structures which contain gradual variations in the waveguide parameters along the propagation direction. However, actual optical systems contain abrupt discontinuities of the parameters in many places at which an appreciable amount of incident power would be reflected. If one intends to apply the *BPM* to waveguide problems which include such discontinuities, some appropriate modification should be made in order to obtain trustworthy results [2]-[5], since reflected waves are not taken into consideration in the original *BPM* concept.

In the present paper, we modify the *BPM* to make it applicable to the characterization of waveguide discontinuities or butt-joints where the waveguide parameters, such as the waveguide thickness and refractive indices, abruptly change by a significant amount [6]. Variation in the waveguide thickness implies the axial displacement between the waveguides interconnected. Two methods of modification proposed in this paper take the Fresnel reflection into consideration. The main algorithm in the *BPM* repeats the same numerical procedure of the Fourier transformation of the modal field and its inverse transformation. One of the two modifications is dealt with in the spatial domain and the other is treated in the spectral domain. Both modifications can easily be incorporated with original *BPM* computing programs. Numerical results for an abrupt discontinuity in film thickness of a slab waveguide are compared with those given by the accurate analytical method [7] and the overlap integral of modal fields [8]-[11]. Some numerical results for slab waveguide butt-joints with axial displacement are presented, which are also compared with the results obtained from the overlap integral [8]-[11]. These numerical results are in good agreement, which verifies that the present methods are applicable to the analysis of abrupt discontinuities or butt-joints of slab waveguides.

2. Analytical Method

Suppose an incoming slab waveguide is butt-jointed, with the axial displacement Δs, to an outgoing one having a different film thickness or refractive indices, as shown in Fig. 1. The profile of the refractive index may be steplike or graded in both waveguides. Discontinuities in the refractive index at the interface, for instance $|n_{o1}-n_{o2}|$, may be considerably large. However, it is assumed that both waveguides satisfy the weakly guiding condition individually. Then the

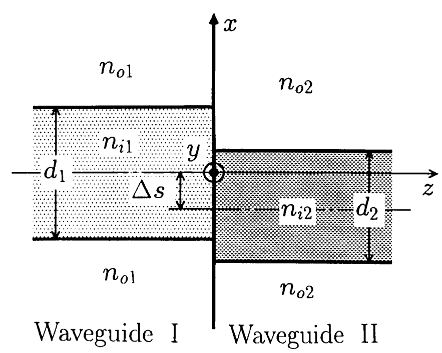

Fig. 1 Butt-Joint between weakly guiding slab waveguides.

Manuscript received October 20, 1992.
Manuscript revised February 25, 1993.
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original *BPM* analysis is applicable to each one of the waveguides. Therefore, if the effects of the Fresnel reflection at the interface are taken into account reasonably, the behavior of optical waves transmitted through and reflected at the junction can be analyzed correctly on the basis of the *BPM* concept.

2. 1 Method I (in the spatial domain)

The optical wave in the incoming waveguide can be traced up to the junction by means of the *BPM*, as mentioned above. The field distribution obtained just in front of the junction is the modal field of a normal mode in the incoming waveguide if the single mode excitation is achieved. Since $\Delta n_2/n_{o_2} \ll 1$ where Δn_2 represents the maximum difference of the refractive index in the outgoing waveguide, the modal profile would not be deformed significantly during passing through the junction. Therefore, the field profile just behind the junction can be obtained by multiplying the incident field by the transmission coefficient

$$t=\frac{2\sqrt{N_1N_2}}{N_1+N_2}. \tag{1}$$

This is an intuitive assumption which needs verifying through the numerical check. We define N_1 and N_2 in the above equation as

$$N_j^2=\frac{\int (n_jE_1)^2 ds}{\int (E_1)^2 ds}, \quad j=1, 2 \tag{2}$$

where E_1 is the transverse component of the electric field in the incoming wave guide and n_j represents the index distribution. The surface integrals must be carried out over the whole cross section. Definition (2) can be regarded as an approximation of the first order to the scalar variational expression for the propagation constant of dielectric waveguides. Resuming the *BPM* analysis by taking the resultant field as an initial field, the optical wave in the outgoing waveguide can be traced.

Under the weakly guiding condition, it is assumed that field distributions of the reflected waves are analogous to those of the incident waves in order to satisfy the boundary condition at the interface. It is also assumed that the incoming waveguide is terminated by a dielectric material with a refractive index N_2. Therefore, the power of the reflected guided waves can be estimated by $|r|^2$ [11], where

$$r=\frac{N_1-N_2}{N_1+N_2}. \tag{3}$$

So far as the transmission coefficient (1) is concerned, we may use the so-called effective refractive indices β_1/k_0 and β_2/k_0 in place of N_1 and N_2, respectively, where β_1 and β_2 are the propagation constants of the normal modes and k_0 is a free-space wavenumber. However, this substitution, if used in the reflection coefficient (3), would lead to unreasonable results [12]. The reflected power estimated with β_1 and β_2 does not definitely depend on the axial displacement because it has nothing to do with the determination of β_1 and β_2. More specifically, in the case of a butt-joint between identical waveguides no amount of incident power would be reflected even if there exists some axial displacement, which obviously contradicts with physical evidence.

2. 2 Method II (in the spectral domain)

The *BPM* conceptually replaces a weakly guiding waveguide by a sequence of thin lenses located at equal spaces in a homogeneous material having a proper reference refractive index. Therefore, the waveguide butt-joint shown in Fig. 1 is regarded as an interface between different dielectric materials containing different sequences of thin lenses. The basic algorithm of the *BPM* is a consecutive procedure of plane-wave expansions of the modal field by means of the Fourier transformation and reconstructions of it by the inverse transformation along the lens sequence [1]. At the interface between the two waveguides each expanded plane wave is partly transmitted with the transmission coefficient [13]

$$t'=\frac{2\sqrt{n_{o_1}\cos\alpha\sqrt{n_{o_2}^2-n_{o_1}^2\sin^2\alpha}}}{n_{o_1}\cos\alpha+\sqrt{n_{o_2}^2-n_{o_1}^2\sin^2\alpha}}, \tag{4}$$

where n_{o_1} and n_{o_2} are the reference refractive indices of the incoming and outgoing waveguides, which usually take the refractive indices of the substrates as in the present paper. For a weakly guiding waveguide, the reference refractive index may take such a different value as the refractive index of the film or some intermediate value [1]-[4]. The remainder is reflected with the reflection coefficient

$$r'=\frac{n_{o_1}\cos\alpha-\sqrt{n_{o_2}^2-n_{o_1}^2\sin^2\alpha}}{n_{o_1}\cos\alpha+\sqrt{n_{o_2}^2-n_{o_1}^2\sin^2\alpha}}, \tag{5}$$

where α is the incident angle measured from the normal of the interface and takes a discrete value which is determined by the discrete Fourier transformation. These coefficients are valid for *TE* waves in slab waveguides, since the boundary conditions at the interface are automatically satisfied by the discrete Fourier transformation in both the incoming and outgoing waveguides. For the *TM* mode analysis in slab waveguides, these coefficients should be replaced with those valid for the other polarization.

Recently, reflection operators have been proposed for the *BPM* analysis and mainly applied to the reflected waves from abrupt discontinuities [2]-[4]. Besides containing the reflection coefficient (5), these operators contain the space-dependent reflection coefficient which is defined by the difference between the space distributions of the refractive index on both sides of the discontinuity. However, this coefficient also seems to be rather an intuitive approximation.

Dominant components of the plane waves constructing the modal field in any type of weakly guiding structure are incident almost normally on the interface. Transmission and reflection coefficients of these plane waves are closely identical for both polarizations. This means that the transmission and reflection coefficients (4) and (5) are applicable to an arbitrarily polarized plane wave having a predominant expansion coefficient without causing any appreciable errors. Consequently, multiplying the amplitude of each plane-wave component in the incoming waveguide by t' at the interface and converting the propagation constant of the plane-waves into $n_{o_2}k_0$ in the outgoing waveguide, we can estimate the field distribution at any place in the outgoing waveguide.

In the same manner we can estimate the power of the reflected waves by multiplying each plane-wave component by r' at the interface and reversing the propagation direction. In the present method, the effects of interference are not taken into consideration between the counterpropagating waves in the incoming waveguide. Concerning the reflected waves, however, this does not matter since our particular interest is not in the field distributions but in their power.

Both modifications of the *BPM* in this paper would be expected to give correct results for a discontinuity problem where an air gap exists between the waveguides connected to each other, as long as the discontinuity in the refractive index is not extremely large.

3. Numerical Results

3. 1 Step Discontinuities

To verify the present modifications of the *BPM*, at first, we apply them to a step discontinuity in film thickness of a symmetric slab waveguide. It is assumed in Fig. 1 that the ratio of the film thickness on both sides of the discontinuity is $d_2/d_1=0.5$ and that the refractive indices of the film and substrate are $n_{o_1}=n_{o_2}=1.0$ and $n_{i_1}=n_{i_2}=1.01$, respectively.

In this case, an accurate analytical solution for the transmitted and reflected waves has been given by Morita [7]. In order to estimate the transmitted power of the guided waves it is considerably a simple way to apply the following overlap integral of the modal profiles;

Table 1 Transmitted and reflected power of the guided waves at the step discontinuity as a function of k_0d_2. The waveguide parameters are chosen so that $d_2/d_1=0.5$, $n_{o_1}=n_{o_2}=1.0$, and $n_{i_1}=n_{i_2}=1.01$.
P_T: Transmitted power of the guided waves.
P_R: Reflected power of the guided waves.

	$k_0d_2=k_0d_1/2$	10.0	20.0	40.0
P_T	Reference [7]	0.990	0.957	0.863
	Eq.(6)	0.990	0.957	0.863
	Method I	0.996	0.969	0.865
	Method II	0.996	0.969	0.865
P_R	Reference [7]	0.260×10^{-5}	0.235×10^{-5}	0.167×10^{-5}
	Eq.(3)	0.242×10^{-5}	0.221×10^{-5}	0.167×10^{-5}

$$P_T=\frac{4\beta_1\beta_2}{(\beta_1+\beta_2)^2}\cdot\frac{\left\{\int E_1\cdot E_2^*dx\right\}^2}{\int|E_1|^2dx\cdot\int|E_2|^2dx} \tag{6}$$

where E_1 and E_2 are the transverse electric field components of the normal mode under consideration and the asterisk indicates complex conjugation [8]-[11].

Table 1 summarizes the power of the transmitted and reflected guided waves estimated by Method I, II, and (3) as a function of k_0d_2. For the sake of comparison, the results given by Morita [7] and the overlap integral (6) are also shown in this table. The results obtained by Method I, II, and (3) are in good agreement with those by the accurate methods; relative differences between them are less than only 2%.

3. 2 Waveguide Butt-Joints

We also apply our methods to butt-joints between slab waveguides with the axial displacement Δs. The refractive indices of the waveguides are chosen so that $n_{o_1}=1.0$, $n_{i_1}=1.01$, $n_{o_2}=1.99$, and $n_{i_2}=2.0$. The incoming waveguide is supposed to be a single-mode waveguide whose film thickness is $d_1=3\lambda$, where λ is a free-space wavelength. We make a couple of choices for the outgoing waveguide in the following: a single-mode waveguide and a multi-mode one.

3. 2. 1 Single-Mode Outgoing Waveguide

The film thickness of the outgoing waveguide is chosen to be $d_2=2\lambda$ so that only the dominant mode can be supported. Figure 2 shows the power-coupling efficiency of the structure. The ordinate is the transmitted power normalized relative to the input power. The abscissa represents the normalized displacement $2\Delta s/(d_1+d_2)$ between the waveguides. Open circles show the results by Method I and triangles represent those by Method II. The numerical results obtained by the overlap in integral (6) are also shown in the

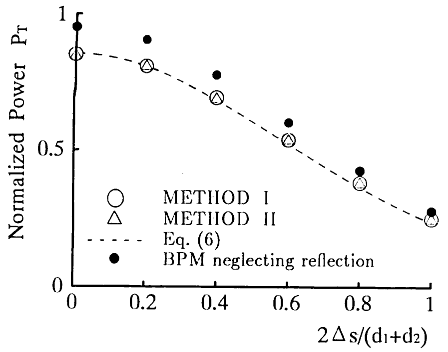

Fig. 2 Normalized transmitted power at the butt-joint between single-mode waveguides as a function of normalized axial displacement. The waveguide parameters are chosen so that $d_1=3\lambda$, $d_2=2\lambda$, $n_{o_1}=1.0$, $n_{i_1}=1.01$, $n_{o_2}=1.99$, and $n_{i_2}=2.0$.

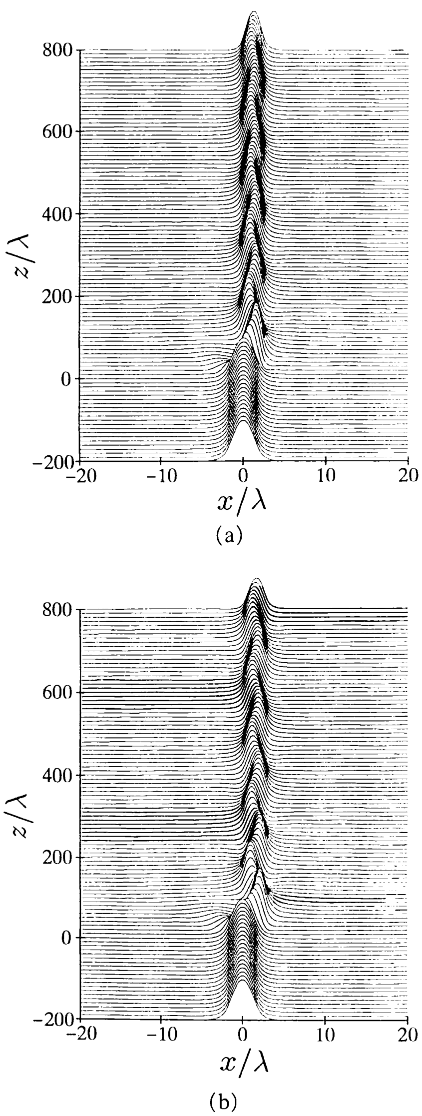

Fig. 3 Distribution of the field intensity along the butt-jointed waveguides obtained by Method II. The normalized axial displacement $2\Delta s/(d_1+d_2)$ is 0.4 in (a) and 0.6 in (b). The other parameters are the same as Fig. 2.

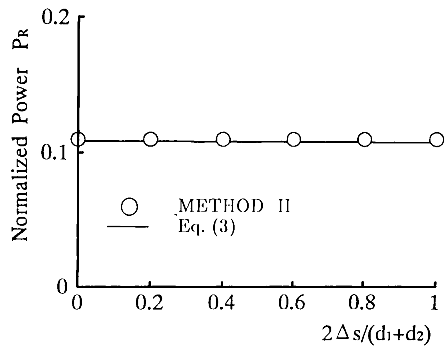

Fig. 4 Normalized reflected power vs. normalized axial displacement for the butt-joint between single-mode waveguides. The parameters are the same as Fig. 2.

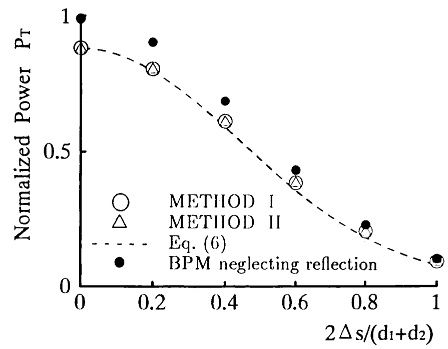

Fig. 5 Normalized transmitted power of the dominant mode at the butt-joint between a single-mode and a multi-mode waveguide as a function of normalized axial displacement. The waveguide thickness is chosen as $d_1=3\lambda$ for the incoming waveguide and $d_2=5\lambda$ for the outgoing one. The other parameters are the same as Fig. 2.

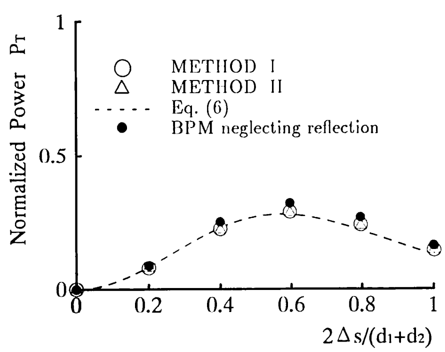

Fig. 6 Normalized transmitted power of the second mode at the butt-joint between a single-mode and a multi-mode waveguide as a function of normalized axial displacement. The parameters are the same as Fig. 5.

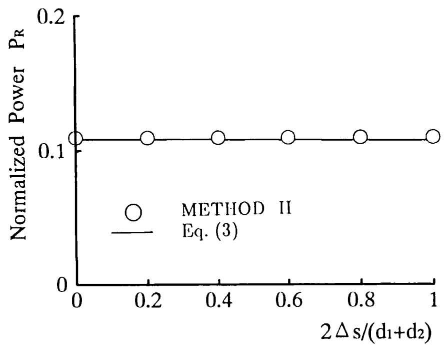

Fig. 7 Normalized reflected guided-wave power vs. normalized axial displacement for the butt-joint between a single-mode and a multi-mode waveguide. The parameters are the same as Fig. 5.

figure by the dashed line.

As shown in the figure, the results obtained by Method I and II are in excellent agreement, differing from the dashed line only by 2% at most. The results obtained from the original *BPM* without considering the Fresnel reflection are also shown in this figure and the maximum error is more than 10%.

Upon neglecting the reflected waves in Method II, we obtain the distributions of the field intensity along the butt-jointed slab waveguides which are illustrated in Fig. 3, where the axial displacement $2\Delta s/(d_1+d_2)=$ 0.4 in (a) and 0.6 in (b). We have obtained almost the same results by Method I, which are not shown in this paper. These illustrations are great helps to our visual understanding on the modal conversion at the discontinuity.

Figure 4 shows the normalized power of the reflected guided waves at the junction. The open circles and solid line represent the results obtained by Method II and (3), respectively. The ordinate and abscissa are the same as Fig. 2. We can hardly recognize the dependence on axial displacement. Both results are in good agreement; the relative differences between them are less than only 2%.

3. 2. 2 Multi-Mode Outgoing Waveguide

Finally, we consider the butt-joints between a single-mode and a multi-mode slab waveguide. The film thickness of the outgoing waveguide is assumed to be $d_2=5\lambda$ so that the lowest two modes can be supported. Figures 5 and 6 show the normalized transmitted power of the dominant and second order modes as a function of the axial displacement, respectively. The ordinates and abscissae are the same as Fig. 2. We can recognize good agreement in the results by Method I and II. The errors from the dashed line which illustrates the results given by (6) are about 2% also in this case. The results obtained from the original *BPM* without any modification contain as much discrepancy as those in Fig. 2.

The normalized power of the reflected guided waves is shown in Fig. 7. The results by Method II and formula (3) are represented in the figure by the open circles and solid line, respectively. At most only 2% of the relative difference exists in numerical data between both results.

4. Conclusions

In the present paper, two methods of modification to the *BPM* are made for the application of it to the analysis of discontinuities in weakly guiding structures. The modification is essentially based on the Fresnel reflection at the interface. One is introduced in the spatial domain and the other in the spectral domain. It is clarified numerically that these methods present satisfactorily accurate results for both the transmitted and reflected guided-wave power at the abrupt discontinuities or butt-joints of slab waveguides which either support the dominant mode or higher order modes. Our methods can easily be extended to the analysis of a waveguide butt-joint which contains an air gap at the junction.

Acknowledgments

The authors would like to thank M. Nakatsu of Kobe Steel Co., LTD. (KOBELCO) for his support in the numerical calculation.

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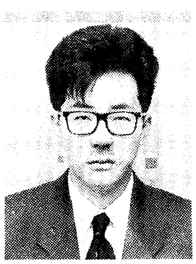

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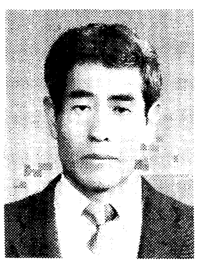

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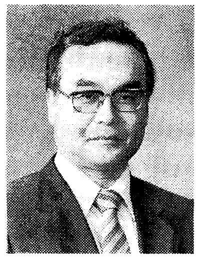

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