LETTER

Exact Determination of Optical Flow by Pixel-Based Temporal Mutual-Correlation Analysis

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SUMMARY An exact method determining optical flow from a sequence of images is presented. We adopt a pixel-based analysis calculating mutual-correlation functions between temporal change of brightness at a target pixel and that at neighboring ones. Local velocity at the target pixel is deceided analytically from lag times of the correlation functions.

1. Introduction

Determining optical flow (velocity field of image points or image displacement field) is interested from viewpoint detecting the 3-dimensional structure and motion from a sequence of digital images[(1)-(3)]. There have been presented many methods estimating optical flow[(4)-(8)]. The main problem in the analysis is determining the correspondence of image events in the two frames; it is usually called the matching problem[(8),(9)]. Several approaches are attempted to avoid the problem [(8)-(12)]. Recently, we have proposed a new technique determining optical flow with pixel-based temporal correlation analysis[(13)-(15)]. In the latest report, we present an improved method that introduce a non-linear interpolation for evaluating a truer lag time and a truer direction having maximum mutual-correlation coefficient[(15)]. The method is useful for automatic analysis of velocity field, however, the obtained is no more than an approximate solution and the resolution power of moving direction is not more than enough.

In this letter, we propose an exact method determining optical flow. Under some simple assumptions and some geometrical considerations, speed and direction of local velocity are decided analytically from lag times of the mutual-correlation functions. The validity of the method is confirmed by computer simulation.

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2. Basic Procedure of the Analysis

We have shown that the local velocity in a dynamic scene can be evaluated by pixel-based temporal correlation analysis [(13),(14)]. When we assume the constant-velocity motion of particles in a localized area (see 5×5 pixels-area in Fig. 1(a) or 3×3 pixels-area in Fig. 1(b)), we can estimate approximately the local velocity at a target pixel (the center pixel in the localized area) by the following steps (1-4).

(Step 1) The mutual-correlation functions $M_0^k(\tau)$ between the temporal brightness change of the target pixel $A_0(t)$ and that of the neighboring pixels (16 pixels in Fig. 1(a) or 8 pixels in Fig. 1(b)) $A_k(t)(k=1, 2, 3, \cdots, 1', 2', 3', \cdots)$ are calculated as

$$M_0^k(\tau)=(1/T)\int_0^T(A_0(t)-\bar{A}_0)\times(A_k(t+\tau)-\bar{A}_k)/S_N\,dt, \qquad (1)$$

where $\bar{A}_k$ is the mean value of $A_k(t)$. Since the brightness change of the pixels along the particle movement must be similar not only the relative time course but also the absolute amplitude, in this letter, normalized factor S_N is defined as

$$S_N=\max(S_0,\ S_k), \qquad (2)$$

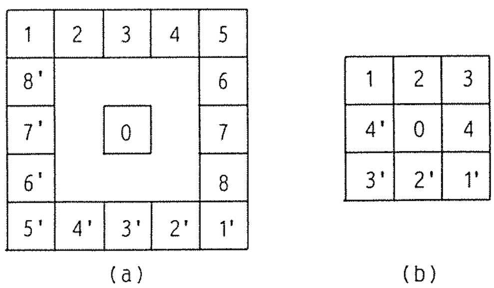


Fig. 1 A target pixel of velocity analysis and neighboring pixels. The local velocity at the target pixel (0) is evaluated by analyzing mutual-correlation functions between a temporal brightness change of the target pixel and that of the neighboring pixels. Motion of constant velocity is assumed in the localized area (5×5 or 3×3 pixels-area is used for the analysis).

$$S_k=(1/T)\int_0^T (A_k(t)-\bar{A}_k)^2 dt. \tag{3}$$

Owing to Eq.(2) the difference in absolute brightness amplitude between the target pixel and a neighboring one is able to have influence in the maximum value of the mutual-correlation function $M_0^k(\tau)$.

(Step 2) Product of mutual-correlation functions $M_0^k(\tau)$ and $M_0^{k'}(\tau)$ is calculated as

$$\gamma_0^{k,k'}(\tau)=\sqrt{M_0^k(\tau)\times M_0^{k'}(-\tau)}, \tag{4}$$

where k and k' are the pixels of diagonal pair (see Fig. 1). It should be noted that if the same object passes from k' to k through the target pixel the paired correlation functions $M_0^k(\tau)$ and $M_0^{k'}(\tau)$ have a symmetric property ($M_0^k(\tau)=M_0^{k'}(-\tau)$), and $\gamma_0^{k,k'}(\tau)$ has a maximum value at a lag time corresponded to the moving speed.

(Step 3) The pair of pixels k and k' having maximum correlation of $\gamma_0^{k,k'}(\tau)$ is selected as a probable candidate of direction of the local velocity.

(Step 4) The speed of the local velocity V can be estimated by the lag time τ_0^k of the product mutual-correlation function $\gamma_0^{k,k'}(\tau)$ and the distance d_0^k between the target pixel and the k-th pixel as

$$V=d_0^k/\tau_0^k. \tag{5}$$

Repeating the above procedure to all over the pixels, we can obtain the velocity-field (optical flow) of the sequential images. Algorithm of the above procedure is simple, however, estimated direction and speed of the local velocity are suffered by digital error. Especially, the estimated direction is restricted to 16 (or 8) directions (i. e. a resolution power of moving direction is about ±10 (or ±20) degrees).

3. Improvement of The Analysis

3.1 Estimation of True Lag Time by a Quadratic Interpolation Method

In the basic procedure, the velocity is determined by the direction and the lag time of the product mutual-correlation function having maximum correlation coefficient γ_0. To improve the analysis we first introduce an interpolation method for estimating a true lag time. The lag time estimated by the basic procedure is a digitized quantity (see Fig. 2). It is expected that the true lag time can be evaluated by detecting a lag time τ_m having true maximum correlation. Strictly speaking, without the knowledge of brightness distribution of moving ogject, we can not estimate exactly the true lag time. Here, we assume that the correlation function near the true lag time is able to evaluate by simple interpolation method with quadratic function;

$$\gamma_0^{k,k'}(\tau)=\gamma_m-a(\tau-\tau_m)^2, \tag{6}$$

where γ_m is the maximum value of the interpolated correlation function (see Fig. 2).

3.2 Exact Determination of Local Velocity

By making use of the true lag time τ_m, we can estimate exactly the local velocity at the target pixel. Now we make the second assumption: The moving object has central symmetry of brightness distribution.

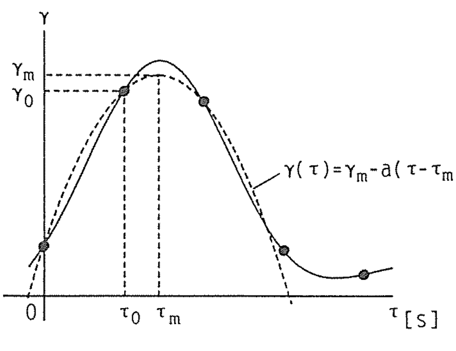


Fig. 2 Estimation of true lag time by a quadratic interpolation method.

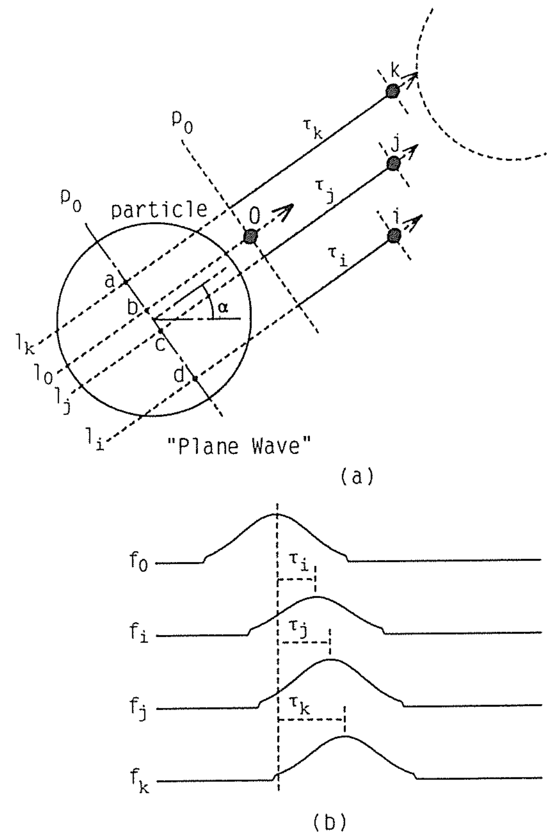


Fig. 3 Relation between bag time of the mutual-correlation function and particle motion. (a) If the particle has central symmetry of brightness distribution, passing of particle is equivalent to the passing of a "Plane Wave" (line p_0 show a ridge of the wave). (b) Temporal brightness change at respective pixels.

Under this assumption, it is easily understood that the lag time at the nearest pixel site by the moving line l_0 (j-th site in Fig. 3) and that at its neighboring pixel sites (i-th and k-th sites in Fig. 3) are determined by passing time of the maximum brightness points (a, b, c and d in Fig. 3(a)) locating on the center line which perpendicular to the moving line. For example, the lag time τ_j is equal to a period from a moment the point b passes the site 0 till a moment the point c passes the site j. We can determine the other lag times in the same manner (see Fig. 3(b)). It should be noted that the passing of the moving object corresponds to the propagation of a "Plane Wave". The propagation vector (wave vector) of the "Plane Wave" is parallel to the moving line l_0 of the object. Both "Plane Wave" and the object bring the same result on the lag time of the correlation function. Therefore, from a simple geometric relations (see Fig. 4) we can estimate the propagation speed V of the "Plane Wave". The following equations are indicated by the geometric relations in the figure;

$$V \times \tau_j = d_j \times \cos(\theta_j - \alpha), \quad (7)$$

$$V \times \tau_k = d_k \times \cos(\theta_k - \alpha), \quad (8)$$

where α is the angle of true direction of moving object. θ_j and θ_k are the coordination angles of the site j and that of site k, respectively. $d_j(d_k)$ is the distance between the target pixel (site 0) and the neighboring pixel of site $j(k)$). From Eqs. (7) and (8), it can be reduced analytically as

$$V = \left(\frac{d_j}{\tau_j}\right)\left(\frac{d_k}{\tau_k}\right) \sin|\theta_k - \theta_j| \Big/ \sqrt{\left(\left(\frac{d_j}{\tau_j}\right)^2 + \left(\frac{d_k}{\tau_k}\right)^2 - 2\left(\frac{d_j d_k}{\tau_j \tau_k}\right)\cos(\theta_k - \theta_j)\right)}, \quad (9)$$

$$\tan\phi = \left(\frac{d_k \cos\theta_k}{\tau_k} - \frac{d_j \cos\theta_j}{\tau_j}\right) \Big/ \left(\frac{d_k \sin\theta_k}{\tau_k} - \frac{d_j \sin\theta_j}{\tau_j}\right), \quad (10)$$

$$\alpha = -\phi \text{ or } \pi - \phi. \quad (11)$$

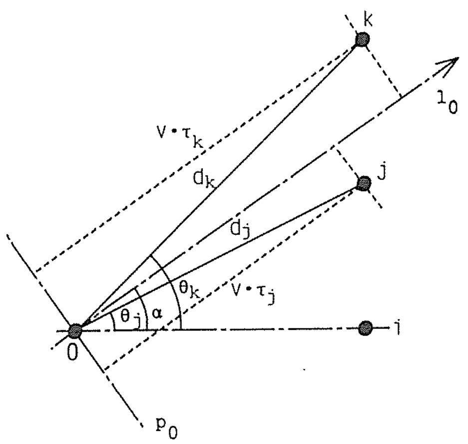


Fig. 4 Estimation of exact velocity at the target pixel. From the geometrical relations in the figure, we can estimate the moving velocity.

As mentioned above, under some assumptions, we obtain exact and analytical solution of the local velocity (optical flow) from a sequence of images.

4. Evaluation of the Analysis by Computer Simulation

The validity of the proposed method is confirmed by using artificial scenes. A sequential image of moving particles with 64×64 pixels and 128 frames are created by computer simulation. Gray-level distribution of each particle is assumed as

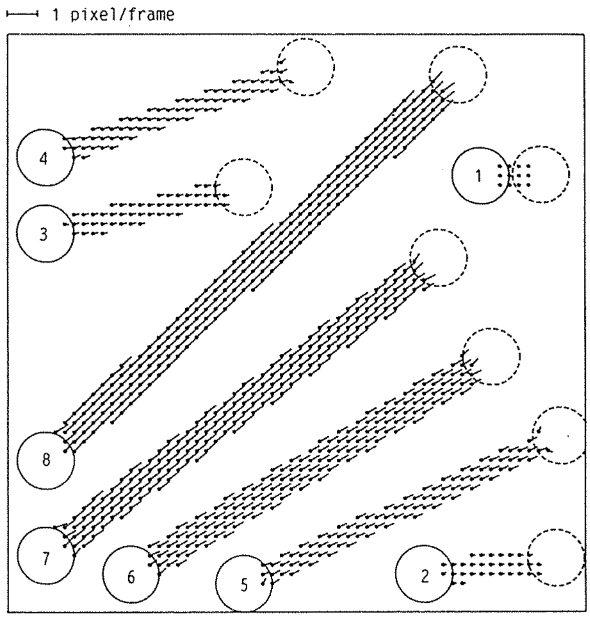
$$G(r) = \begin{cases} G_0 \exp((-r/r_c)^2) & : r < 1.5r_c \\ 0 & : r > 1.5r_c, \end{cases} \quad (12)$$

where $G_0=220$, and $r_c=2$ are adopted in the simulation. A unit of the distance r is equal to the length of one pixel. Number of moving particles in the scene is fixed 8. The data of speed and direction of respective particles are shown in Table 1.

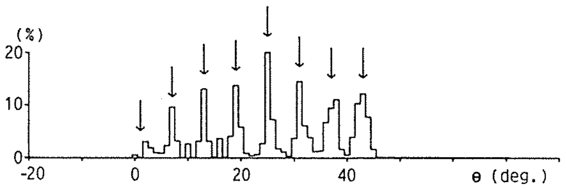
The results of the analysis are presented in Fig. 5. Fiqure 5(a) illustrates the obtained optical flow. Simulated motion of the particles are also illustrated on the same figure (from start position (solid circle) to end position (broken circle)). The given particle-motions are well represented in the analyzed optical flow. Quantitative evaluations are given in Fig. 5(b), which shows histograms of speed-distribution and angle-distribution of obtained flow-vector. Histograms are normalized by particle speed, because higher speed particle draws longer locus in the image. True speeds and true directions are indicated by arrow in the same figures. The results are well consistent. Derivations of speed and direction analyzed are within 0.01 pixel/frame (ΔV) and 0.5 degree ($\Delta\theta$), respectively. These resolution powers are apparently improved than that in the analysis proposed in the latest report ($\Delta V=0.02$ pixel/frame, $\Delta\theta=3$ degree)[15].

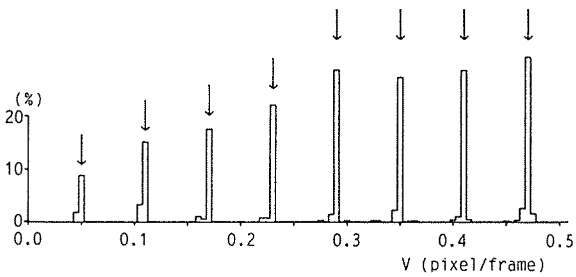
Tabble 1 Data of speed and direction of respectine moving particles in the coputer simulation.

particle NO.	speed v (pixel/frame)	direction θ (degree)
1	0.05	1.0
2	0.11	7.0
3	0.17	13.0
4	0.23	19.0
5	0.29	25.0
6	0.35	31.0
7	0.41	37.0
8	0.47	43.0



(a)





(b)

Fig. 5 Result of analysis of the artificial image. (a) Obtained optical flow. Given motion of particles are illustrated by solid circle (start position) and broken circle (end position). (b) Histograms of speed-distribution and angle-distribution of obtained optical flow. True data are indicated by arrows in the figure.

5. Concluding Remarks

In this letter, we propose the exact method determining optical flow. The validity of the method is confirmed by computer simulation. Comparing to the interpolation technique proposed in our latest report, the resolution power of optical flow is improved. A detailed comparison between our improved method and others will be reported in near future. An application to real world scenes will also be presented in the next report.

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