PAPER

Velocity-Field Measurement by Pixel-Based Temporal Mutual-Correlation Analysis of Dynamic Image

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SUMMARY A new dynamic image processing technique is proposed for measuring the velocity-field of the translational motion of crowded particles. The local velocity at a target pixel-site is estimated by mutual-correlation analysis between the temporal brightness change of the target pixel and that of neighbouring pixels. The next-nearest 16 pixels are selected as the neighbouring pixels in this study. The direction and the speed of the particle motion are evaluated by the mutual-correlation function having maximum value and the lag time of the function, respectively. The usefulness of the proposed technique is examined by a simulation experiment (unidirectional motion) and an actual scene analysis (vortex motion). Obtained velocity-field of these motions are visualized with orientated lines, and the resulting fields are consistent with expected fields.

1. Introduction

Many efforts have been devoted to the velocity-field analysis of dynamic scene[1]. In recent study, Imaichi and Ohmi[2] have investigated the unsteady twin-vortex flow and Kārmān vortices behind a circular cylinder, and they have calculated distribution of stream function, vorticity and pressure through flow-visualization and image processing. In their analysis, particle velocity is computed from the length of every clearly drawn particle path on the photograph and from the exposure time of the camera. Their technique is very effective, however, the measurement of particle path is done by the manual operation.

Many different approaches to a fully automated method evaluating the velocity of moving objects have been proposed by many authors[3]. One of these approaches is based on the matching technique between successive temporal frames[4]. Generally, two-dimensional cross-correlation[5] or mean square difference have been used as a measure of similarity [4],[6],[7], and the differential images between consecutive frames have been used to detect moving objects[6],[8],[9]. The motion-vectors of respective moving objects are obtained by these method, however, it seems to be not suitable for the analysis of point-velocity. Velocity information at a point is developed by relating the time variation of image intensity caused by motion to the spatial variation of intensity over object surfaces[9]. Various reports for the determination of displacement vector field following the approach are reviewed by Nagel[1]. In recent investigations, Horn and Shunk[10], and Terzopoulos[11] developed and sophisticated the approach. Although a usefulness of the approach is demonstrated with simulation images, it is assumed that the image brightness is differentiable (i. e., the apparent velocity of the brightness pattern varies smoothly almost every where in the image). This constraint seems to be rather strict for the actual scene analysis.

Fundamentally, the conventional methods introduced above are equivalent to the method pursuiting the coordinate of the moving objects with time. Recently, we have introduced a new conception of a fixed coordinate analysis[12]-[14]. We believe that a temporal brightness change caused by passing particles at the fixed coordinate must have a information of the velocity of moving objects. In this paper, we will propose a quite different approach to the determination of displacement vector field. The method is based on a mutual-correlation analysis between temporal brightness change of a target pixel and that of neighbouring pixels. A usefulness of the proposed technique is examined by the simulation experiment and the actual scene analysis. It is also confirmed that the proposed analysis is not affected by the smoothness of the brightness pattern and the size of moving objects.

2. Theory of Velocity Measurement by Mutual-Correlation Analysis

Let's consider a digital dynamic image with $M \times M$ pixels and N frames as shown in Fig. 1(a). To begin with, we assume a unidirectional motion of crowded particles, i. e, a uniform velocity-field with constant velocity is supposed. In this case, we can estimate the local velocity of crowded particles at the target pixel-

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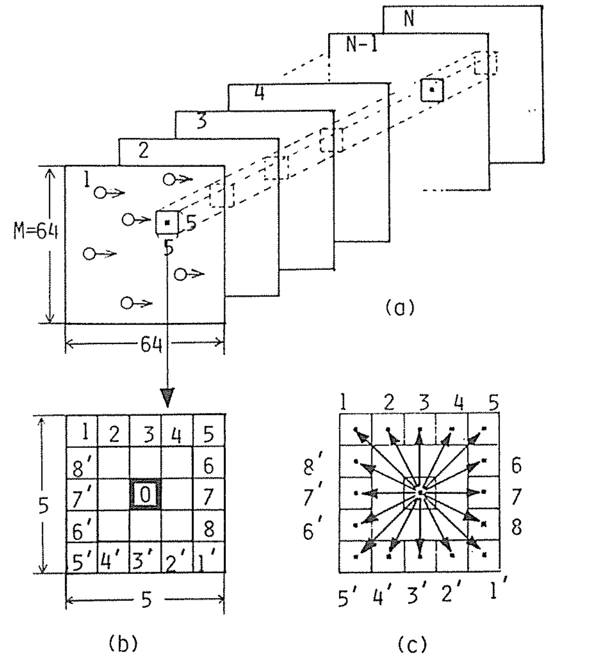


Fig. 1 Configuration of dynamic image. (a) A digital dynamic image with $M \times M$ pixels and N frames. (b) A target pixel (0) and its next-nearest neighbouring 16 pixels. (c) A definition of 16 directions.

site (i, j) by the following procedures.

First, mutual-correlation functions $M_0^k(\tau)$ between the temporal brightness change of the target pixel $A_0(t)$ (labeled as 0-th site) and that of the neighbouring pixels $A_k(t)$ (k-th site; $k \neq 0$) are calculated as

$$M_0^k(\tau)=(1/T)\cdot\int_{-T}^{T}\{A_0(t)-\bar{A}_0\}\{A_k(t+\tau)-\bar{A}_k\}dt \tag{1}$$

For a digitized time-series the function can be reduced as

$$M_0^k(n\Delta t)=\begin{cases}\dfrac{1}{N-|n|}\displaystyle\sum_{m=1}^{N-|n|}[A_0\{(m+|n|)\Delta t\}-\bar{A}_0] \\ \times\{A_k(m\Delta t)-\bar{A}_k\} \qquad : n\leqq 0 \\ \dfrac{1}{N-n}\displaystyle\sum_{m=1}^{N-n}\{A_0(m\Delta t)-\bar{A}_0\} \\ \times[A_k\{(m+n)\Delta t\}-\bar{A}_k] \; : n>0\end{cases} \tag{2}$$

Here, n and m are the integer, Δt is the sampling time ($m\Delta t=t$, $n\Delta t=\tau$), and $\bar{A}_0$ is a mean value of $A_0(t)$. In this study, the next-nearest 16 pixels as shown in Fig. 1(b) are selected as the neighbouring pixels. Number of the neighbouring pixels are also given in the figure. Then, 16 mutual-correlation functions $M_0^k(\tau)$ are calculated and 16 directions (k=1, 1′, 2, 2′, 3, 3′, 4, 4′, 5, 5′, 6, 6′, 7, 7′, 8, and 8′) are considered as the candidate directions of moving particles (see Fig. 1(c)).

Second, the most probable direction of the particle movement is decided using the 16 mutual-correlation functions. Suppose that a particle passes from $k=7'$ site to $k=7$ site through the target pixel, then a pair of mutual-correlation functions M_0^7 and $M_0^{7'}$ have more symmetric property and stronger correlation than the other correlation functions as shown in Fig. 2(a). In an ideal case (i. e., the same particle passes through 7′, 0, and 7-th sites by turns), the pair of mutual-correlation functions should satisfy the symmetric relation:

$$M_0^7(\tau)=M_0^{7'}(-\tau) \tag{3}$$

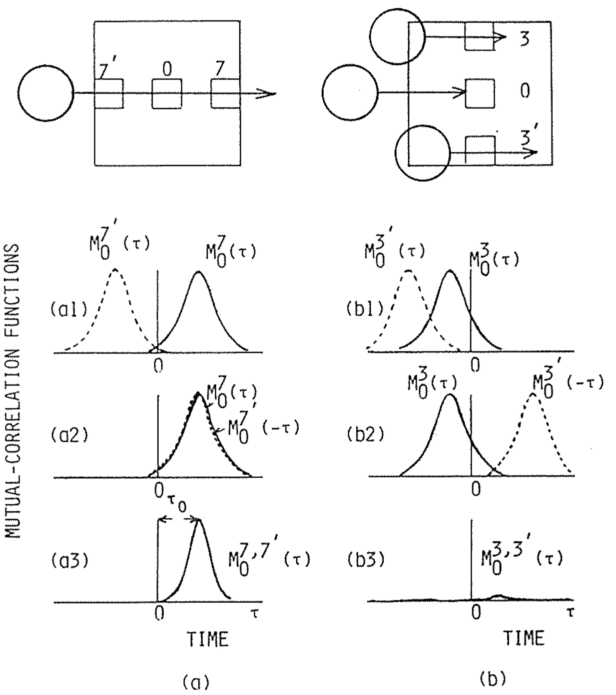


Fig. 2 Principle of the image processing. (a) A case when the same particle passes through 7′, 0, and 7-th sites by turns. A pair of mutual-correlation functions M_0^7 and $M_0^{7'}$ show symmetric property (a1). Then, the product mutual-correlation function $M_0^{k,k'}(\tau)$ $(=M_0^7(\tau)\times M_0^{7'}(\tau))$ shows strong correlation at lag time τ_0. (b) Another case when different particles pass through 3, 0, and 3′-th sites. The product ccrrelation function $M_0^{3,3'}$ shows weak correlation.

On the other hand, if the mutual-correlation is calculated between the paired pixels through which the same particle does not pass (see Fig. 2(b)), the correlation functions M_0^k and $M_0^{k'}$ generally show weak correlation and don't satisfy the symmetric relation of Eq. (3). Consequently, in order to detect the most probable direction, we propose a product mutual-correlation function $M_0^{k,k'}$ defined as

$$M_0^{k,k'}(\tau)=M_0^k(\tau)\times M_0^{k'}(-\tau) \tag{4}$$

If the considered pixel-sites k, 0, and k' are on a locus of the same particle, $M_0^{k,k'}(\tau)$ should have a sharp peak at a lag time $\tau=\tau_0$ (see Fig. 2(a 3)). Unless the considered pixel-sites are on the same locus of the particle, $M_0^{k,k'}(\tau)$

should have small values to every τ (see Fig. 2(b 3)). Now we can determine the direction of the particle movement at the target pixel-site. The paired pixel-sites k and k' having maximum value of $M_0^{k,k'}(\tau)$ indicate the most probable candidate for the moving direction. If $\tau_0>0$, the particle is toward from k' to k-th site, and if $\tau_0<0$ one is toward from k to k'-th site. With these manner, the moving direction D_{ij} at the target pixel-site is deceided.

Third, the speed of the motion can be estimated by the lag time τ_0. Let R_0^k the distance between the target pixel and k-th pixel having maximum value of $M_0^{k,k'}(\tau)$. The speed at the target pixel V_{ij} is evaluated by

$$V_{ij}=R_0^k/\tau_0 \tag{5}$$

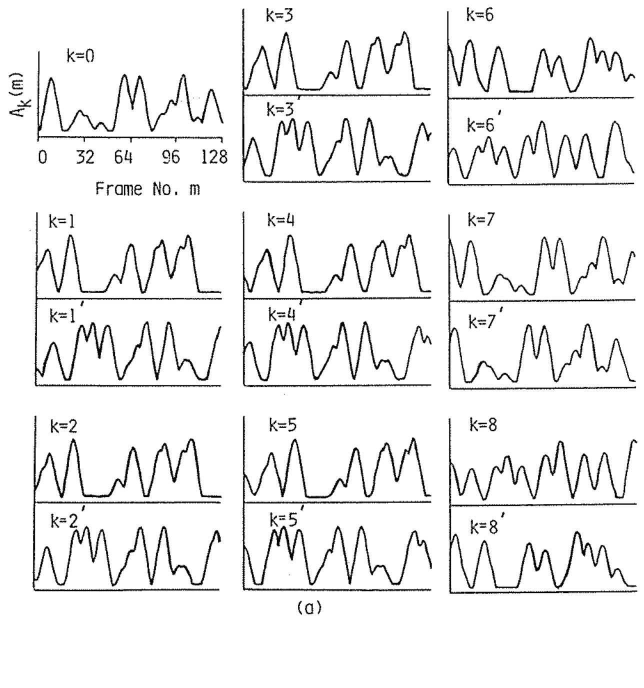
Following above procedures, we can obtain the speed V_{ij} at the target pixel-site (i, j). The velocity-field of the dynamic scene is obtained by calculating the speed V_{ij} and the direction D_{ij} to all over the pixels in the image.

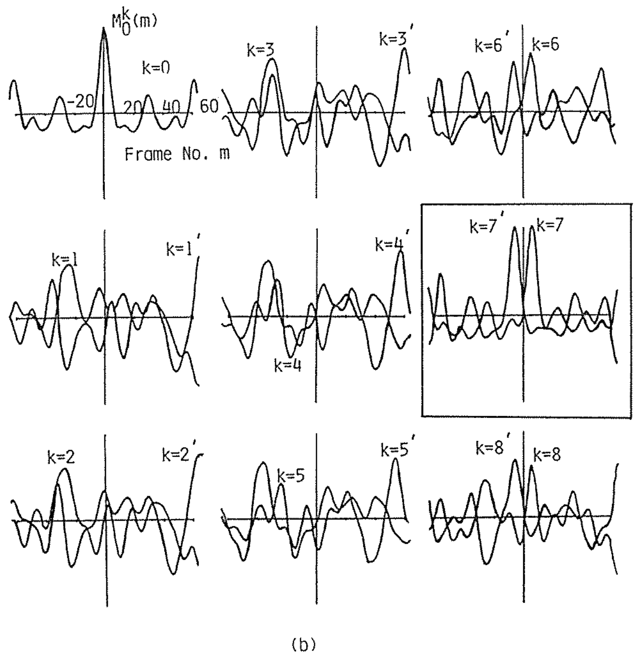
3. Simulation Study on Velocity-Field Analysis of Unidirectional Motion of Crowded Particles

To clarify the usefulness of the proposed theory, we first analyse a unidirectional motion of crowded particles. The dynamic scene of the motion with 64×64 pixels and 128 frames are created by computer simulation technique. In the simulation, the unidirectional motion of 300 particles with constant velocity is considered. The direction of the motion is selected toward from 7' to 7-th site as shown in Fig. 2. The size of one particle is about 5×5 pixel area, and Gaussian-like distribution of brightness is assumed to each particle. Namely, the gray-level $G(r)$ at a distance r from the center of the particle is given as

$$G(r)=\begin{cases} G_0\exp(-(r/r_c)^2) & : r\leq 1.5r_c \\ G_c & : r>1.5r_c \end{cases} \tag{6}$$

where $G_0=222$, $G_c=20$, and $r_c=2$ are adopted in this





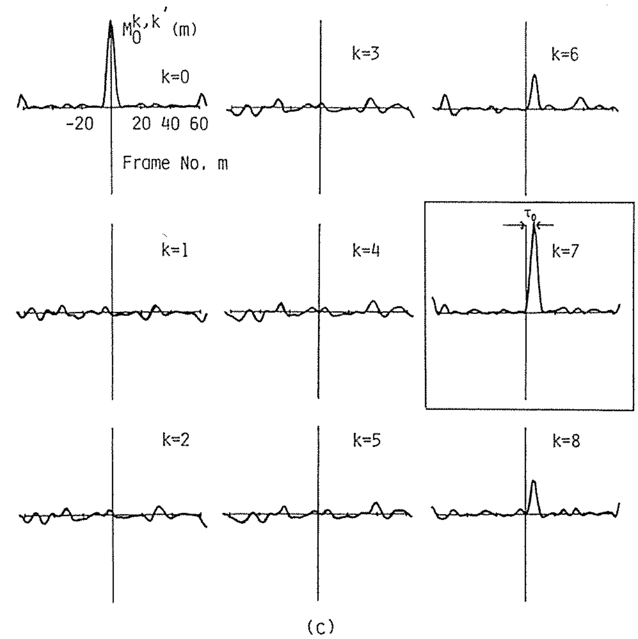


Fig. 3 Simulation study of the velocity-field analysis. (a) Temporal brightness changes of target pixel ($k=0$) and its neighbouring pixels ($k\neq 0$). (b) Mutualcorrelation functions $M_0^k(\tau)$ between the temporal brightness change of the target pixel (32, 32) and that of the neighbouring pixels. (c) Product mutualcorrelation functions $M_0^{k,k'}(\tau)$. The correlation functions have maximum value at $k=7$ with lag time τ_0. The speed and the direction of the motion are evaluated by the correlation function $M_0^{7,7'}(\tau)$.

simulation. And, one pixel-size is selected as a unit of the distance r.

The results of the analysis are illustrated in Fig. 3. Figure 3(a) shows the temporal change of the gray-level at the target pixel $((i, j)=(32,32))$ and at its neighbouring 16 pixels. The number of pixel-site is defined in Fig. 1(b). Sixteen mutual-correlation functions $M_0^k(\tau)$ between the target pixel ($k=0$) and the other neighbouring pixels ($k\neq 0$) are calculated by Eq. (2). The results are shown in Fig. 3(b). The autocorrelation function $M_0^0(\tau)$ is also illustrated in the same figure as suffix 0. In these figure it should be noted that:

1) Strong correlation is observed in the paired mutual-correlation functions $M_0^7(\tau)$ and $M_0^{7'}(\tau)$, and the two functions show symmetrical form with each other.

2) In the other direction, some mutual-correlation functions show large value, but the symmetric relation is not seen between the paired correlation functions $M_0^k(\tau)$ and $M_0^{k'}(\tau)$ ($k\neq 7$).

These facts indicate that the direction 7′ to 7 is the probable candidate of the moving direction.

Figure 3(c) shows the product mutual-correlation functions $M_0^{k,k'}(\tau)$. A square of the autocorrelation function is also illustrated in the figure. As might be expected the product mutual-correlation function has maximum value at 7-th direction, and the lag time τ_0 is positive. This indicates that the most probable candidate of the moving direction is toward from 7′ to 7. Using the lag time τ_0 at $k=7$, the speed of the particle movement at the target pixel (32, 32) is evaluated by Eq. (5).

In these manner, the velocity of all pixel-sites are analysed and the velocity-field of the dynamic scene is visualized. Figure 4 shows examples of the velocity-field of the simulation images with solid circles and oriented lines. The length and the orientation of the line represent the speed and the moving direction, respectively. The direction of the motion is toward from the solid circle to the oriented line. The location of the solid circle corresponds to the center of each pixel. To avoid unnecessary complication the velocity-fields are illustrated every 5 pixels. Images in Fig. 4 show analysed results of unidirectional motion with different translational speed: (a) 0.18 (pixel/frame), (b) 0.36 (pixel/frame), (c) 0.53 (pixel/frame) and (d) 0.71 (pixel/frame). The resulting images are well consistent to the given simulation models. In the slowest speed, however, several false estimation of the velocity are recognized. The cause of the false estimation may be attributed to the finite length of observed time $T(=n\Delta t)$.

Figure 5 summarizes the relation between the given speed (horizontal axis) and analysed mean speed (vertical axis). The results are well consistent to the theoretical value in which the digital errors are estimated correctly. The errors are originated in a digitized time lag caused by a finite sampling speed. To estimate correctly the more rapid movement, we should make more precise estimation of the lag time τ_0.

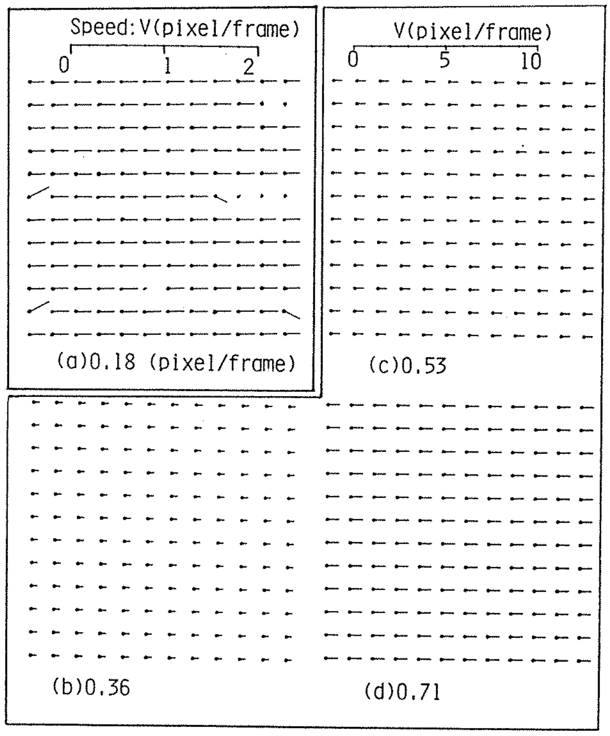


Fig. 4 Velocity-field representation of the simulation images with orientated lines. The length and the orientation of the lines represent the speed and the direction of particle movement, respectively. Given speeds are also shown in the figure.

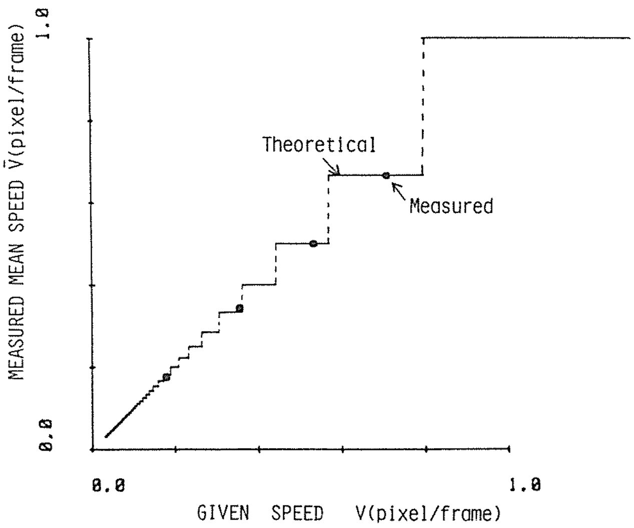


Fig. 5 Relation between the given speed and the measured mean speed. Stepwise line shows the theoretical value considering the digital error in the measurement.

The simulation experiments for moving objects having different size and different brightness pattern are also carried out. It is found that the proposed analysis is not affected by the smoothness of the brightness pattern and the size of moving objects.

4. Analysis of Actual Dynamic Scene of Vortex Motion

As the actual example, we pick up a vortex motion of crowded particles. Polystyrene dust is used as the visualizing material of the vortex motion which is caused by drawing motion of water through a drainage hole (5mm ϕ). The vortex motion was picked up in the latest report of our study in which the detailes of the experiment were described[14]. In the vortex motion, the velocity-field is not uniform but it depends on a position of the vortex. Nevertheless, the proposed mutual-correlation analysis can be applied to the velocity-field measurement under the condition that the particles move unidirectionally with constant velocity in a local region. In the present analysis, the constant-velocity motion of particles is required at least in the local area of 5×5 pixels. Under this assumption the velocity-field of the vortex is analysed.

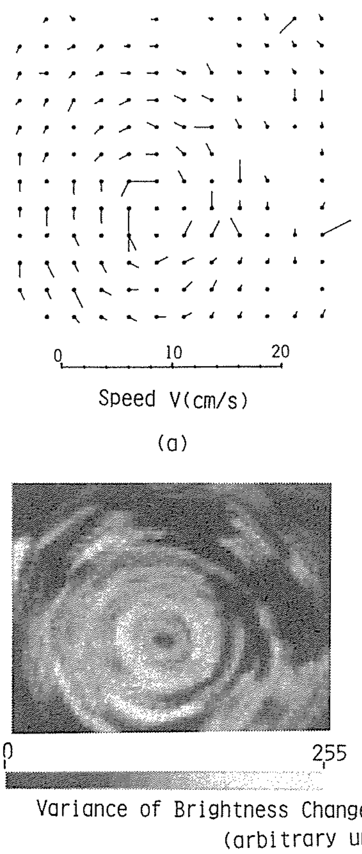


Fig. 6 Result of the velocity-field analysis of the vortex motion. (a) Representation of the velocity-field with orientated lines. Vacant pixel-sites show the position where the velocity can not evaluate because of no passing of particles. (b) "The variance image" of the vortex motion. Black regions in the image indicate no particle movement at the regions. The regions well coincide with the vacant pixel-sites in the upper figure (a).

The result of the analysis is shown in Fig. 6. The velocity-field of the vortex is also illustrated by orientated lines (Fig. 6(a)). Vacant pixel-sites in the figure indicate the positions where the velocity can not estimate because of no passing of moving particles. Figure 6(b) shows "the variance image" of the vortex[14]. The variance of temporal brightness change of the target pixel is calculated from autocorrelation function $M_0^0(\tau)$. Bright region in the figure indicates the existence of particle movement at the position. The regions of no particle movement (black regions in Fig 6(b)) are coincide with those in Fig. 6(a).

From Fig. 6(a), we can recognize that the flow speed of the particles increases as the observed position approaches to the center of the vortex and the direction of the vortex motion is counter clockwise. Fundamentally, these results are well consistent to visual observation, however, several false estimation of the motion vector is observed in the analysed image. The cause of the false estimation is not clarified, however, it may be due to the finite observation time T, the fixed sampling speed and digital processing of the image. Physical characteristics of the vortex motion will be discussed in the next report.

5. Concluding Remarks

The new dynamic image processing technique based on the mutual-correlation analysis is proposed for the measurement of the velocity-field of the crowded particles. The passing velocity of particles at the target pixel-site can be evaluated by the following procedures.

1) The mutual-correlation functions $M_0^k(\tau)$ between the temporal brightness change of the target pixel and that of the neighbouring 16 pixels are calculated.

2) The most probable candidate of the direction of the motion is decided by analysing the product mutual-correlation function $M_0^{k,k'}(\tau)$ $(=M_0^k(\tau)\times M_0^{k'}(-\tau))$ of the paired pixels k and k'.

3) The speed of the motion can be estimated by the lag time τ_0 of the product mutual-correlation function.

The usefulness of the proposed technique is confirmed by the simulation experiment and the actual scene analysis. It is also found that the proposed analysis is not affected by the smoothness of the brightness pattern and the size of moving object. Fundamental technique to evaluate the velocity-field has been established in this study, however, several problems are left unsolved. One is the false estimation of the motion vector observed occasionally in the actual scene analysis. And, the resolving power of the moving direction and that of speed are the other problems. The direction is restricted to digitized 16 directions in this study. We are now proceeding to develop the proposed technique. In our latest research, a method for more precise estimation of the motion vector (speed and direction) has been established. The detailes about the method are also reported in near future.

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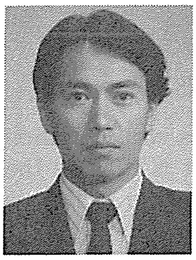

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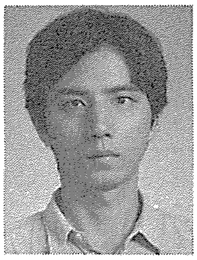

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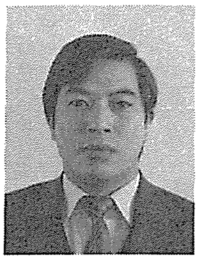

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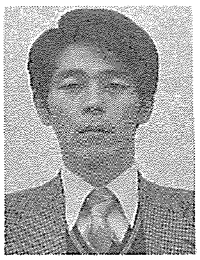

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