Viscoelastic Study of the Asphalt Mixtures (1 st Report)

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Abstract

In constructing the road, asphalt pavement is most useful one. And then, when we consider of the structure of road, all pavement slabs are viscoelastic bodies, but in the past, almost investigators analyzed the stress distribution of pavement slabs by treating them as elastic bodies. So, the author studied behaviors of viscoelastic bodies and at first considered asphalt mixtures to be visco elastic bodies.

The combination of a spring and a dashpot connected in series is known as Maxwell element. In generalized Maxwell element, it is reported by Mr. Kenji Ishihara that the definition of (transient creep function with discrete spectrum) may be given to the ratio σ/ε in

$$\sigma=\left[\sum_{i=-n}^{n}\nu_i\left\{\frac{\tau_i}{t}\left(1-e^{-t/\tau_i}\right)\right\}\right]\varepsilon \tag{1}$$

where $\varepsilon=\varepsilon_0 t$

The compressive tests subjected to a linearly increasing strain $\varepsilon_0 t$ were tried and their stress-strain curves were drawn at different rates of strains.

And we could decide the visco-elastic constants of asphalt mixtures by their stress-strain curves. The results of experiments and the method of deciding the viscoelastic constants are mentioned in this paper.

1. Introduction

The definition of Rheology takes in a very broad range of phenomena, which might include the elastic deformation of ideal solids, where the strains observed are proportional to the stress applied, (Hooke's law of elasticity), and the viscous flow of ideal liquids, where the rate of deformation or flow is likewise proportional to the applied force or stress (Newton's law of viscosity). However, these two types of behavior are treated in detail in the classic fields of elasticity theory and hydrodynamics, including the more complicated behavior where the simple laws of proportionarity no longer hold and where such phenomena as turbulent flow are encountered.

Rheology is therefore ordinarily regarded as applying especially to the more complicated types of deformation and flow behavior that do not follow the simple laws of elasticity theory and hydrodynamics. This still includes a wide variety of systems, since ideal solids and liquids are only simple approximate idealizations of certain classes of real materials.

The most important rheological systems are those that show a combination of viscous and elastic effects. The British physicist, James Maxwell, recognized that asphalt behaves in this way; in 1867 he suggested that a mechanical analog model consisting of an elastic spring connected in series with a viscous unit (a dashpot) could provide a useful representation of such a viscoelastic system.

The German physicist, Woldemar Voigt, and the British physicist, Lord Kelvin, suggested the alternate possibility of connecting the elastic and viscous elements in parallel. This type of representa-

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tion has since been elaborated into complicated networks that must be described by mathematical distribution functions. Nonlinear spring and dashpots, which do not obey Hooke's and Newton's laws, have also been employed on occasion. A wide variety of time-dependent behavior can be represented in this way; the corresponding mathematical theory is identical with that used to describe electric networks made up of capacitors and resistors in similar series and parallel combinations.

Materials such as steel and glass, often regarded as elastic at ordinary temperatures, on close observation are found also to exhibit small amounts of viscous behavior. This can be observed as creep (increase of deformation at constant stress) or stress relaxation (decay of elastic stress at constant deformation), and these effects may be important under certain conditions of use.

However, the great impetus to development of the theory of visco-elasticity has come from the natural and synthetic rubber industry and more recently from the plastic and synthetic textiles industries. These deal with materials that can be classified chemically as polymers, or long-chain molecules; under processing and fabrication conditions, and also occasionally under use conditions, they show elastic and viscous behavior to almost equal degrees, unlike the conventional solids or liquids, which show predominantly elasticity or viscosity, respectively.

The development of a theory that would allow a proper understanding and treatment of this type of behavior therefore became of crucial importance, and the theory evolved allows a description of the behaviors of visco-elastic materials over wide ranges of temperature, time, and stress level. The theory is largely phenomenological, providing only a mathematical description of the phenomena observed. Theoretical interpretation in terms of the molecular or microscopic structure of these material is less well developed, and much work remains to be done in this area.

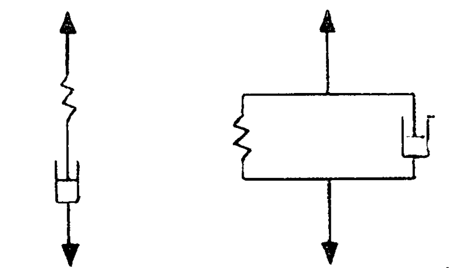

Fig.1 Analog model of viscoelastic systems

The rheological behavior of systems that do not follow simple visco-elastic laws is also frequently of interest and importance. These include such suspentions as printing ink and blood; solids that show plastic yield under stress; such composite systems as soil, solid rocket propellants, and the asphalt and crushed rock mixtures used for road surfacing; and glaciers and rocks, the slow deformation of which are sometimes measurable only in geologic time. The theory for these systems, however, in general remains to be developed.

2. Theoretical consideration

The combination of a spring and a dashpot connected in series is known as Maxwell element. In Fig.2 is shown the generalized Maxwell model elements all linked in parallel. In this model, the strains associated

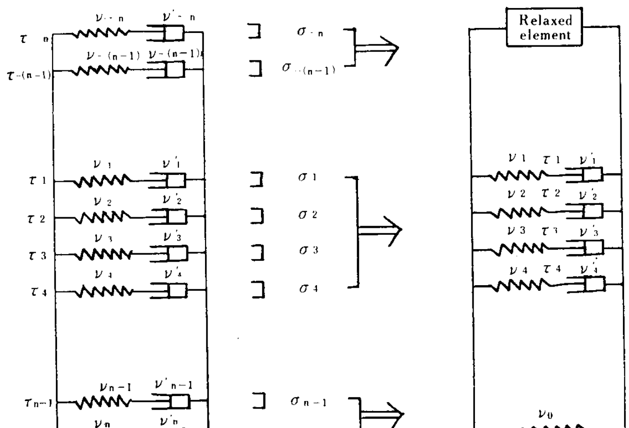

Fig.2 Generalized Maxwell model

with all elements are equal, but the total stress is divided among the elements. The stress component on the i-th Maxwell element is related to the common strain by the differential equation,

$$\frac{d\varepsilon}{dt}=\frac{1}{\nu_i}\frac{d\sigma_i}{dt}+\frac{\sigma_i}{\nu_i'} \qquad (2)$$

$$\sigma=\sum_{i=-n}^{n}\sigma_i \qquad (i=-n,\ -(n-1),\ \cdots\cdots 1,2,3\cdots\cdots n) \qquad (3)$$

Where ν_i denotes the spring constant, and ν_i' is the constant of spring. While the Voigt specification allows a simple prediction of the strain as a function of time for a known stress, the Maxwell model is designed to allow an equally simple prediction of the stress as a function of time when a known strain sequence is imposed upon the material. Let us consider the stress-strain relation which is applicable to the generalized Maxwell body subjected to a linearly increasing strain. If Eq. (2) are integrated under the condition of zero strain at time $t=0$, the resulting stress component is expressed by.

$$\sigma=\left[\sum_{i=-n}^{n}\nu_i\left\{\frac{\tau_i}{t}\left(1-e^{-t/\tau_i}\right)\right\}\right]\varepsilon \qquad (4)$$

where $\varepsilon=\varepsilon_0 t$,

Here the definition of "transient creep function with descrete spectrum" may be given to the ratio σ/ε in Eq. (4). Now we consider of simple model showed in Fig.3.

In case of Fig.3, it is reported by Mr. Kenji Ishihara that Eq. (4) becomes to

$$\sigma=\nu_0\varepsilon+\varepsilon_{04}\nu_4\tau_4\left(1-e^{-\varepsilon/\tau_4\varepsilon_{04}}\right) \qquad (5)$$

where $\varepsilon=\varepsilon_{04}t$ is used to eliminate time parameter t. In order to compare theory with experiment, the author applied the method of least squares to Eq. (5) by putting,

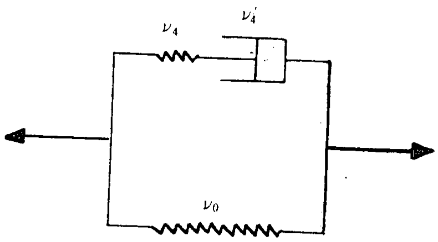

Fig.3 Three element model

$$\tau_4=\nu_4'/\nu_4,\quad \nu_4'\varepsilon_{04}=a,\quad \nu_4=b,\quad \nu_0=c$$

Eq. (5) becomes

$$\sigma=c\varepsilon+a\left(1-e^{-\frac{b}{a}\varepsilon}\right) \qquad (6)$$

The deviation of Eq. (6) being

$$\delta_i=c\varepsilon_i+a\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)-\sigma_i \qquad (7)$$

The sum of squares to be minimized being

$$\sum_{i=1}^{n}\delta_i^2=\sum_{i=1}^{n}\left\{c\varepsilon_i+a\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)-\sigma_i\right\}^2 \qquad (8)$$

Partial differentiation of Eq. (8) with respect to a gives

$$\frac{\partial\sum_{i=1}^{n}\delta_i^2}{\partial a}=2\sum_{i=1}^{n}\left\{c\varepsilon_i+a\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)-\sigma_i\right\}\left\{\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)+a\left(-\frac{b\varepsilon_i}{a^2}\right)e^{-\frac{b}{a}\varepsilon_i}\right\} \qquad (9)$$

In order to minimize the sum of squares, let us put Eq. (9)$=0$ and then

$$\sum_{i=1}^{n}\left\{c\varepsilon_i+a\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)-\sigma_i\right\}\left\{\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)-\frac{b}{a}\varepsilon_i e^{-\frac{b}{a}\varepsilon_i}\right\}=0$$

$$a\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\left\{1-e^{-\frac{b}{a}\varepsilon_i}-\frac{b}{a}\varepsilon_i e^{-\frac{b}{a}\varepsilon_i}\right\}=\sum_{i=1}^{n}(\sigma_i-c\varepsilon_i)\left\{1-e^{-\frac{b}{a}\varepsilon_i}-\frac{b}{a}\varepsilon_i e^{-\frac{b}{a}\varepsilon_i}\right\} \qquad (10)$$

Next, partial differentiation of Eq. (8) with respect to b gives

$$\frac{\partial \sum_{i=1}^{n} \delta_i^2}{\partial b} = 2\sum_{i=1}^{n}\left\{c\varepsilon_i + a\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)-\sigma_i\right\}\times a\times\frac{\varepsilon_i}{a}e^{-\frac{b}{a}\varepsilon_i}=0$$

$$\sum_{i=1}^{n} a\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i e^{-\frac{b}{a}\varepsilon_i} = \sum_{i=1}^{n}(\sigma_i - c\,\varepsilon_i)\ \varepsilon_i\ e^{-\frac{b}{a}\varepsilon_i}$$

$$a\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i e^{-\frac{b}{a}\varepsilon_i} = \sum_{i=1}^{n}\varepsilon_i\,\sigma_i\, e^{-\frac{b}{a}\varepsilon_i} - c\sum_{i=1}^{n}\varepsilon_i^2 e^{-\frac{b}{a}\varepsilon_i} \tag{11}$$

$$a=\frac{\sum_{i=1}^{n}\varepsilon_i\,\sigma_i\, e^{-\frac{b}{a}\varepsilon_i} - c\sum_{i=1}^{n}\varepsilon_i^2 e^{-\frac{b}{a}\varepsilon_i}}{\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i\ e^{-\frac{b}{a}\varepsilon_i}} \tag{12}$$

and also, partial differentiation of Eq. (8) with respect to c gives

$$\frac{\partial \sum_{i=1}^{n} \delta_i^2}{\partial c} = 2\sum_{i=1}^{n}\left\{c\varepsilon_i + a\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)-\sigma_i\right\}\varepsilon_i = 0$$

$$c\sum_{i=1}^{n}\varepsilon_i^2 = \sum_{i=1}^{n}\varepsilon_i\left\{\sigma_i - a\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\right\}$$

$$c\sum_{i=1}^{n}\varepsilon_i^2 = \sum_{i=1}^{n}\varepsilon_i\,\sigma_i - a\sum_{i=1}^{n}\varepsilon_i\left(1-e^{-\frac{b}{a}\varepsilon_i}\right) \tag{13}$$

from (13) $\times \sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i\, e^{-\frac{b}{a}\varepsilon_i}$

$$c\sum_{i=1}^{n}\varepsilon_i^2\times\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i\ e^{-\frac{b}{a}\varepsilon_i} = \sum_{i=1}^{n}\varepsilon_i\,\sigma_i\times\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i\, e^{-\frac{b}{a}\varepsilon_i}$$
$$-a\sum_{i=1}^{n}\varepsilon_i\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\times\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i\, e^{-\frac{b}{a}\varepsilon_i} \tag{14}$$

and from (11) $\times\sum_{i=1}^{n}\varepsilon_i\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)$

$$a\sum_{i=1}^{n}\varepsilon_i\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\times\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i\, e^{-\frac{b}{a}\varepsilon_i} = \sum_{i=1}^{n}\varepsilon_i\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\times\sum_{i=1}^{n}\varepsilon_i\,\sigma_i\, e^{-\frac{b}{a}\varepsilon_i}$$
$$-c\sum_{i=1}^{n}\varepsilon_i\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\times\sum_{i=1}^{n}\varepsilon_i^2\, e^{-\frac{b}{a}\varepsilon_i} \tag{15}$$

from Eqs. (14) and (15), we obtain the next equations

$$c\sum_{i=1}^{n}\varepsilon_i^2\times\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i\, e^{-\frac{b}{a}\varepsilon_i} = \sum_{i=1}^{n}\varepsilon_i\,\sigma_i\times\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i\, e^{-\frac{b}{a}\varepsilon_i}$$
$$-\sum_{i=1}^{n}\varepsilon_i\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\times\sum_{i=1}^{n}\varepsilon_i\,\sigma_i\, e^{-\frac{b}{a}\varepsilon_i} + c\sum_{i=1}^{n}\varepsilon_i\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\times\sum_{i=1}^{n}\varepsilon_i^2\, e^{-\frac{b}{a}\varepsilon_i}$$

$$c=\frac{\sum_{i=1}^{n}\varepsilon_i\sigma_i\times\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i\, e^{-\frac{b}{a}\varepsilon_i} - \sum_{i=1}^{n}\varepsilon_i\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\times\sum_{i=1}^{n}\varepsilon_i\sigma_i e^{-\frac{b}{a}\varepsilon_i}}{\sum_{i=1}^{n}\varepsilon_i^2\times\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i\, e^{-\frac{b}{a}\varepsilon_i} - \sum_{i=1}^{n}\varepsilon_i\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\times\sum_{i=1}^{n}\varepsilon_i^2\, e^{-\frac{b}{a}\varepsilon_i}} \tag{16}$$

Substitution of Eq. (12) into Eq. (10) results in

$$\frac{\sum_{i=1}^{n}(\sigma_i - c_i)\varepsilon_i\, e^{-\frac{b}{a}\varepsilon_i}}{\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\varepsilon_i\, e^{-\frac{b}{a}\varepsilon_i}}\times\sum_{i=1}^{n}\left(1-e^{-\frac{b}{a}\varepsilon_i}\right)\left\{1-e^{-\frac{b}{a}\varepsilon_i}-\frac{b}{a}\varepsilon_i\, e^{-\frac{b}{a}\varepsilon_i}\right\}$$
$$=\sum_{i=1}^{n}(\sigma_i - c\varepsilon_i)\left\{1-e^{-\frac{b}{a}\varepsilon_i}-\frac{b}{a}\varepsilon_i\, e^{-\frac{b}{a}\varepsilon_i}\right\} \tag{17}$$

By substitution of proper value b/a and the relation of stress-strain into Eq. (16), we can obtain the value of c, and again we substitute the values of c and others into Eq. (17). The same thing is repeated at defferent values of b/a until Eq. (17) comes into existence. We can decide visco-elastic constants from the values of b/a which makes Eq. (17) come into existence. Nearly all the stress-strain curves in our experiment are such as shown in Fig.4. In case of the stress-strain curve which is shown in Fig.4, the larger ε becomes, the nearer σ draws on a constant value. So, when $\varepsilon \to \infty$, tan $\phi=0$, and then

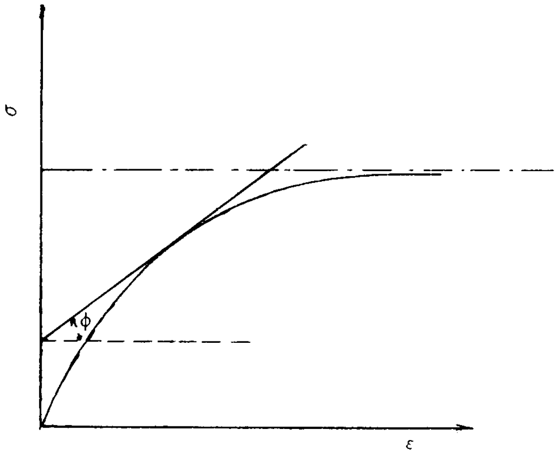

Fig.4 Stress-strain curve of our experiment

$$\tan \phi = \frac{d\sigma}{d\varepsilon} = \nu_0 + \nu\, e^{-\frac{\varepsilon}{\varepsilon_0 \tau}}$$

If, $\varepsilon \to \infty$

$$\lim_{\varepsilon \to \infty} \left(\nu_0 + \nu\, e^{-\frac{\varepsilon}{\varepsilon_0 \tau}} \right) = \nu_0$$

so, it becomes

$$\nu_0 = 0$$

In case of $\nu_0 = 0$, ν_0, namely $c=0$, therefore, Fig. (17) becomes

$$\frac{\sum_{i=1}^{n} \sigma_i \varepsilon_i e^{-A\varepsilon_i}}{\sum_{i=1}^{n} \varepsilon_i e^{-A\varepsilon_i}\left(1-e^{-A\varepsilon_i}\right)} \sum_{i=1}^{n}\left(1-e^{-A\varepsilon_i}\right)\left\{1-e^{-A\varepsilon_i}-A\varepsilon_i e^{-A\varepsilon_i}\right\} = \sum_{i=1}^{n} \sigma_i \left\{1-e^{-A\varepsilon_i}-A\varepsilon_i e^{-A\varepsilon_i}\right\} \quad (18)$$

where $A=b/a$.

From the upper results, we can use Eq. (18) in case of Maxwell model, and Eq. (17) in case of three element model. In this paper, the author regarded asphalt mixtures as visco-elastic bodies which obey Maxwell model, so the author used Eq. (18).

3. Results of the experiment

The results of compressive test subjected to a linearly increasing strain $\varepsilon_0 t$ is shown in Fig.5~13. These figures show the stress-strain curves at different amount of filler and rate of deformation. From these figures, the author decided the viscoelastic constant by using Eq. (18).

The results are shown in Fig.14~22. In Fig.5~13, the faster the rate of deformation becomes, the larger tangent of the stress-strain curves become, then it must be said that if the rate of deformation becomes faster and faster, the behavior of asphalt mixture looks like that of elastic body.

It is also said in Fig.14~22, that is to say, the constant of dashpot becomes smaller and smaller with the rate of deformation becoming fast and the spring constant becomes larger and larger.

And also, Fig.14~22 show that there are maximum point of viscoelastic constant in the middle of filler, so it is really declared that we can use the standard proportion of asphalt mixture as the most suitable one.

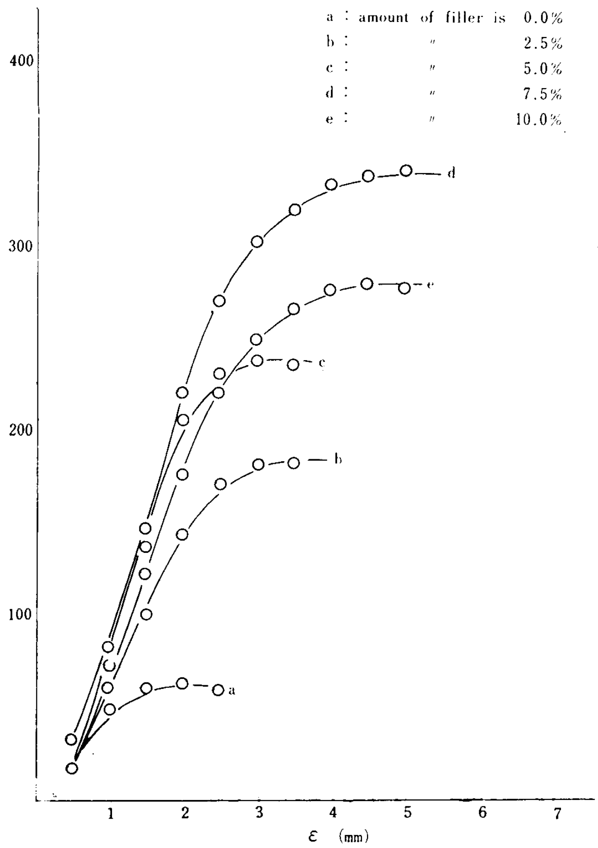

Fig.5 Coarsegraded type asphalt concrete at the rate of deformation being 1mm/min

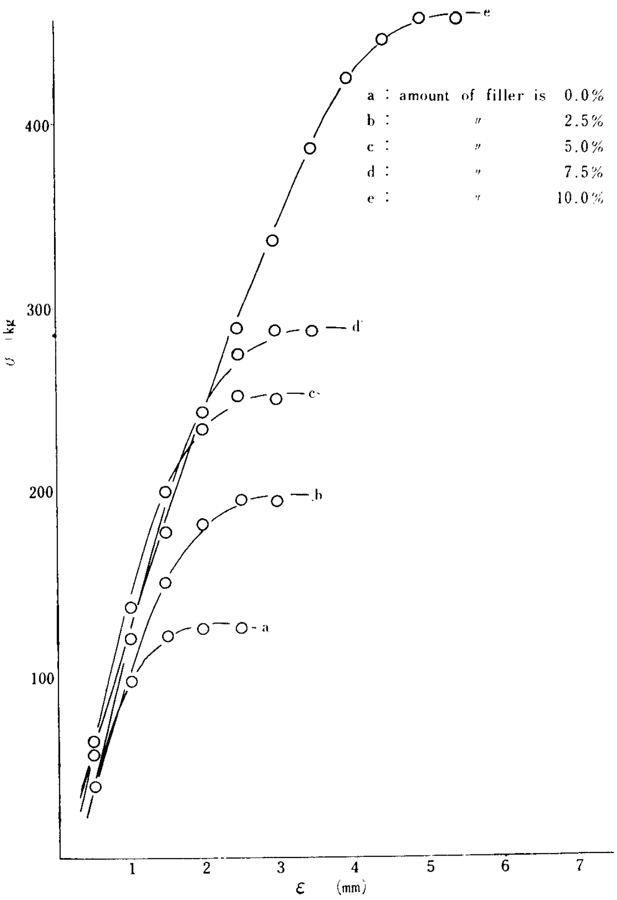

Fig.6 Coarsegraded type asphalt concrete at the rate of deformation being 5mm/min

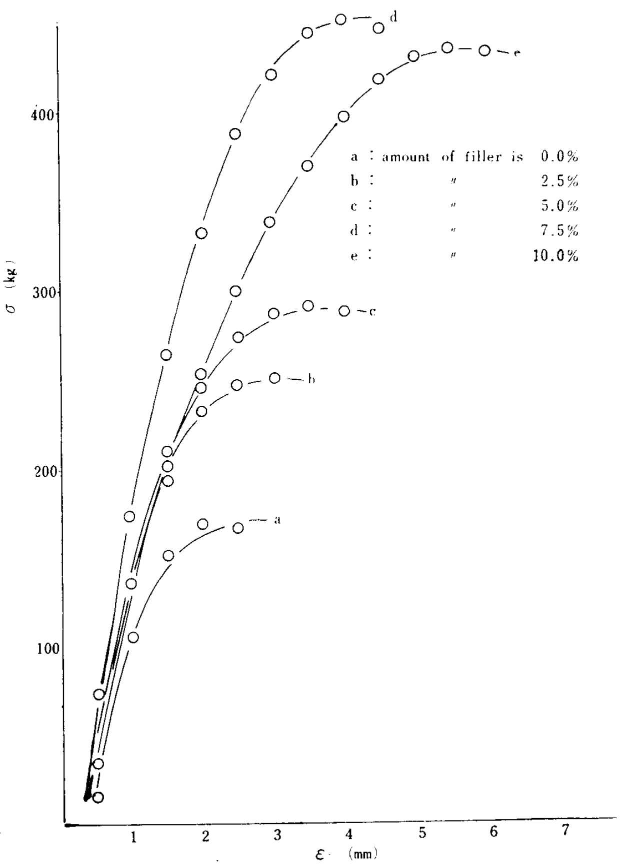

Fig.7 Coarsegraded type asphalt concrete at the rate of deformation being 10mm/min

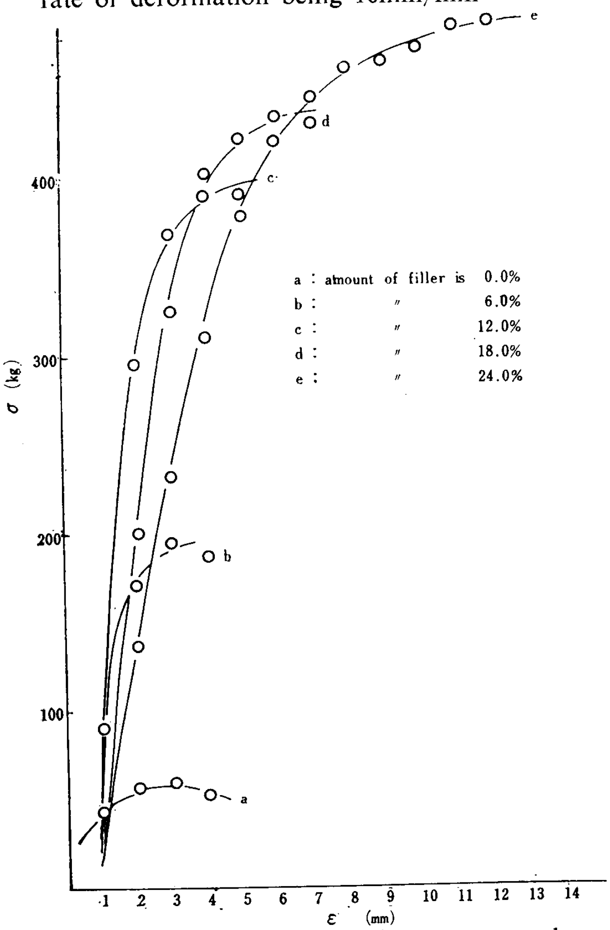

Fig.8 Densegraded type asphalt concrete at the rate of deformation being 1mm/min

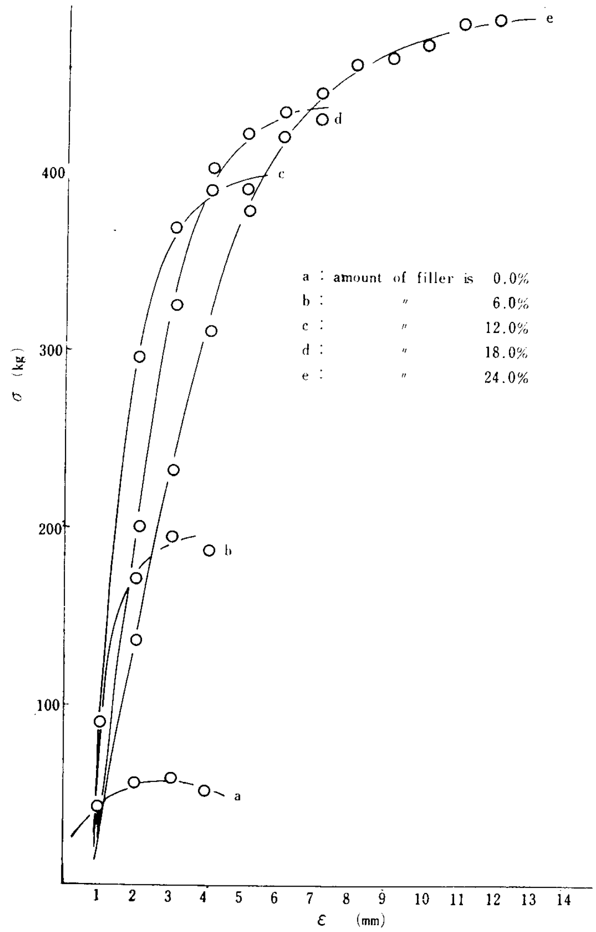

Fig.9 Densegraded type asphalt concrete at the rate of deformation being 5mm/min

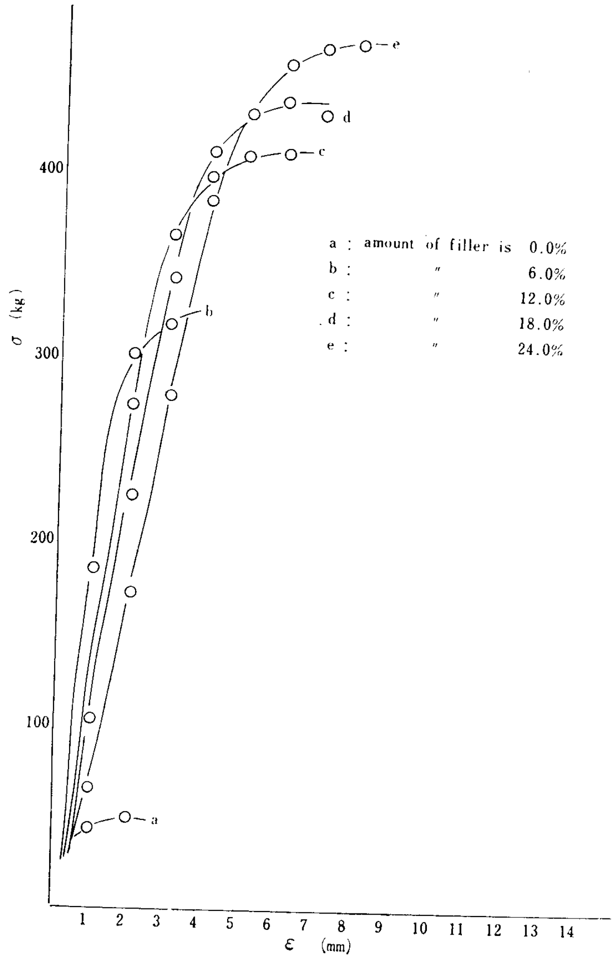

Fig.10 Densegraded type asphalt concrete at the rate of deformation being 10mm/min

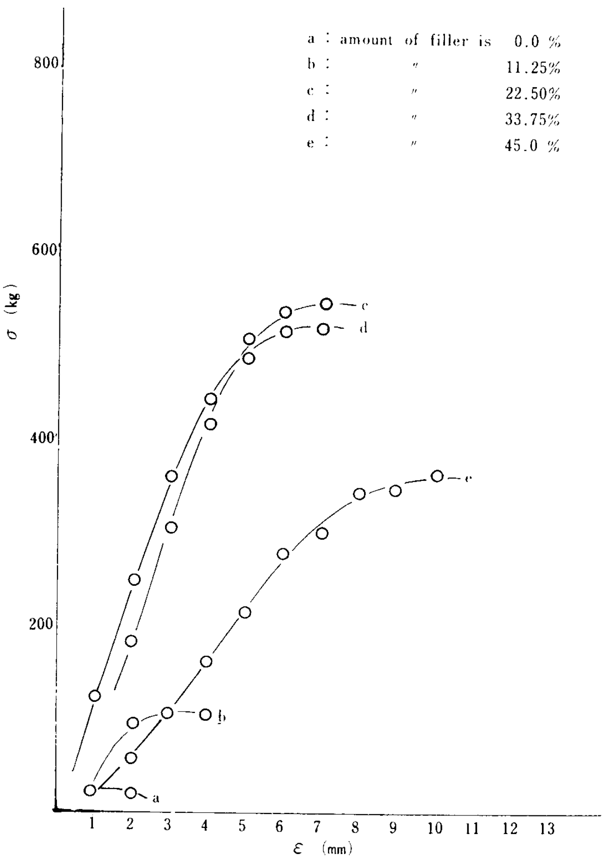

Fig.11 Topeka at the rate of deformation being 1mm/min

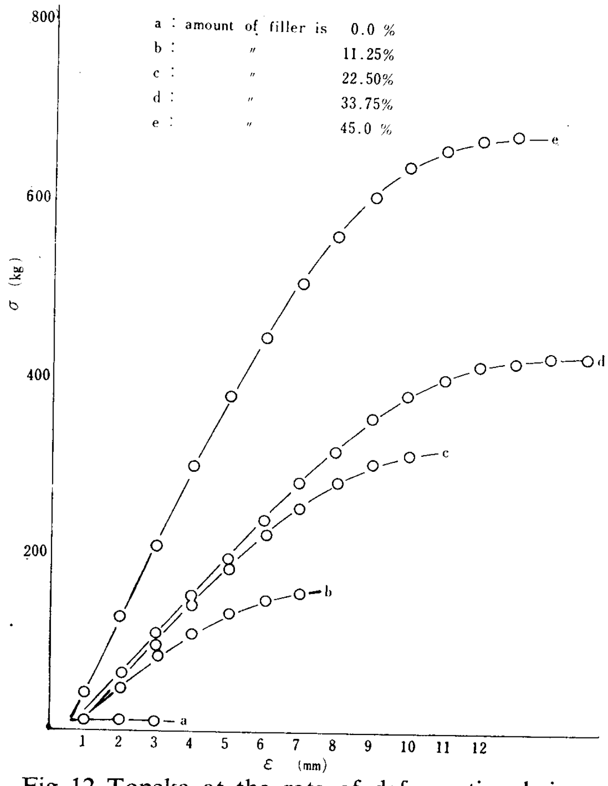

Fig.12 Topeka at the rate of deformation being 5mm/min

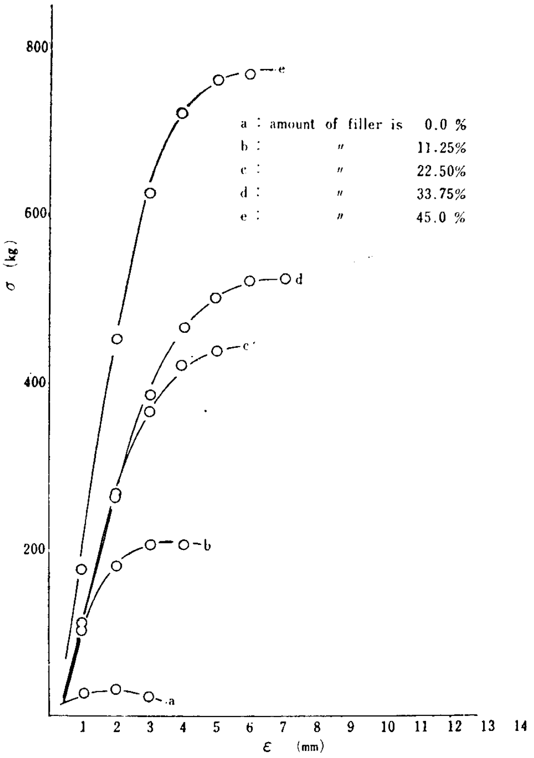

Fig.13 Topeka at the rate of deformation being 10mm/min

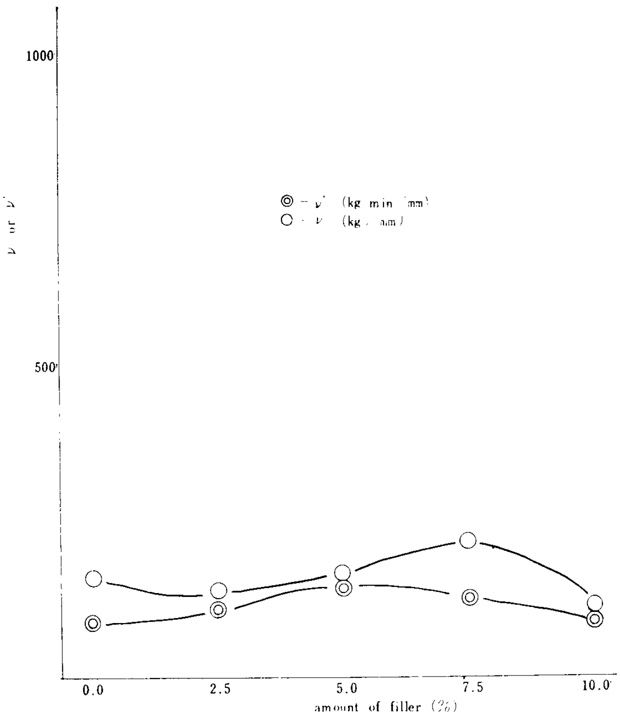

Fig.15 Coarsegraded type asphalt concrete at the rate of deformation being 5mm/min

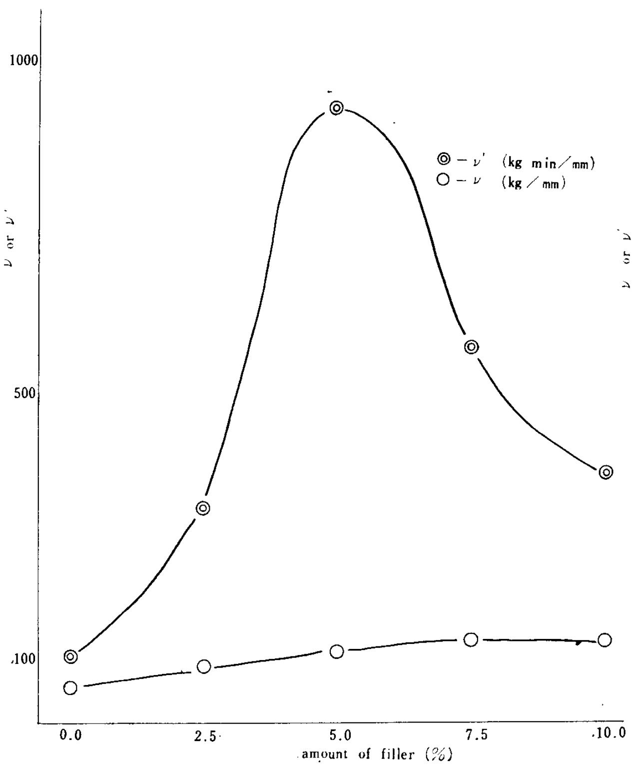

Fig.14 Coarsegraded type asphalt concrete at the rate of deformation being 1mm/min

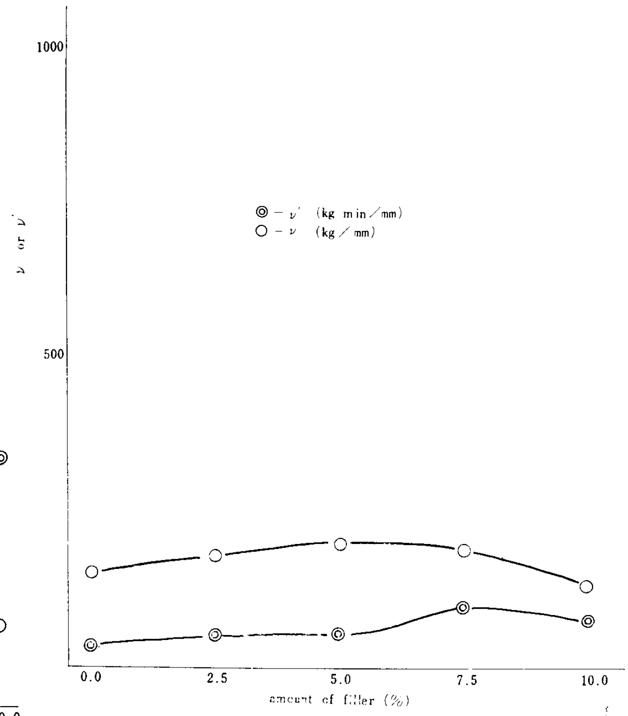

Fig.16 Coarsegraded type asphalt concrete at the rate of deformation being 10mm/min

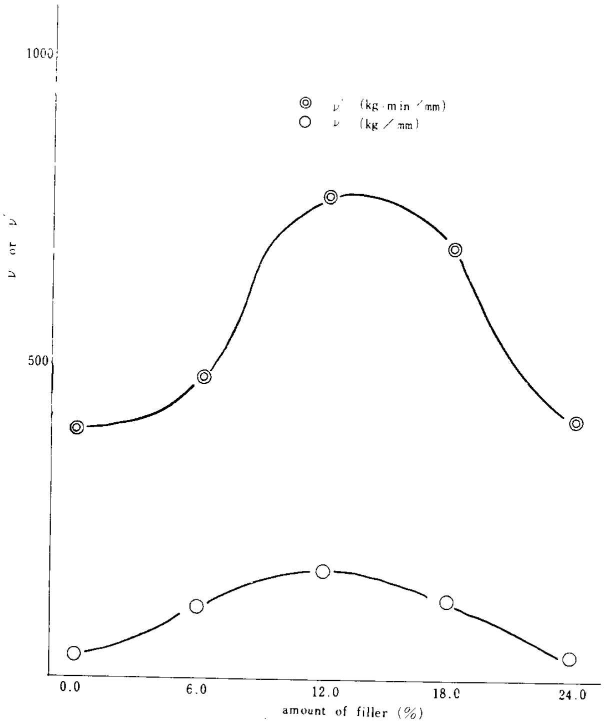

Fig.17 Densegraded type asphalt concrete at the rate of deformation being 1mm/min

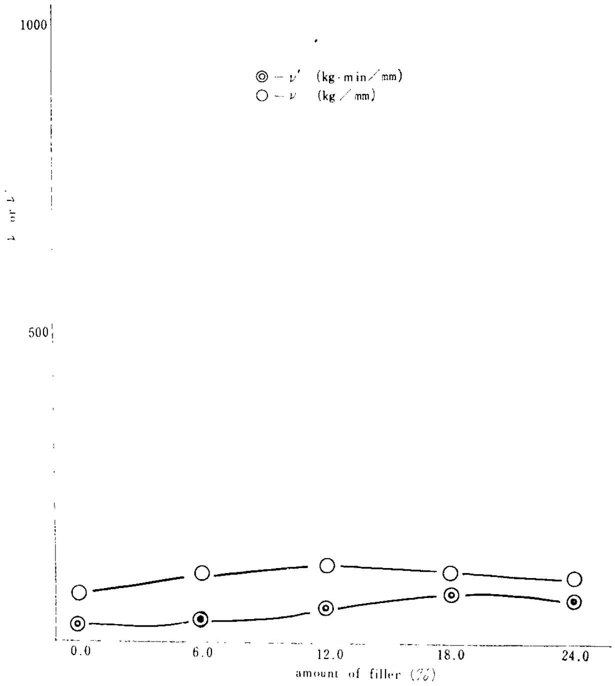

Fig.19 Densegraded type asphalt concrete at the rate of deformation being 10mm/min

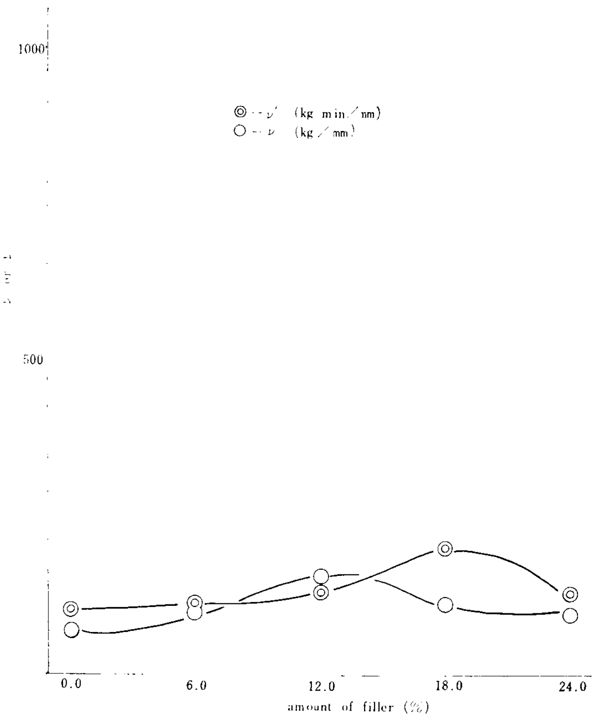

Fig.18 Densegraded type asphalt concrete at the rate of deformation being 5mm/min

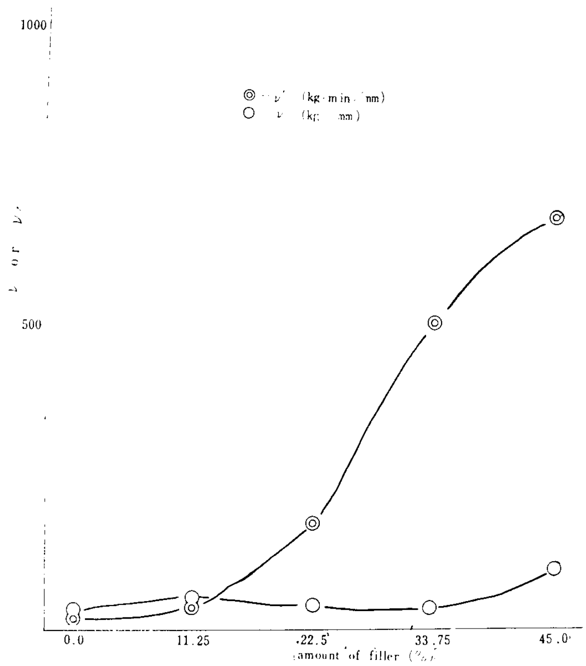

Fig.20 Topeka at the rate of deformation being 1mm/min

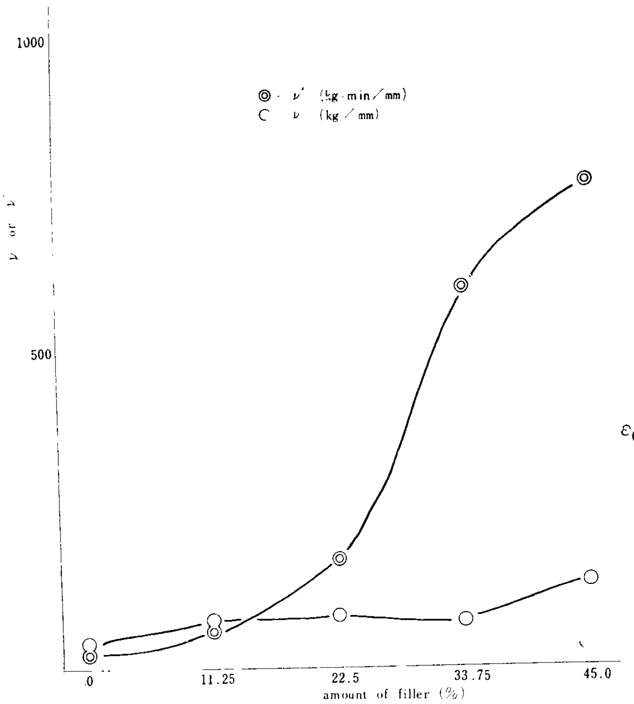

Fig.21 Topeka at the rate of deformation being 5mm/min

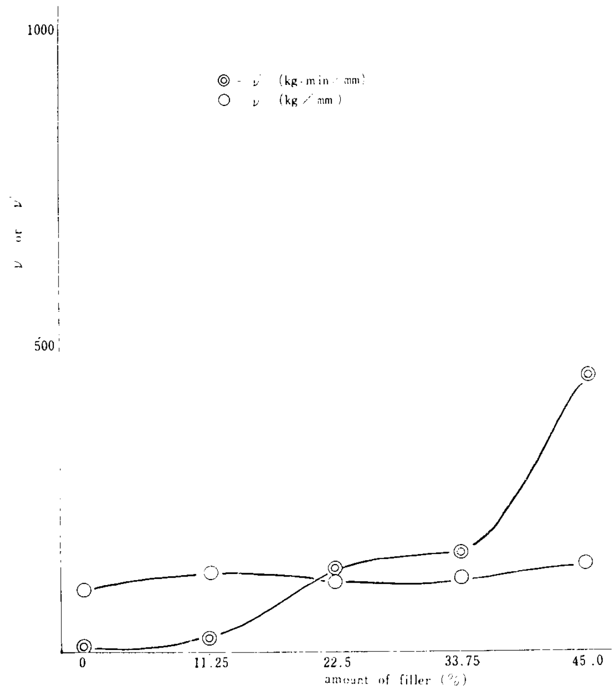

Fig.22 Topeka at the rate of deformation being 10mm/min

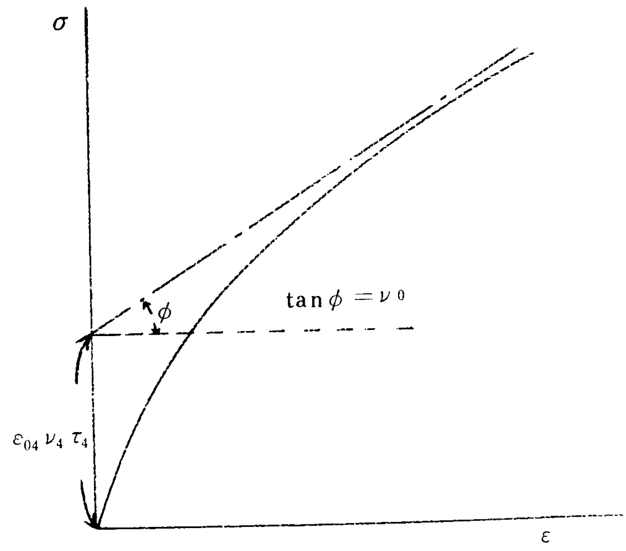

Fig.23 Stress-strain curve of quick deformation

4. Consideration

In case of the rate of deformation becoming faster and faster, the stress-strain curve of asphalt mixture becomes more larger tangent curve, such as Fig.23. This figure shows that the larger ε becomes, the neare tan ϕ draws on a constant value, so when $\varepsilon \to \infty$, $\tan\phi \fallingdotseq 0$. In this case, we can use Eq.(5), then

$$\lim_{\varepsilon\to\infty} \tan\phi = \lim_{\varepsilon\to\infty} \frac{d\tau}{d\varepsilon}$$

$$= \lim_{\varepsilon\to\infty} (\nu_0 + \nu_4 e^{-\varepsilon/\tau_4 \varepsilon_{04}})$$

$$= \nu_0$$

$$\sigma = \nu_0 + \varepsilon_{04}\nu_4\tau_4 - \varepsilon_{04}\nu_4\tau_4 e^{-\varepsilon/\tau_4\varepsilon_{40}}$$

And, the point on which the tangent of the stress-strain curve intersects with σ-axis is A σ-coordinate of which is $\varepsilon_{04}\nu_4\tau_4$.

Generally the pavement slab receives rapid load, so it must be useful consideration that more fast rate of deformation is allowed in our experiment. In such case, three element model must be considered, but it is very difficult for us to measure the rapid rate of deformation without strain gauge. And the behavior of asphalt mixture depends so much upon the temperature of test piece, then in our experiment, the temperature of before breaking is probably different from that of after breaking. From now on, the author intends to be carefull of those points.

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