Research Contribution

Closure Properties of Alternating One-Way Multihead Finite Automata with Constant Leaf-Sizes

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Previous papers introduced alternating multihead finite automata with constant leaf-sizes (AMHFACLs) and investigated several properties of these automata. Leaf-size, in a sense, reflects the number of processors that run in parallel in scanning a given input word. AMHFACLs are more realistic parallel computation models than ordinary alternating multihead finite automata, because of the restriction that the number of processors running in parallel should be constant. This paper examines the closure properties of the class of sets accepted by one-way AMHFACLs and one-way alternating simple multihead finite automata with constant leaf-sizes in the operations of taking union, intersection, complementation, concatenation, Kleene closure, reversal, and ε-free homomorphism.

1. Introduction

We previously introduced alternating multihead finite automata with constant leaf-sizes (AMHFACLs) and investigated several properties of these automata [1]. The main results were as follows: (1) two-way sensing AMHFACLs can be simulated by two-way nondeterministic simple multihead finite automata, (2) for one-way AMHFACLs, $k+1$ heads are better than k, and (3) for one-way alternating simple multihead finite automata with constant leaf-sizes (ASPMHFACLs), sensing versions are more powerful than non-sensing versions.

Leaf size, in a sense, reflects the number of processors that run in parallel in scanning a given input. AMHFACLs are more realistic parallel computation models than ordinary alternating multihead finite automata, because of the restriction that the number of processors running in parallel should be constant. It is interesting to examine the properties of AMHFACLs and ASPMHFACLs, because they have two kinds of parallelism: constant leaf-size and the number of heads.

In this paper, we examine the closure properties of the class of sets accepted by AMHFACLs and ASPMHFACLs in the operations of taking union, intersection, complementation, concatenation, Kleene closure, reversal, and ε-free homomorphism.

Section 2 explains the terminology and notation used in this paper. In Sections 3 and 4, we investigate the closure properties of AMHFACLs and ASPMHFACLs, respectively.

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2. Preliminaries

The reader is referred to King [2] for formal definitions of an alternating multihead finite automation (AMHFA). An alternating *simple* multihead finite automation [6] (ASPMHFA) is an AMHFA with the restriction that one head (called the 'reading head') can sense input symbols, while the others (called the 'counting heads') can detect only the left endmarker "¢" and the right endmarker "$". When the heads of an AMHFA (ASPMHFA) are allowed to sense the presence of other heads in the same input position, we call it a 'sensing' AMHFA (ASPMHFA).

A one-way AMHFA is defined in the usual way. A semi-one-way ASPMHFA is an ASPMHFA whose reading head can move only in one direction, but whose conting heads can move in two directions. A one-way ASPMHFA is an ASPMHFA whose reading and counting heads can move in one direction.

A *step* of an AMHFA (ASPMHFA) M consists of reading a symbol from the string input by each head, moving the heads in the specified directions (note that any of the heads can remain stationary during a move), and entering a new state, in accordance with the transition function. If one of the heads falls off the input words, then M can make no further move.

For any AMHFA (ASPMHFA) M, let $T(M)$ be the set of inputs accepted by M. In this paper, to represent the different kinds of one-way ASPMHFAs (resp. AMHFAs, sensing AMHFAs) systematically, we use the notation Xk-HFA (resp. Xk-HFA, XSNk-HFA), $k \geq 1$, where

(1) $X \in \{D, N, A, U\}$
D: deterministic
N: nondeterministic
A: alternation
U: alternating automaton with only universal states
(2) $Y \in \{SP, SNSP\}$
SP: simple
SNSP: sensing simple
(3) $k-H$: k-head (the number of heads is k).
Furthermore,

$\mathscr{L}$[XYk-HFA]$=\{T \mid T=T(M)$ for some XYk-HFA $M\}$

$\mathscr{L}$[XSNk-HFA]$=\{T \mid T=T(M)$ for some XSNk-HFA $M\}$

$\mathscr{L}$[Xk-HFA]$=\{T \mid T=T(M)$ for some Xk-HFA $M\}$.

Definition 2.1 Let $L: N \rightarrow R$ be a function, where N denotes the set of all positive integers and R denotes the set of all nonnegative real numbers. For each tree t, let LEAF(t) denote the leaf size of t (that is, the number of leaves of t). We say that for $X \in \{A, U\}$, $Y \in \{SP, SNSP\}$, an XYk-HFA (XSNk-HFA, Xk-HFA) M is $L(n)$ leaf-size bounded if when we give an input x of length n to M there is no computation tree of M on x such that LEAF(t)$> \lceil L(n) \rceil$ [1].

For each $X \in \{A, U\}$, $Y \in \{$SP, SNSP$\}$, $k \geqq 1$, we let XYk-HFA($L(n)$) (resp. Xk-HFA($L(n)$), XSNk-HFA($L(n)$)) denote $L(n)$ leaf-size bounded XYk-HFA (resp. Xk-HFA, XSNk-HFA). Let $\mathscr{L}$[XYk-HFA($L(n)$)] $=\{T \mid T=T(M)$ for some XYk-HFA($L(n)$)$M\}$ for $X \in \{A, U\}$ and $Y \in \{SP, SNSP\}$. $\mathscr{L}$[Xk-HFA($L(n)$)] and $\mathscr{L}$[XSNk-HFA($L(n)$)] are defined similarly.

3. Multihead Finite Automata

In this section, we will investigate the closure properties of the class of sets accepted by AMHFACLs. The following lemma is shown in Matsuno *et al.* [1].

Lemma 3.1 For each $r \geqq 1$, let

$$A(r)=\{w_1 2 w_2 2 \cdots 2 w_{2r} \mid \forall i(1 \leqq i \leqq 2r)[w_i \in \{0,1\}^*] \ \& \ \forall j(1 \leqq j \leqq r)[w_j = w_{2r+1-j}]\}$$

and for each string $x = w_1 2 w_2 2 \cdots 2 w_{2r}$ in $A(r)$ and for each $i(1 \leqq i \leqq r)$, let the pair of w_i and w_{2r+1-i} be called twins of x. Then, Ak-HFA(s) can compare all twins of a string in $A(r)$ if and only if $r \leqq k(k-1)s/2$.

Lemma 3.2 Let
$T_1(b)=\{w_1 2 w_2 2 \cdots 2 w_{2b} \mid \forall i(1 \leqq i \leqq 2b)[w_i \in \{0,1\}^* 3 \{0,1\}^*] \ \& \ \exists i, j[w_i = x3y \ \& \ w_j = x3z \ \& \ y \neq z]\}$.
Then, for each $r \geqq 1$,
(1) $T_1(r(r-1)/2+1) \in \mathscr{L}$[N2-HFA], and
(2) $\overline{T_1(r(r-1)/2+1)} \notin \mathscr{L}$[NSN$r$-HFA].

Proof. (1): The proof of (1) is omitted, since it is obvious.

(2): Suppose that for some $r \geqq 1$, there exists an NSNr-HFA M accepting $\overline{T_1(r(r-1)/2+1)}$. Let $T_1'(b)$ $=\{w_1 2 w_2 2 \cdots 2 w_{2b} \mid \forall i(1 \leqq i \leqq 2b)[w_i \in \{0,1\}^* 3 \{0,1\}^* \ \& \ w_i = w_{2b+1-i} = B(\min(i, 2b+1-i))3y \ \& \ y \in \{0,1\}^*\}$[2] and let $T_1''(b)=\{w_1 2 w_2 2 \cdots 2 w_{2b} \mid \forall i(1 \leqq i \leqq 2b)[w_i \in \{0,1\}^* 3 \{0,1\}^* \ \& \ w_i = B(\min(i, 2b+1-i))3y \ \& \ y \in \{0,1\}^*\}$.

It is easily seen that $T_1(b)$ is accepted by a (one-head) finite automation, and therefore $T_1''(b)$ is a regular set.

On the other hand, it is easily seen that $T_1'(b) = \overline{T_1(b)} \cap T_1''(b)$. From these facts and the fact that $\mathscr{L}\{$NSNr-HFA] ($r \geqq 1$) is closed under intersection with regular sets, it follows that $T_1'(r(r-1)/2+1)$ is accepted by M.

We can prove that $T_1'(r(r-1)/2+1) \notin \mathscr{L}$[NSN$r$-HFA] by using the same technique as in the proof of Theorem 1 [3]. This is a contradiction.

Q.E.D.

Theorem 3.1 *For each $k \geqq 2$ and each $s \geqq 1$, $\mathscr{L}$[Ak-HFA(s)] and $\mathscr{L}$[ASNk-HFA(s)] are not closed under complementation.*

Proof. It is shown in Theorem 4.4 in [1] that $\mathscr{L}$[ASNk-HFA(s)]$\subsetneq \mathscr{L}$[NSN(ks)-HFA] ($k \geqq 2$, $s \geqq 1$). From this fact and Lemma 3.2, we can show that $\overline{T_1(ks(ks-1)/2+1)} \notin \mathscr{L}$[ASNk-HFA($s$)]. This completes the proof of the theorem.

Q.E.D.

Lemma 3.3 For each $r \geqq 2$ and each $i(1 \leqq i \leqq r(r+1)/2)$, $T_2(r, i)=\{w_1 2 w_2 2 \cdots 2 w_p \mid p = r(r+1) \ \& \ i(1 \leqq i \leqq p)[w_i \in \{0,1\}^* \ \& \ w_i = w_{p+1-i}]\}$ and $T_2(r) = \{w_1 2 w_2 2 \cdots 2 w_p \mid p = r(r+1) \ \& \ \forall i(1 \leqq i \leqq p)[w_i \in \{0,1\}^* \ \& \ w_i = w_{p+1-i}]\}$. Then,
(1) for each $r \geqq 2$ and each $i(1 \leqq i \leqq r(r+1)/2)$, $T_2(r, i) \in \mathscr{L}$[D2-HFA] and
(2) $T_2(ks, 1) \cap T_2(ks, 2) \cap \cdots \cap T_2(ks, (ks+1)ks/2) = T_2((ks+1)ks/2) \notin \mathscr{L}$[ASNk-HFA($s$)] ($k \geqq 2$, $s \geqq 1$).

Proof. (1): Obvious.

(2): It has been shown [3] that $T_2((k+1)k/2) \notin \mathscr{L}$[NSNk-HFA]. From this fact and Theorem 4.4 [1] (see above), we can obtained the above lemma.

Q.E.D.

We shall formulate two sufficient conditions for a language L to be in $\mathscr{L}$[USNk-HFA(s)] for $k \geqq 2$ and $s \geqq 1$. For each input x given to an alternating automaton with only universal states, there exists only one computation tree corresponding to the input x. Thus, informally, an alternating automaton with only universal states can be considered as a 'deterministic parallel machine.' We shall need the following languages for an arbitrary natural number f.

$C_f(n)=\{ucw_1cw_2c \cdots cw_fcw_fc \cdots cw_2cw_1 \mid |u| = |w_i| = n \ \& \ u, w_i \in \{a, b\}^*$ for each $i(1 \leqq i \leqq f)\}$;[3]

$D_f(n)=\{v_1 d v_1 d \mid v_1 \in C_f(n)\}$;

$E_f(n)=\{ucw_1cw_2c \cdots cw_fcw_{f+1}c \cdots cw_{2f-1}cw_{2f} \mid |u| = |w_i| = n \ \& \ u, w_i \in \{a, b\}^*$ for each $i(1 \leqq i \leqq 2f)$ &

[1] $\lceil r \rceil$ means the smallest integer greater than or equal to r.

[2] $B(i)$ denotes the binary representation of i.

[3] For any word w, $|w|$ denotes the length of w.

$\exists j(1 \leqq j \leqq f)[w_j \neq w_{2f+1-j}]\}$ and

$F_f(n) = \{v_1 c w_1 c w_2 c \cdots c w_f c w_f c \cdots c w_2 c w_1 d v_2 c w_1 c w_2 c \cdots c w_f c w_f c \cdots c w_2 c w_1 d \mid |v_1| = |v_2| = |w_i| = n \;\&\; v_1, v_2, w_i \in \{a, b\}^*$ for each $i(1 \leqq i \leqq f)$ and $v_1 \neq v_2\}$.

Let $C_f = \bigcup_{1 \leqq n < \infty} C_f(n)$; $D_f = \bigcup_{1 \leqq n < \infty} D_f(n)$; $E_f = \bigcup_{1 \leqq n < \infty} E_f(n)$ and $F_f = \bigcup_{1 \leqq n < \infty} F_f(n)$ for arbitrary $f = 1, 2, 3, \ldots$.

Lemma 3.4 Let L be an arbitrary set fulfilling the following conditions:

(1) $L \supseteq C_f \cup D_f$

(2) $L \cap (E_f \cup F_f) = \phi$.

Let $f = k(k-1)s/2$, where $k \geqq 2$ and $s \geqq 1$. Then L is not in $\mathscr{L}$[USNk-HFA(s)].

Proof. The proof is an extension of the proof on Theorem 1 in Hromkovic [4]. Let us assume that there exists a USNk-HFA(s') which recognizes a set L satisfying (1) and (2) where $1 \leqq s' \leqq s$. We need the following notations:

A *configuration* of M working on the input word w is a $(k+1)$-tuple $(q, i_1, i_2, \ldots, i_k)$, where q is the state of the finite state control and i_j is the position of the j-th head on the input word w.

A *prominent configuration* is a configuration of the computation tree on the input word x in $C_f \cup D_f \cup E_f \cup F_f$, from which M moves one of its heads on the symbol c, d, or \$.

The subsequence of prominent configuration of the j-th path of the computation tree on the word x is called a *j-pattern* of x (denoted by $P_j(x)$).

For each word x given to USNk-HFA(s') M, we let $(P_1(x), P_2(x), \ldots, P_{s'}(x))$ denote the *pattern* of M.

Let M be a USNk-HFA(s') with t states that recognizes a set L satisfying (1) and (2). We shall consider the initial part of computation tree of M on the word y in $C_f(n) \cup D_f(n)$, which begins in the initial configuration and ends in a prominent configuration, in which one of the heads had read the whole subword $y_1 = u c w_1 c w_2 c \cdots c w_f c w_f c \cdots c w_2 c w_1$ of the input word y (see Fig. 1). (In other words, the initial part of the computation tree is the part that is the same for words y_1 and $y_1 d y_1 d$, because M does not know whether it is working on the word y_1 in $C_f(n)$ or on the word $y_1 d y_1 d$ in $D_f(n)$.)

Now, let us consider the number of all patterns of the initial part of the computation tree on the word y in $C_f(n) \cup D_f(n)$, which number we denote $p(n)$. If we note that $|y_1| = k(k-1)s(n+1)+n$, we can easily see that the number of all configurations on word y_1 is bounded by

$$t[(k(k-1)s+1)(n+1)]^k.$$

Thus we obtain the following inequality:

$$p(n) \leqq [[t[(k(k-1)s+1)(n+1)]^k]^{k((k-1)ks+1)}]^s,$$

because the leaf-size of the computation tree is bounded by $s(\geqq 1)$, and for each $j\,(1 \leqq j \leqq s)$, no j-pattern of the initial part of the computation tree can consist of more than $k((k-1)ks+1)$ prominent configurations, as shown in Fig. 2.

Since the number of all words y_1 from $C_f(n)$ is

$$2^{[(k-1)ks/2+1]n}$$

there exists a pattern σ of the initial part of the computation tree such that at least

$$2^{[(k-1)ks/2+1]n}/p(n)$$

different words y_1 from $C_f(n)$ have the same pattern σ as the initial part of the computation tree.

Now we distinguish the following two cases according to the last prominent configuration $(q, i_1, i_2, \ldots, i_k)$ of each path of the pattern σ.

(1) $i_j > n$ for all j in $\{1, 2, \ldots, k\}$, that is, all heads have read the initial subword $u \in \{a, b\}^*$.

(2) There exists some j in $\{1, 2, \ldots, k\}$ such that $i_j \leqq n$, that is, at least one head has not read the initial subword $u \in \{a, b\}^*$. We shall show below that both (1)

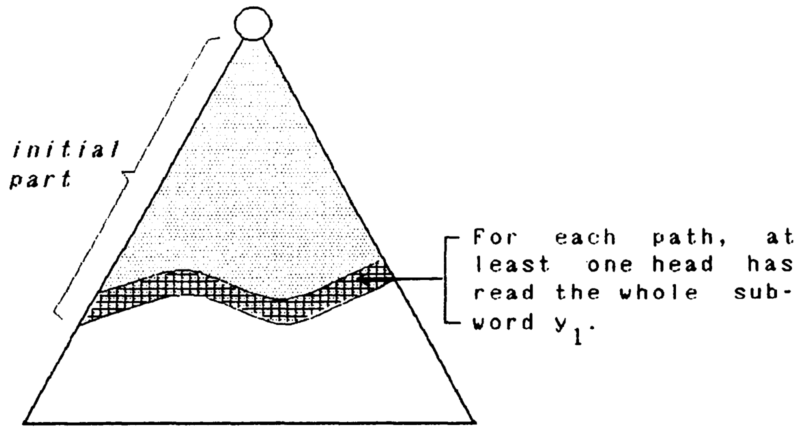

Fig. 1 Initial part of the computation tree on the word y in $C_f(n) \cup D_f(n)$.

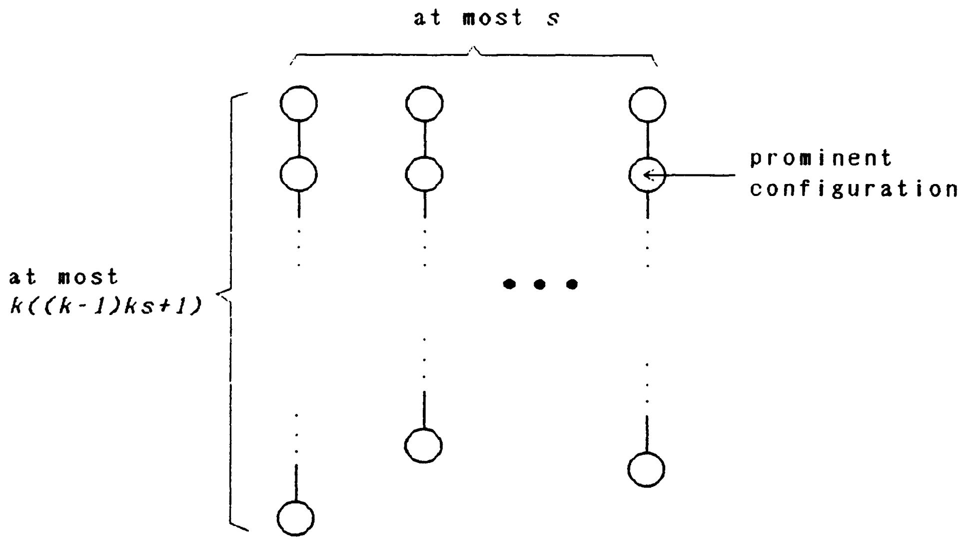

Fig. 2 Pattern of initial part of computation tree on y in $C(n) \cup D(n)$.

and (2) lead to contradictions.

(1) In this case we shall consider input words $y_1 d y_1 d$ in $D_f(n)$, where y_1 has the pattern σ. Noting that there exist at least $2^{((k-1)ks/2+1)n}/p(n)$ different words y_1 with the pattern σ, we see that there exist at least

$$m=[2^{((k-1)ks/2+1)n}/p(n)]\cdot[1/2^{(k-1)ks/2}]$$

different words y_1 with the pattern σ differing from each other only in the initial subword u; that is, there exist at least m words $y_1=ucx$, where $x=w_1cw_2c\cdots cw_fcw_fc\cdots cw_2cw_1$ is fixed, with the pattern σ.

It is obvious that $m=2^n/p(n)$ is greater than 2 for a sufficiently large value of n, since $p(n)$ is bounded by a polynomial. This means that for a sufficiently large n there exist two words from $C_f(n)$ $v_1=u_1cx$ and $v_2=u_2cx$, where $u_1\neq u_2$, with the same pattern σ as the initial part of the computation tree. Since M accepts the word $y=v_1dv_1d=u_1cxdu_1cxd$ in $D_f(n)$ and the state set of M consists of only universal states, it follows that M must also accept the word $y'=u_2cxdu_1cxd$, which clearly belongs to $F_f(n)$.

(2) We shall consider the input word y in $C_f(n)$ in this case. Let us consider all accepting computation trees on all

$$2^{(k(k-1)s/2+1)n}/p(n)$$

different words $y=y_1$ in $C_f(n)$ that have the same pattern σ as the initial part of the computation tree.

Let $p'(n)$ be the number of all possible patterns of accepting computation trees on words y in $C_f(b)$. We obtain the following inequality,

$$p'(n)\leqq[[t[(k(k-1)s+1)(n+1)]^k]^{k((k-1)ks+1}]^s.$$

From this fact it follows that there exist at least

$$2^{(k(k-1)s/2+1)n}/p'(n)$$

different words y in $C_f(n)$ with the same pattern σ' containing the pattern σ as an initial subsequence.

From Lemma 3.1 and assumption (2), we can see that for each input word in $C_f(n)$, there must be an i_0 such that both subwords w_i of the words $y=ucw_1cw_2c\cdots cw_{i_0}c\cdots cw_fcw_fc\cdots cw_{i_0}c\cdots cw_2cw_1$ are never read by any two heads at the same time. This means that there exist at least

$$m=[2^{(f+1)n}/p'(n)]\cdot[1/2^{(f-1)n}2^n]=2^n/p'(n)$$

different words in $C_f(n)$ that have the same pattern σ', which differ from each other only in the subword w_{i_0}.

It can be seen that $m\geqq 2$ for a sufficiently large value of n, and so there exist two words in $C_f(n)$

$$v_1=ucw_1cw_2c\cdots cw_{i_0}c\cdots cw_fcw_fc\cdots cw_{i_0}c\cdots cw_2cw_1$$

$$v_2=ucw_1cw_2c\cdots cw'_{i_0}c\cdots cw_fcw_fc\cdots cw'_{i_0}c\cdots cw_2cw_1$$

with the same pattern σ' as in the accepting computation tree, where $w_{i_0}\neq w'_{i_0}$.

By an argument similar to that in the proofs of Theorem 1 in Yao and Rivest [3] and Theorem 1 in Hromkovic [4], it can be shown that M must also accept the word

$$y'=ucw_1cw_2c\cdots cw_{i_0}c\cdots cw_fcw_fc\cdots cw'_{i_0}c\cdots cw_2cw_1,$$

which belongs to $E_f(n)$. This is a contradiction.

Q.E.D.

Lemma 3.5 Let L be an arbitrary set fulfilling the following conditions:

(3) $L\supseteq\{e\}\cdot C_f\cup\{e\}\cdot D_f$

(4) $L\cap(\{e\}\cdot E_f\cup\{e\}\cdot E_f)=\phi$.

Let $f=k(k-1)s/2$, where $k\geqq 2$ and $s\geqq 1$. Then the set L

is not in $\mathscr{L}$ [USNk-HFA(s)].

Proof. It is an easy matter to show that if there exists a set L satisfying the conditions of Lemma 3.4 such that $L \in \mathscr{L}$[USNk-HFA(s)], then there exists a set L' fulfilling the conditions of Lemma 3.4 such that $L' \in \mathscr{L}$[USNk-HFA(s)].

Q.E.D.

The following theorem can be directly obtained from Lemma 3.3 above:

Theorem 3.2 *For each $k \geqq 2$ and $s \geqq 1$, $\mathscr{L}$[Ak-HFA(s)] and $\mathscr{L}$[ASNk-HFA(s)] are not closed under intersection.*

Theorem 3.3 *For each $k \geqq 2$ and $s \geqq 1$, $\mathscr{L}$[Uk-HFA(s)] and $\mathscr{L}$[USNk-HFA(s)] are not closed under the following operations:*

(1) Intersection
(2) Concatenation
(3) Reversal
(4) Kleene closure
(5) Union
(6) ε-free homomorphism.

Proof. (1): Obvious from Lemma 3.3.

(2): Let us consider the following languages:

$L_1 = \{a, b\}^* c \cup \{\varepsilon\}$,

$L_2 = \{udud \mid u \in \{a, b, c\}^*\} \cup \{\varepsilon\}$,

$G_f = \{w_1 c w_2 c \cdots c w_f c w_f c \cdots c w_2 c w_1 \mid w_i \in \{a, b\}^*$ for $1 \leqq i \leqq f\} \cup \{\varepsilon\}$ for $i = 1, 2, 3, \ldots$

Clearly, $L_1 \in \mathscr{L}$[D1-HFA], $L_2 \in \mathscr{L}$[D2-HFA], and $G_f \in \mathscr{L}$[Uk-HFA(s)] for $f \leqq k(k-1)s/2$. On the other hand, by an argument similar to that in the proof of Theorem 2 in Hromkovič [4], we can see that the set $L_1 L_2 G_{k(k-1)s/2}$ is not in $\mathscr{L}$[USNk-HFA(s)].

(3): The set $L_2 \cup \{a, b\}^* c G_{(k-1)ks/2}$ does not belong to $\mathscr{L}$[USNk-HFA(s)], since it fulfills the conditions of Lemma 3.4, but $L_2^R \cup \{a, b\}^* c G_{(k-1)ks/2}$ belongs to $\mathscr{L}$[Uk-HFA(s)].

(4): Let us consider the set $L_3 = \{e\} \cdot L_2 \cup \{a, b\}^* c G_{(k-1)ks/2} \cup \{e\}$, which belongs to $\mathscr{L}$[Uk-HFA(s)]. By an argument similar to that in the proof of Theorem 4 in Hromkovič [4], we can see that L_3^* satisfies conditions (3) and (4) of Lemma 3.5, which implies that L_3^* is not in $\mathscr{L}$[USNk-HFA(s)].

(5): It can easily be seen that the set L_2 and $\{a, b\}^* c G_{(k-1)ks/2}$ belongs to $\mathscr{L}$[Uk-HFA(s)] for each $k \geqq 2$ and $s \geqq 1$ and that the set $L_2 \cup \{a, b\}^* c G_{(k-1)ks/2}$ fulfills the conditions of Lemma 3.4.

(6): Clearly, the set $L_4 = \{e\} \cdot L_2 \cup \{g\} \cdot \{a, b\}^* c G_{(k-1)ks/2}$ belongs to $\mathscr{L}$[(*Uk-HFA(s)*] for each $k \geqq 2$ and $s \geqq 1$. Let us define an ε-free homomorphism h as follows: $h(e) = h(g) = e$, $h(a) = a$, $h(b) = b$, $h(c) = c$, $h(d) = d$. Then $T(L_4)$ satisfies conditions (3) and (4) of Lemma 3.5.

When leaf-size is not restricted, the following result holds:

Theorem 3.4 *For each $k \geqq 2$, $\mathscr{L}$[Uk-HFA] and $\mathscr{L}$[USNk-HFA] are not closed under complementation.*

Proof. Let us suppose that $\mathscr{L}$[Uk-HFA] is closed under complementation. From Theorem 1 in Sakurayama *et al.* [5], we find that $\mathscr{L}$[Uk-HFA] $=$ co$-\mathscr{L}$[Nk-HFA] for $k \geqq 1$. It follows that for some set L, if $L \in \mathscr{L}$[Nk-HFA], then $\bar{L} \in \mathscr{L}$[Uk-HFA], and $\bar{\bar{L}} = L \in \mathscr{L}$[Uk-HFA] from the assumption above. Thus, $\mathscr{L}$[Nk-HFA] $\subseteq \mathscr{L}$[Uk-HFA]. On the other hand, from Corollary 3 (3) in Sakurayama *et al.* [5], we can show that $\mathscr{L}$[Uk-HFA] is not comparable with $\mathscr{L}$[Nk-HFA] for each $k \geqq 2$. This is a contradiction. The case of $\mathscr{L}$[USNk-HFA] is proved by using a similar argument to the one above.

Q.E.D.

4. Simple Multihead Finite Automata

The closure properties under Boolean operations of ASPMHFAs are given in Matsuno *et al.* [6]. In this section, we first summarize the closure properties under Boolean operations of ASPMHFACLs derived from those results. The following theorem is obvious:

Theorem 4.1 *For each $Y \in \{SP, SNSP\}$, $k \geqq 1$, and $s \geqq 1$, $\mathscr{L}$[AYk-HFA(s)] is closed under union.*

Theorem 4.2 *For each $Y \in \{SP, SNSP\}$, $k \geqq 1$, and $s \geqq 1$, $\mathscr{L}$[UYk-HFA(s)] is not closed under union.*

Proof. The proof can be found from Lemma 6.5 in Matsuno *et al.* [6].

Q.E.D.

Theorem 4.3 *For each $X \in \{A, U\}$, $Y \in \{SP, SNSP\}$, $k \geqq 2$, and $s \geqq 1$, $\mathscr{L}$[XYk-HFA(s)] is not closed under complementation and intersection.*

Proof. The proof can be found from Lemmas 6.2, 6.3, and 6.4 in Matsuno *et al.* [6].

Q.E.D.

We next investigate the closure properties of ASPMHFA with only universal states under operations of concatenation, Kleene closure, reversal, and ε-free homomorphisms.

Lemma 4.1 Let $T_3 = \{x \in \{0, 1\}^+ \mid (|x| \geqq 3)$ & ($|x|$ is odd) & (the center symbol in x is '1')$\}$ and $T_4 = \{a\}^*$. Then,

(1) $T_3, T_4, T_4 T_3 \in \mathscr{L}$[DSP2-HFA]

(2) $T_3 T_4 = (T_4 T_3)^R \notin \mathscr{L}$[USNSPk-HFA($s$)].

Proof. (1): Obvious.

(2): Let $T_5 = T_3 T_4$. Suppose that there exists a USNSPk-HFA(s) M which accepts T_5. Let u be the number of states (of the finite control) of M, and let R be the reading head of M. For each $n \geqq 1$, let

$$V(n) = \{0^n w 0^{r_1} a^{r_2} \mid (w \in \{0, 1\}^*) \;\&\; (|w| = n) \;\&\; (r_1, r_2 \geqq 1) \;\&\; (r_1 + r_2 = 2n)\}.$$

For each $x = 0^n w 0^{r_1} a^{r_2}$ in $V(n)$, let $SC(z)$ be the multi-set of semi-configurations[4] of M as follows:

$SC(z) = \{(q, i_1, i_2, \ldots, i_{k-1}) \mid c = (z, 2n+1, (q, i_1, i_2, \ldots,$

[4]Semi-configuration of M is a k-tuple $(q, i_1, i_2, \ldots, i_{k-1})$ where q is a state of finite control of M and i_j ($1 \leqq j \leqq k-1$) is the position of the j-th counting head.

i_{k-1})) is a configuration of M just after the point where R reads the initial segment $0^n w$ of $\not{c}\ z\$$}.

Then, the following proposition must hold:

Proposition 4.1 For any two words z and z' in $V(n)$ whose initial segments $0^n w's$ (of length $2n$) are different, $SC(z) \neq SC(z')$.

(For the apposite case, suppose that $z = 0^n w 0^{r_1} a^{r_2}$, $z' = 0^n w' 0^{r_1} a^{r_2}$ $(w \neq w')$, and $SC(z) = SC(z')$. Let $w = w_1 1 w_2$, $w' = w_1 0 w_2$ $(|w_1| = |w_2| = t$: $0 \leqq t \leqq n-1)$. We then consider the two words $z = 0^n w_1 1 w_2 0^p a^r$ and $z_1 = 0^n w_1 0 w_2 0^p a^r$ $(p = n + t - |w_2|$, $r = 2(n-t)-1)$ in $V(n)$. Clearly, $z \in T_5$, and so z is accepted by M. It follows that z_1 must be also accepted by M. This contradicts the fact that z_1 is not in T_5.)

Clearly $t(n) < u(4n+2)^{k-1}$, where $t(n)$ is the number of possible semi-configurations of M just after the point where R reads the initial segments $0^n w's$ (of length $2n$) of words in $V(n)$. For each z in $V(n)$, the leaf-size of the computation tree of M on z is at most $s(\geqq 1)$. Thus, for each z in $V(n)$, $|SC(z)| \leqq s$. Therefore, letting $S(n) = \{SC(z) | z \in V(n)\}$, it follows that for some constants c and c'

$$|S(n)| < ct(n)^s < c' n^{(k-1)s}.^5$$

Cleardy, $|V(n)| = 2^n$. From these facts, it follows that for large n, $|S(n)| < |V(n)|$. Therefore for large n, there must be two words z and z' in $V(n)$ whose initial segments $0^n w's$ are different such that $SC(z) = SC(z')$. This contradicts Proposition 4.1.

Q.E.D.

Theorem 4.5 *For each* $k \geqq 2$, $Y \in \{SP, SNSP\}$, $\mathscr{L}$*[UYk-HFA(s)] is not closed under the following operations:*

(1) Concatenation with a regular set
(2) Reversal
(3) Kleene closure
(4) ε-free homomorphism.

Proof. (1), (2): Obvious from Lemma 4.1.

(3): Let $T_6 = T_3 \cup T_4$. It is easy to see that $T_6 \in \mathscr{L}$[DSP2-HFA]. On the other hand, $T_6 \cap (\{0, 1\}^+ \{a\}^*) = T_3 T_4 \notin \bigcup_{1 \leq k < \infty} \bigcup_{1 \leq s < \infty} \mathscr{L}$[USNSPk-HFA(s)] (from Lemma 4.1). It follows from this fact and the fact that [USNSPk-HFA(s)] is closed under union with a regular set (which is easy to prove) that $T_6 \notin \bigcup_{1 \leq k < \infty} \bigcup_{1 \leq s < \infty} \mathscr{L}$[USNSPk-HFA(s)]. This completes the proof of (3).

(4): Let $T_3' = \{x \in \{0, 1\}^+ | (|x| \geqq 3)$ & $(|x|$ is odd) & (x has exactly one '2' as the center symbol of x}. Then it is readily proved that $T_3' T_5 \in \mathscr{L}$[DSP2-HFA]. On the other hand, let h be the ε-free homomorphism defined by $h(0)=0$, $h(1)=1$, $h(2)=1$, and $h(a)=a$. Then $h(T_3' T_5) = T_3 T_5 \notin \bigcup_{1 \leq k < \infty} \bigcup_{1 \leq s < \infty} \mathscr{L}$[USNSPk-HFA(s)] $(s \geqq 1)$. This completes the proof of (4).

Q.E.D.

Theorem 4.6 *For each* $k \geqq 2$ *and* $s \geqq 1$, $\mathscr{L}$*[ASPk-HFA(s)] is not closed under concatenation.*

Proof. *For each* $l \geqq 2$, let $L_l = \{a^n b^n | n \geqq 1\}^l$. The set L_{k-1} is accepted by NSPk-HFA M', which acts as follows:

Let R be the reading head of M' and $C_1, C_2, \ldots, C_{k-1}$ be the counting heads of M'. Suppose that an input $a^{n_1} b^{m_1} a^{n_2} b^{m_2} \cdots a^{n_{k-1}} b^{m_{k-1}} \$$ is presented to M'.

For each $i (1 \leqq i \leqq k-1)$, M moves C_i on the left end of subword a^{n_i} nondeterministically. Then, for each $i (1 \leqq i \leqq k-1)$, M moves C_i two cells to the right for every one right move of R on the subword b^{m_i}. $C_1, C_2, \ldots, C_{i-1}$ are moved one cell to the right simultaneously with R, and $C_{i+1}, C_{i+2}, \ldots, C_{k-1}$ remain stationary at the left ends of subwords $a^{n_{i+1}}, a^{n_{i+2}}, \ldots, a^{n_{k-1}}$, respectively. Moving each head in this way, M accepts the input if and only if all heads reach the right endmarker \$ at the same time. it is obvious that $T(M) = L_{k-1}$.

If it can be shown that $L_{((k-1)s+2)((k-1)s+1)s} \notin$ [ASPk-HFA(s)], then we have completed the proof of the theorem. The proof is an extension of Theorem 1 in [7].

For the apposite case let us suppose that there exists an ASPk-HFA(s) M $(k \geqq 2$, $s \geqq 1)$ accepting $L_{((k-1)s+2)((k-1)s+1)s}$, which has m states. (Without loss of generality we assume that the input tape of M has no left endmarker.) For each input word w in $L_{((k-1)s+2)((k-1)s+1)s}$, there exists an accepting computation tree of M denoted by $T_M(w)$. We divide input word w into s subwords. That is,

$$w = w_1 w_2 \cdots w_s.$$

Without loss of generality, we assume that each node of $T_M(w)$ that is labeled by a configuration[6] with a universal state has exactly two children. Then, because of the bounded leaf-size s, there are at most $s-1$ nodes labeled by configurations with a universal state in $T_M(w)$. From this fact and the fact that word w has s subwords $w_i's$, there is a subword w_i in the word w such that on each computation path of $T_M(w)$, there is a sequence of steps which implies that M never enters a universal state while reading the subword w_i. We let such subword w_i be w_f, let $e(1 \leqq e \leqq s)$ be the number of sequences of steps during M reads the subword w_f, and let $S(1), S(2), \ldots, S(e)$ be these e sequences of steps.

Let the subword $w_i (1 \leqq i \leqq s)$ be as follows:

$$w_i = y_1 y_2 \cdots y_{(k-1)s+1} \ (1 \leqq i \leqq s)$$
$$y_i = y = x_1 x_2 \cdots x_{(k-1)e+2} \ (1 \leqq i \leqq (k-1)s+1)$$
$$x_i = x = a^n b^n \ (1 \leqq i \leqq (k-1)e+2).$$

For each $i (1 \leqq i \leqq e)$ and each $j (1 \leqq j \leqq (k-1)s+1)$, let $N_i(j)$ be the number of counting heads that reach the right endmarker \$ while the reading head R reads the y_j in w_f, in the i-th sequence $S(i)$ of $T_M(w)$. Since M has only $(k-1)$ counting heads and leaf-size s, it follows that $N_1(j_0) = N_2(j_0) = \cdots = N_e(j_0) = 0$ for some

[5]For any set S, $|S|$ denotes the number of elements of S.

[6]A configuration of M on w is a $(k+1)$-tuple of a state of finite control and k head positions.

$j_0 (1 \leqq j_0 \leqq (k-1)e+2)$.

Consider the case when in $T_M(w)$ R reads the subword y_{j_0} such that $N_1(j_0)=N_2(j_0)=\cdots=N_e(j_0)=0$. We select an arbitrary number i_0 $(1 \leqq i_0 \leqq (k-1)e+2)$ and let

$$x_{i_0}=a_{i_01}a_{i_02}\cdots a_{i_0n}b^n \ (a_{i_0j}=a, \ 1 \leqq j \leqq n).$$

For each $j(1 \leqq j \leqq e)$ and each symbol a_{i_0l}, let $q^j_{i_0l}$ be the state of M when R moves onto a_{i_0l} on the j-th sequence $S(j)$. For each symbol a_{i_0l}, we consider the e-tuple of states as follows:

$$(q^1_{i_0l}, q^2_{i_0l}, \ldots, q^e_{i_0l})=Q_{i_0l}.$$

We call Q_{i_0l} above a *multi-state* of M.

A *j-configuration* of M is a $(k+1)$-tuple $(q^j_i, b^i_{j1}, \ldots, b^i_{jk})$ (denoted by c_i), where q^j_i is the state of finite control of M and b^i_{jl} is the position of the l-th head in $S(j)$. A *j-increment* is a $(k+1)$-tuple $(q^j_i, h^i_{j1}, \ldots, h^i_{jk})$, where q^j_i is the state of finite control of M and each h^i_{jk} is either 0 or 1. (Informally, the j-increment describes moving the heads at one step of computation in the j-th sequence $S(j)$.) Let $c^j_f, c^j_{f+1}, \ldots, c^j_g$ be the subsequence of $S(j)$, where c^j_f is a j-configuration when R reads the symbol a_{i_01} and c^j_g is a j-configuration when R reads the symbol a_{i_0n} of $x_{i_0}=a_{i_01}a_{i_02}\cdots a_{i_0n}b^n$. We say that the sequence of j-increments $d^j_f, d^j_{f+1}, \ldots, d^j_{g-1}$, where

$$d^j_i=(q^j_i, b^i_{j1}-b^i_{j1}, \ldots, b^i_{jk}-b^i_{jk})$$
$$\text{if} \quad c^j_i=(q^j_i, b^i_{j1}, \ldots, b^i_{jk}) \quad \text{and}$$
$$c^j_{i+1}=(q^j_{i+1}, b^{i+1}_{j1} \ldots, b^{i+1}_{jk})$$

for each i $(1 \leqq i \leqq g-1)$, is the sequence of j-increments of $c^j_f, c^j_{f+1}, \ldots, c^j_g$. Figure 3 summarizes the relationship between j-configurations and j-increments in each path $S(j)$.

We let

$$\begin{array}{l} d^1_{f_1}, d^1_{f_1+1}, \ldots, d^1_{g_1-1} \\ d^2_{f_2}, d^2_{f_2+1}, \ldots, d^2_{g_2-1} \\ \cdot \\ \cdot \\ \cdot \\ d^e_{f_e}, d^e_{f_e+1}, \ldots, d^e_{g_e-1} \end{array} \qquad (1)$$

where $d^j_{f_j}, d^j_{f_j+1}, \ldots, d^j_{g_j-1}$ is a subsequence of j-increments in $S(j)$ and $d^j_{f_j}$ $(d^j_{g_j-1})$ is the j-increment when M reads the symbol a_{i_01} (a_{i_0n}). Let

$$\begin{array}{l} d^1_{\alpha_1}, d^1_{\alpha_1+1}, \ldots, d^1_{\beta_1} \\ d^2_{\alpha_2}, d^2_{\alpha_2+1}, \ldots, d^2_{\beta_2} \\ \cdot \\ \cdot \\ \cdot \\ d^e_{\alpha_e}, d^e_{\alpha_e+1}, \ldots, d^e_{\beta_e} \end{array} \qquad \begin{array}{l} (f_i \leqq \alpha_i < \beta_i \leqq g_{i-1} \\ \quad : \forall i (1 \leqq i \leqq e)) \end{array}$$

be a subsequence of (1), and be denoted by *segment*. If the length of $d^3_{\alpha_3}, \ldots, d^3_{\beta_3}$ is the shortest of any $d^i_{\alpha_i}, \ldots, d^i_{\beta_i}$ then let the length of $d^3_{\alpha_3}, \ldots, d^3_{\beta_3}$ be the *length of the segment*.

For each symbol $a_{i_0l_1}$ $(a_{i_0l_2})$ $(l_1 < l_2)$, let $d^j_{l_{1j}} (d^j_{l_{2j}})$ be the j-increment when M reads symbol $a_{i_0l_1}$ $(a_{i_0l_2})$, and let $Q_{i_0l_1}=Q_{i_0l_2}$. Then we say that the segment

$$\begin{array}{l} d^1_{l_{11}}, d^1_{l_{11}+1}, \ldots, d^1_{l_{21}} \\ d^2_{l_{12}}, d^2_{l_{12}+1}, \ldots, d^2_{l_{22}} \\ \cdot \\ \cdot \\ \cdot \\ d^e_{l_{1e}}, d^e_{l_{1e}+1}, \ldots, d^e_{l_{2e}} \end{array} \qquad \begin{array}{l} (f \leqq l_{1j} < l_{2j} \leqq g-1 \\ \quad : \forall i (1 \leqq i \leqq e)) \end{array}$$

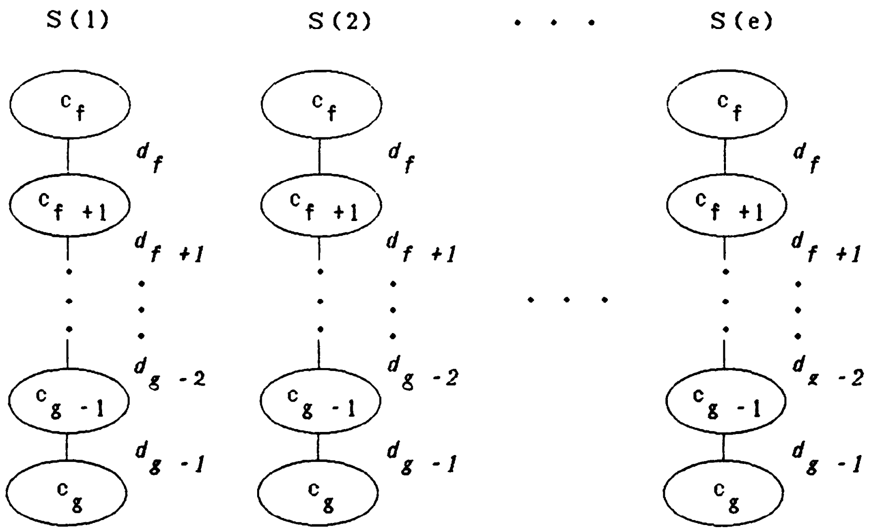

Fig. 3 J-configurations and j-increments.

is a *Q-cycle*. Furthermore, we say the following $(k-1)e+1$-tuple

$$\left(\sum_{l=l_{11}}^{l_{21}} h_{11}^l, \sum_{l=l_{11}}^{l_{21}} h_{12}^l, \ldots, \sum_{l=l_{11}}^{l_{21}} h_{1k}^l, \sum_{l=l_{12}}^{l_{22}} h_{22}^l, \sum_{l=l_{12}}^{l_{22}} h_{23}^l, \ldots, \sum_{l=l_{12}}^{l_{22}} h_{2k}^l, \ldots, \sum_{l=l_{1e}}^{l_{2e}} h_{e2}^l, \sum_{l=l_{1e}}^{l_{2e}} h_{e3}^l, \ldots, \sum_{l=l_{1e}}^{l_{2e}} h_{ek}^l\right)$$

is a *parameter of this Q*-cycle.

Fact 4.1 If (1) can be written in the form s_1, p_1, s_2, p_2, s_3, where s_1, s_2, s_3 are the segments and p_1, p_2 are the Q-cycle (for some multi-state Q), then there is an accepting computation tree of M that is constructed by replacing s_1, p_1, s_2, p_2, s_3 of $T_M(w)$ by s_1, p_1, p_2, s_2, s_3 (see Fig. 4).

Since every segment with a length of at least m^e+1 contains a Q-cycle, from Fact 4.1, we have the following:

Fact 4.2 There is a permutation of (1) which can be written in the form

$$s_1, p_1, s_2, p_2, \ldots, s_r, p_r, s_{r+1}, \tag{2}$$

where $r \leqq m^e$, each s_i is a segment with length at most m^e, each p_i can be written in the form $p_i = p_i^1, p_i^2, \ldots, p_i^{l_i}$, where each p_i^j is a Q-cycle with length at most m^e and there is an accepting computation tree of M on w that is constructed by replacing (1) of $T_M(w)$ by $s_1, p_1, s_2, p_2, \ldots, s_r, p_r, s_{r+1}$.

Fact 4.3 Let p_i's be the Q-cycles from Fact 4.2. For (2), there is a parameter $v=(v_1, v_2, \ldots, v_{(k-1)e+1})$ with $v_1>0$ and $0 \leqq v_i \leqq m^e$ for each $i(1 \leqq i \leqq (k-1)e+1)$, such that the number of Q-cycles p_i^j with parameter v is at least $(n-(m^e+1)m^e)/(m^e(m^e+1)^{(k-1)e+1})$.

Proof. Since the reading head crosses the i_0-th subword a^n of word y_{j_0} during the part of the computation corresponding to (2), there are n increments (in (2)) in which the reading head is moved to the right. Clearly, at least n-$(m^e+1)m^e$ increments from these n increments are contained in the cycles p_i^j, because $r \leqq m^e$ and the length of each s_i is at most m^e (see Fact 4.2). This implies that the number of Q-cycles p_i^j with parameters whose first component is greater than zero is at least $(n-(m^e+1)m^e)/m^e$. Since the number of all different parameters, for the cycles with length at most m^e, is at most $(m^e+1)^{(k-1)e+1}$, there is a parameter v such that the number of cycles p_i^j with parameter v is at least $(n-(m^e+1)m^e)/(m^e(m^e+1)^{(k-1)e+1})$. ∎

Since the number i_0 $(1 \leqq i_0 \leqq (k-1)e+2)$ was selected arbitrarily, from Fact 4.3, we find that there is an accepting computation tree of M on w with the sequence

$$u_1, z_1, u_2, z_2, \ldots, u_{(k-1)e+2}, z_{(k-1)e+2}, u_{(k-1)e+3} \tag{3}$$

where, for each $i(1 \leqq i \leqq (k-1)e+2)$, z_i is the segment corresponding to the part of this accepting computation tree at which the reading head reads the i-th subword a^n of word y_j, z_i is of the form (2), and each u_i is a segment. Further, from Fact 4.3, there are parameters $v^i=(v_1^i, v_2^i, \ldots, v_{(k-1)e+1}^i)$ for each $i(1 \leqq i \leqq (k-1)e+2)$, with $v_1>0$ and $0 \leqq v_j^i \leqq m^e$ for each $i(1 \leqq i \leqq (k-1)e+2)$ and each $j(1 \leqq j \leqq (k-1)e+1)$, such that the number of cycles with parameter v_i is at least $(n-(m^e+1)m^e)/(m^e(m^e+1)^{(k-1)e+1})$ in segment z_i for each $i(1 \leqq i \leqq (k-1)e+2)$. Clearly there are rational numbers $r_1, r_2, \ldots, r_{(k-1)e+2}$ such that

$$\sum_{i}^{(k-1)e+2} r_i v^i = 0, \tag{4}$$

where

$0=(\underbrace{0, 0, \ldots, 0}_{k})$ and $r_i \neq 0$ for some $i(1 \leqq i \leqq (k-1)e+2)$,

because the vectors $v^i \neq 0$ are linearly independent. Without loss of generality we can assume that the r_i's in (4) are integers.

Let w be the word as above. Now we consider the word

$$w' = y_1 \cdots y_{y_{i_0}-1} y_{j_0} y_{j_0} y_{j_0+1} \cdots y_{(k-1)s+1},$$

where

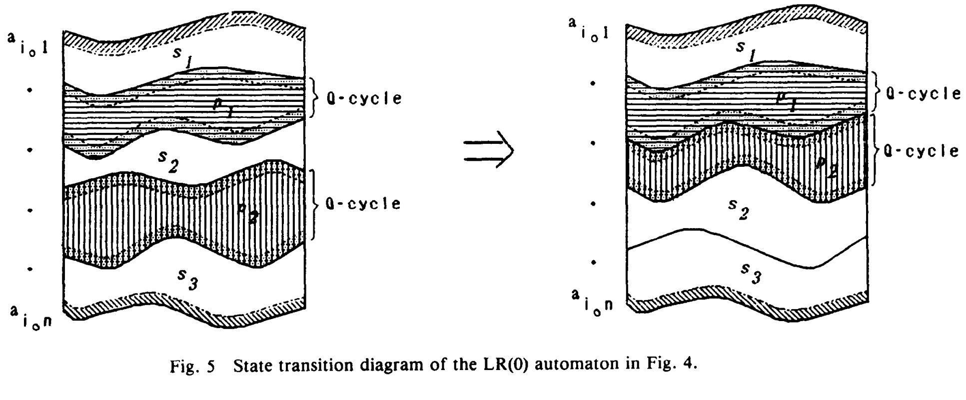

Fig. 5 State transition diagram of the LR(0) automaton in Fig. 4.

$w = y_1 \cdots y_{(k-1)s+1}$,

$y_{j_0} = a^n b^n a^n b^n \cdots a^{n_{(k-1)e+2}} b^n$ and $n_i = n + r_i v_1^i$

for each $i (1 \leqq i \leqq (k-1)e+2)$. (Note that all $n_i > 0$ when n is large enough.) Since $r_i \neq 0$ for some i and $v_1^i > 0$ for all i (see above), we find that $n_i \neq n$ for some i, and therefore $w' \notin L_{((k-1)e+1)((k-1)s+1)s}$. On the other hand, from (3), (4) and

$$|y'_{j_0}| = 2((k-1)e+2)n + \sum_{i=1}^{(k-1)e+2} r_i v_1^i$$
$$= 2((k-1)e+2)n + 0 = |y_{j_0}|,$$

we know that there is an accepting computation tree of M on w' with the sequence $u_1, z'_1, u_2, z'_2, \ldots, u_{(k-1)e+2}, z'_{(k-1)e+2}, u_{(k-1)e+3}$ where segment z_i is obtained by inserting (if $r_i > 0$) or by deleting (if $r_i < 0$) r_i cycles with parameter v^i from segment z_i. Therefore, w' is accepted by M. This is a contradiction.

Q.E.D.

Theorem 4.7 *For $k \geqq 2$ and $s \geqq 1$, $\mathscr{L}$[ASPk-HFA(s)] and $\bigcup_{1 \leq r < \infty} \bigcup_{1 \leq t < \infty} \mathscr{L}$[ASPr-HFA(t)] is not closed under Kleene closure.*

Proof. Let $T_7 = \{a^n b^n | n \geqq 1\}$. It is easily seen that $T_7 \in \mathscr{L}$[DSP2-HFA]. On the other hand, it is shown in Lemma 4.4 in Matsuno *et al.* [1] that $T_7^* \notin \bigcup_{1 \leq k < \infty} \bigcup_{1 \leq s < \infty} \mathscr{L}$[ASPk-HFA(s)]. This completes the proof of the theorem.

Q.E.D.

5. Conclusions

The closure properties of AMHFACLs and ASPMHFACLs are summarized in Tables 1 and 2, respectively. It is easy to show that ordinary alternating automata (that is, state sets containing existential states) are closed under union. From these tables, we notice that A(SP)MHFACLs with only universal states are not closed for almost any operator dealt with in this paper.

Most of the results in Table 2 hold for semi-one-way alternating simple multihead finite automata with constant leaf-sizes. That is, 'no with superscript 2' indicates that the result is only valid for one-way.

In a sense, these tables are generalizations of nondeterministic and deterministic cases. That is, if we fix parameter $s=1$ in these tables (which implies that leaf-size is '1'), the symbols U and A in these tables indicate the deterministic and nondeterministic cases, respectively.

In this paper, we have investigated the closure properties of AMHFACLs and ASPMHFACLs, in some instances improving previously known results. An open problem is how to get the results at symbol '?' in these tables.

Table 1 Multihead Finite Automata.

	U	USN	A	ASN
comple.	?[1]	?[1]	no	no
union	no	no	yes	yes
inter.	no	no	no	no
concate.	no	no	?	?
Kleene	no	no	?	?
revers.	no	no	?	?
ε-free	no	no	?	?

U: $\mathscr{L}$[Uk-HFA(s)], USN: $\mathscr{L}$[USNk-HFA(s)], A: $\mathscr{L}$[Ak-HFA(s)], ASN: $\mathscr{L}$[ASNk-HFA(s)], 1: $\mathscr{L}$[Uk-HFA] and $\mathscr{L}$[USNk-HFA] are not closed under complementation.

Table 2 Simple Multihead Finite Automata.

	USP	USNSP	ASP	ASNSP
comple.	no	no	no	no
union	no	no	yes	yes
inter.	no	no	no	no
concate.	no	no	no[2]	?
Kleene	no	no	no[2]	?
revers.	no	no	?	?
ε-free	no	no	?	?

USP: $\mathscr{L}$[USPk-HFA(s)], ASP: $\mathscr{L}$[ASPk-HFA(s)], USNSP: $\mathscr{L}$[USNSPk-HFA(s)], ASNSP: $\mathscr{L}$[ASPSNk-HFA(s)], 2: Only valid for one-way (not hold for semi-one-way).

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