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LETTER

A Note on Reversal Complexities of Real-Time Counter Machines

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SUMMARY This paper gives a hierarchical property on the number of reversals of real-time counter machines. That is, we show that for any $k \geq 1$, a real-time counter machine with $2k+1$ reversals is more powerful than one with k reversals.

1. Introduction

In Ref. (1), Chan investigated some properties of counter machines with non-constant reversal-bounded counters, and showed that n^{r_1} reversal-bounded one-way deterministic counter machines are more powerful than n^{r_2} reversal bounded ones, where $r_1 > r_2 > 0$.

This short paper investigates a hierarchical properties of real-time one counter machines with constant reversal and shows that for each $k \geq 1$, a real-time counter machine with $2k+1$ reversals is more powerful than one with k reversals.

2. Preliminaries

A counter machine is a pushdown machine whose pushdown store operates as counter, i. e. has a single-letter alphabet. In this paper, we consider a real-time counter machine with constant revernal-bounded counter. A machine is real-time if it reads a new input symbol in every step, and the machine stops immediately after reading the endmarker.

We use the following notations. For each $k \geq 1$, NRTRBCM(k) (DRTRBCM(k)) denotes a nondeterministic (deterministic) real-time k reversal bounded counter machine. For each $X \in \{D, N\}$, we let $\mathcal{L}$ [XRTRBCM(k)] $= \{T | T$ is accepted by some XRTRBCM(k)$\}$.

3. Result

In Ref. (2), Ďuriš and Galil introduce crossing sequences on the working tape of a real-time Turing machine. Note that any crossing sequence at a given boundary on the working tape defines a partition of the input string x into segments $x = x_1 \cdots x_s$. Each time this boundary is crossed, a new segment is determined.

The following lemma is easily proved by using a, modification of the proof of Lemma 1 in Ref. (2).

[Lemma 1] Assume there are two accepting computations by a real-time counter machine M on inputs $x = x_1 x_2 \cdots x_k$ and $y = y_1 y_2 \cdots y_k$ with two identical crossing sequences, and that the k segments of x and y are defined by each crossing sequence. (Note that, the boundaries which determine crossing sequences on inputs x and y are not always located at the same place on the working tape.) Then M also accepts $x_1 y_2 x_3 y_4 \cdots$ (and $y_1 x_2 y_3 x_4 \cdots$).

[Theorem 1] For each $X \in \{D, X\}$ and each $k \geq 1$,

$$\mathcal{L}\,[\mathrm{XRTRBCM}(k)] \subseteq \mathcal{L}\,[\mathrm{XRTRBCM}(2k+1)] .$$

(Proof) For each $r \geq 1$, let

$$S(r) = \{0^{n_1} 1 0^{n_1} 2 0^{n_2} 1 0^{n_2} 2 \cdots 2 0^{n_r} 1 0^{n_r} | \; \forall i (1 \leq i \leq r)[n_i \geq 1]\}.$$

We can easily see that $S(k+1) \in \mathcal{L}$[DRTRBCM($2k+1$)]. We then show that $S(k+1) \notin \mathcal{L}$[NRTRBCM($k$)]. We assure, to the contrary, that $S(k+1)$ is accepted by an NRTRBCM(k) M with a state set Q. Choose a sufficiently large n so that

$$(1) \qquad \frac{n - |Q|}{k+1} \geq (|Q|+1) \cdot (k+1) \cdot |Q|^{k+1} + 1,^{\dagger}$$

and let

$$\hat{S}(n) = \{0^{n_1} 1 0^{n_2} 1 0^{n_2} 2 \cdots b o^{n_{k+1}} 1 0^{n_{k+1}} | \; \forall i (1 \leq i \leq k+1)[|Q|+1 \leq n_i \leq n]\}.$$

(Fact 1) There is a subset S of $\hat{S}(n)$ and $1 \leq i_0 \leq k+1$ such that:

(a) $|S| \geq (|Q|+1) \cdot (k+1) \cdot |Q|^{k+1} + 1$;

(b) for all z, z' in S,

$$z = 0^{n_1} 1 0^{n_1} 2 0^{n_2} 1 0^{n_2} 2 \cdots 2 0^{n_{k+1}} 1 0^{n_{k+1}}$$

and

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†For any set T, $|T|$ denotes the number of elements of T.

$z' = 0^{n'_1}10^{n'_1}20^{n'_2}10^{n'_2}2\cdots20^{n'_{k+1}}10^{n'_{k+1}}$,

$n_i = n'_i$ for $1 \geq i \geq k'+1$ and $i \neq i_0$.

(c) for all strings in S, there is no head reversal when M reads $0^{n_{i_0}}10^{n_{i_0}}$ in the corresponding accepting computations.

(Proof) There are $(n-|Q|)^{k+1}$ strings in $\hat{S}(n)$. Fer each of them, there is $1 \leq i \leq k+1$ such that there is no head reversal when M reads $0^{n_{i_0}}10^{n_{i_0}}$. Therefore, there are $1 \leq i_0 \leq k+1$ and a subset S_1 of $\hat{S}(n)$ such that $|S_1| \geq (n-|Q|)^{k+1}/(k+1)$ and there is no head reversal when M reads $0^{n_{i_0}}10^{n_{i_0}}$ for all strings in S_1. There are $(n-|Q|)^k$ possible k tuples

$$(n_1, n_2, \cdots, n_{i_0-1}, n_{i_0+1}, \cdots, n_{k+1})$$

with $|Q|+1 \leq n_i \leq n$. Hence there is a subset S such that

$$|S| \geq \frac{(n-|Q|)^{k+1}}{k+1} \cdot \frac{1}{(n-|Q|)^k} = \frac{n-|Q|}{k+1}$$

that satisfies (b) and (c). It also satisfies (a) because of Eq. (1).

We let $x = 0^{n_1}10^{n_1}2\cdots20^{n_{i_0-1}}10^{n_{i_0-1}}2$ ($x=\varepsilon$ if $i_0=1$) and $y = 20^{n_{i_0+1}}10^{n_{i_0+1}}2\cdots20^{n_{k+1}}10^{n_{k+1}}$ ($y=\varepsilon$ if $i_0=k+1$). Hence, each string in S is of the form $x0^{n_{i_0}}10^{n_{i_0}}y$ (with the same x and y).

(Fact 2) For all strings in S, during the $|Q|+1$ steps after reading $0^{n_{i_0}}1$, the counter head of M must move (left or right) at least once. (Note that $|Q|+1$ steps include the step that follows reading the 1.)

(Proof) Otherwise, there is $x0^{n_{i_0}}10^{n_{i_0}}y$ in S such that the counter head of M does not move $|Q|+1$ steps after it reads $0^{n_{i_0}}1$. Using a pumping technique $0^{n_{i_0}} = 0^{n_{i_01}}0^{n_{i_02}}, 0^{n_{i_03}}$, $n_{i_02} \neq 0$ ($n_{i_0} \geq |Q|+1$ by the definition of $\hat{S}(n)$) and M also accepts $x0^{n_{i_0}}10^{n_{i_01}}0^{m\cdot n_{i_02}}0^{n_{i_03}}y$ ($n_{i_02} \neq 0$) for all $m \geq 1$. This is a contradiction. ■

Hence, every string in S can be written in the form $x0^{m_1}0^{m_2}10^{m_1}0^{m_2}y$, where $|Q|+1 \leq m_1+m_2 \leq n$, $m_1 \leq |Q|$, and after reading the 1 the counter head of M moves for the first time immediately after the input head completes reading 10^{m_1}. (Possibly $m_1=0$.) This head movement defines a boundary on the counter tape of M, a crossing sequence at that boundary. While M reads an input string in $S(k+1)$, it crosses the boundary defined above at most $k+1$ times. On the other hand, by (c) of Fact 1, M crosses this boundary exactly once while reading $0^{n_{i_0}}10^{n_{i_0}}$. We let this crossing be the p-th crossing ($1 \leq p \leq k+1$) of this boundary. The number of dffifferent crossing sequences of M is at most $|Q|^{k+1}$, and the number of possible $0^{m_1}s$ is $|Q|+1$. Hence by (a) of Fact 1, there are two strings in S:

$$w = x0^{m_1}0^{m_2}10^{m_1}0^{m_2}y, \text{ and}$$

$$w' = x0^{m'_1}0^{m'_2}10^{m'_1}0^{m'_2}y$$

with $m_1 = m'_1$, $m_2 \neq m'_2$, with the same crossing sequences at the corresponding boundaries, and the same p. By (c) of Fact 1, M crosses the corresponding boundary exactly once while reading $0^{n_{i_0}}10^{n_{i_0}}$ for both w and w'. By Lemma 1, M also accepts two mixed versions of w and w'. One is of the form $\tilde{x}0^{m_1}0^{m_2}10^{m_1}0^{m'_2}\tilde{y}$ and one is of the form $\tilde{\tilde{x}}0^{m'_1}0^{m'_2}10^{m'_1}0^{m_2}\tilde{\tilde{y}}$. Both strings are not in $S(k+1)$. This is a contradiction. (Q. E. D.)

4. Conclusions

In this short paper, we show that nearly twice the number of reversals bring out the increase of accepting powers of real-time one counter machines. It is unknown whether or not $k+1$ reversals are more powerful than k reversals for $k \geq 1$.

References

(1) T. Chan: "Reversal complexity of real-time counter machine", STOC, Milwaukee, 1981, pp. 146-157 (1981).

(2) P. Duris and Z. Galil: "Two tapes are better than one for nondeterministic machines", SIAM J. Comput., 13, 2 pp. 219-227 (May 1984).