PAPER

Alternating One-Way Multihead Turing Machines with Only Universal States

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SUMMARY This paper introduces a space bounded alternating one-way multihead Turing machine with only universal states, and investigates fundamental properties of this machine. We show for example that for any function L such that $\lim_{n\to\infty}[L(n)/n]=0$, (1) there is a set in $\mathscr{L}$[U2-HTM(0)], but not in $\bigcup_{1\leq k<\infty}\mathscr{L}$[N$k$-HTM$(L(n))$], and there is a set in $\mathscr{L}$[N2-HTM(0)], but not in $\bigcup_{1\leq k<\infty}\mathscr{L}$[U$k$-HTM$(L(n))$], (2) for each $k\geq 1$, $\mathscr{L}$[Uk-HTM$(L(n))$] $\subsetneq$ $\mathscr{L}$[U$(k+1)$-HTM$(L(n))$], and (3) $\mathscr{L}$[Uk-HTM$(L(n))$] $\cap$ $\mathscr{L}$[Nk-HTM$(L(n))$] $\neq$ $\mathscr{L}$[Dk-HTM $(L(n))$], where $\mathscr{L}$[Uk-HTM$(L(n))$] denotes the class of sets accepted by $L(n)$ space bounded alternating one-way k-head Turing machines with only universal states, and $\mathscr{L}$[Nk-HTM$(L(n))$] ($\mathscr{L}$[Dk-HTM $(L(n))$]) denotes the class of sets accepted by $L(n)$ space bounded nondeterministic (deterministic) one-way k-head Turing machines.

1. Introduction

During the past ten years, many investigations about one-way multihead finite automata (MHFA's) have been made[(1)-(7)]. On the other hand, as a generalization of nondeterministic machines and as a mechanism to model parallel computations, several types of alternating machines were introduced, and many results about them were established[(8)-(11)]. Further, alternating machines with only universal states were also introduced, and several results about them were reported[(12),(13)]. Alternating machines with only universal states are interesting parallel computation models because they can be considered as more realistic parallel computation models than ordinary alternating Turing machines, and might be useful in investigating properties of the complements of languages accepted by nondeterministic machines as is suggested by Theorem 1 in this paper.

In this paper, we introduce space bounded alternating one-way multihead Turing machines with only universal states, and investigate some properties of them. Section 2 gives terminologies and notations necessary for this paper.

In Section 3, we first show that for any function $L(n)$, the class of sets accepted by UMHTM$(L(n))$'s (where UMHTM$(L(n))$ denotes an $L(n)$ space bounded alternating one-way multihead Turing machine with only universal states) is equal to the class of complements of sets accepted by NMHTM$(L(n))$'s (where NMHTM$(L(n))$ denotes an $L(n)$ space bounded nondeterministic one-way multihead Turing machine).

It is shown Ref. (3) that there is a set accepted by a nondeterministic one-way 2 head finite automaton (N2-HFA), but not accepted by any deterministic one-way multihead finite automaton. In contrast to this fact, Section 3 shows that for any $L(n)$ such that $\lim_{n\to\infty}[L(n)/n]=0$, (1) there is a set accepted by an N2-HFA, but not accepted by any UMHTM$(L(n))$, and (2) there is a set accepted by an alternating one-way 2 head finite automaton with only universal states, but not accepted by any NMHTM$(L(n))$.

It is well-known Ref. (3) that for MHFA's, k+1 heads are more powerful than k heads ($k\geq 1$). On the other hand, it is unknown Ref. (10) whether a similar fact holds for alternating one-way multihead finite automata (AMHFA's). Recently[(13)], it was shown that k+1 heads are better than k ($k\geq 1$) for AMHFA's with only universal states. Section 4 strengthens this result, and shows that k+1 heads are better than k ($k\geq 1$) for UMHTM $(L(n))$'s such that $\lim_{n\to\infty}[L(n)/n]=0$. Section 4 also investigates basic closure properties of languages accepted by UMHTM$(L(n))$'s with $\lim_{n\to\infty}[L(n)/n]=0$.

Whether or not NP$\cap$co-NP=P is a very important problem in the computational complexity theory[(15)]. We cannot solve this problem, but in Section 5, we show that $\mathscr{L}$[Nk-HTM$(L(n))$] $\cap$ co-$\mathscr{L}$[Nk-HTM$(L(n))$] is not equal to $\mathscr{L}$[Dk-HTM$(L(n))$] for any $k\geq 2$ and any $L(n)$ such that $\lim_{n\to\infty}[L(n)/n]=0$, where $\mathscr{L}$[Nk-HTM$(L(n))$] ($\mathscr{L}$[Dk-HTM $(L(n))$]) denotes the class of sets accepted by $L(n)$ space bounded nondeterministic (deterministic) one-way k head Turing machines.

2. Preliminaries

We first give full definitions of alternating one-way multihead Turing machines.

[Definition 1] An alternating one-way k-head Turing machine (Ak-HTM) ($k\geq 1$) is a 9-tuple $M=(k, Q, U, \Sigma, \Gamma, \delta, q_0, F, R)$, where

$k\geq 1$ is the number of input heads,

Q is a finite set of states,

$U\subseteq Q$ is the set of universal states,

Σ is a finite input alphabet ($\$\notin\Sigma$ is the right endmarker),

Γ is a finite storage tape alphabet ($B\in\Gamma$ is the blank

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symbol),

δ is the transition function mapping $(Q \times (\Sigma \cup \{\$\})^k \times \Gamma)$ into $2^{Q \times (\Gamma - \{B\}) \times \{\text{stationary, right}\}^k \times \{\text{left, stationary, right}\}}$,

$q_0 \in Q$ is the initial state,

$F \subseteq Q$ is the set of accepting states, and

$R \subseteq Q$ is the set of rejecting states.

A state q in Q-U is said to be existential.

An Ak-HTM M has a read-only input tape with the right endmarker \$ and one semi-infinite storage tape, initially blank. (Of course, M has k input heads on the input tape, and one storage head on the storage tape.) A step of M consists of reading symbols under k input and one storage heads, writing a symbol on the storage tape, moving the input and storage heads in specified directions (note that input heads cannot move to the left), and entering a new state, in accordance with the transition function. If one of the heads of M falls off the input word, then M can make no further move. Furthermore, we assume that when M enters an accepting state or a rejecting state, M can make no further move.

[Definition 2] An instantaneous description (ID) of an Ak-HTM $M=(k, Q, U, \Sigma, \Gamma, \delta, q_0, F, R)$ $(k \geq 1)$ is an element of

$$\Sigma^* \times C_M$$

where $C_M = N^k \times Q \times (\Gamma - \{B\})^* \times N$, and N denotes the set of all positive integers. An element of C_M is called a configuration of M. The first component x of an ID $I=(x, c)$, where $c=((i_1, \ldots, i_k), q, \alpha, j)$† $\in C_M$, represents the input word (excluding the right endmarker \$). The first component $(i_1, \ldots, i_k)$ of the configuration c represents the positions of k input heads, and the second, third, and fourth components of c represent the state of the finite control, the nonblank contents of the storage tape, and the storage head position, respectively. If q is the state associated with an ID I, then I is said to be universal (existential, accepting, rejecting) if q is a universal (existential, accepting, rejecting) state. The initial ID of M on input x is $I_M(x)=(x, ((\underbrace{1, \ldots, 1}_{k}), q_0, \lambda, 1))$, where λ is the null word (i.e., $|\lambda|=0$).

[Definition 3] Given an Ak-HTM $M=(k, Q, U, \Sigma, \Gamma, \delta, q_0, F, R)$, we write $I \vdash I'$ and say I' is a successor of I if an ID I' follows from an ID I in one step, according to the transition function δ. The reflexive transitive closure of $\vdash$ is denoted by $\vdash^*$. A computation path of M on x is a sequence $I_0 \vdash I_1 \vdash \ldots \vdash I_n$ $(n \geq 0)$, where $I_0 = I_M(x)$. A computation tree of M is a finite, nonempty labeled tree with the properties

(1) each node π of the tree is labeled with an ID, $l(\pi)$,

(2) if π is an internal node (a non-leaf) of the tree, $l(\pi)$ is universal and $\{I \mid l(\pi) \vdash I\} = \{I_1, \cdots, I_r\}$, then π has exactly r children $\rho_1, \ldots, \rho_r$ such that $l(\rho_i)=I_i$,

(3) if π is an internal node of the tree and $l(\pi)$ is existential, then π has exactly one child ρ such that $l(\pi) \vdash l(\rho)$.

†We note that for each $1 \leq r \leq k$, $1 \leq i_r \leq |x|+1$, and $1 \leq j \leq |\alpha|+1$, where for any word w, $|w|$ denotes the length of w.

A computation tree of M on x is a computation tree of M whose root is labeled with $I_M(x)$. An accepting computation tree of M on x is a computation tree of M on x whose leaves are all labeled with accepting ID's. We say that M accepts x if there is an accepting computation tree of M on x. Define $T(M)=\{x \in \Sigma^* \mid M \text{ accepts } x\}$.

Nondeterministic and deterministic one-way k head Turing machines $(k \geq 1)$ are special cases of Ak-HTM's. That is, a nondeterministic one-way k head Turing machine (Nk-HTM) is an Ak-HTM which has no universal states, and a deterministic one-way k head Turing machine (Dk-HTM) is an Ak-HTM whose ID's have at most one successor.

In this paper, we are interested in an Ak-HTM $(k \geq 1)$ with only universal states, i.e., with no existential state. We denote such an Ak-HTM by "Uk-HTM".

With each Xk-HTM M, where $X \in \{A, U, N, D\}$, we associate a space complexity function SPACE which takes ID's to natural numbers. That is, for each ID $I=(x, ((i_1, \ldots, i_k), q, \alpha, j))$, let SPACE (I) be the length of α. Let $L: N \to R$ be a function, where R denotes the set of all non-negative real numbers. We say that M is $L(n)$ space bounded if for each n and for each input x of length n, each computation tree of M on x is such that for each node π of the tree, space $(l(\pi)) \leq \lceil L(n) \rceil$†. By X$k$-HTM$(L(n))$ we denote an $L(n)$ space bounded Xk-HTM. For each $X \in \{D, N, A, U\}$, define $\mathscr{L}$[Xk-HTM$(L(n))$]$=\{T \mid T=T(M)$ for some Xk-HTM $(L(n))$ $M\}$.

An alternating (nondeterministic, deterministic) one-way k-head finite automaton $(k \geq 1)$ is an Ak-HTM (Nk-HTM, Dk-HTM) which uses no cell on the storage tape. By Ak-HFA (Nk-HFA, Dk-HFA), we denote an alternating (nondeterministic, deterministic) one-way k-head finite automaton.

Further, by Uk-HFA we denote an Ak-HFA with only universal states.

3. A Relationship among Ak-HTM's, Uk-HTM's, Nk-HTM's and Dk-HTM's

In this section, we are mainly concerned with investigating a relationship among the accepting powers of Ak-HTM's, Uk-HTM's, Nk-HTM's and Dk-HTM's with spaces less than n.

For each $k \geq 1$, an Ak-HTM (Uk-HTM, Nk-HTM, Dk-HTM) M is halting if for each input x, M halts by entering an accepting ID or a rejecting ID in each computation path of M on x.

By using the same technique in the proof of Theorem 1 in Ref. (14), we can easily prove the following fact.

[Fact 1] For each $k \geq 1$, each function $L: N \to R$ and each $X \in \{D, N, A, U\}$, a set T is accepted by an Xk-HTM $(L(n))$ if and only if T is accepted by a halting Xk-HTM $(L(n))$.

For each k and each function $L: N \to R$, let co-$\mathscr{L}$[Nk-HTM$(L(n))$] $=\{\overline{L} \mid L \in \mathscr{L}$[N$k$-HTM$(L(n))$]$\}$. (For any set T, $\overline{T}$ denotes the complement of T.)

†$\lceil r \rceil$ means the smallest integer greater than or equal to r.

Fact 1 is used to prove the following theorem.

[Theorem 1] For each $k \geq 1$ and each function $L:N \to R$,

$$\mathscr{L}[\text{U}k\text{-HTM}(L(n))] = \text{co-}\mathscr{L}[\text{N}k\text{-HTM}(L(n))].$$

(Proof) Let $M = (k, Q, U, \Sigma, \Gamma, \delta, q_0, F, R)$ be a Uk-HTM $(L(n))$ where $k \geq 1$ and $L:N \to R$ is a function. By Fact 1, we can assume without loss of generality that M is a halting Uk-HTM$(L(n))$. We consider an Nk-HTM$(L(n))$ M' obtained from M by letting each universal state of M be an existential state of M' and by interchanging accepting and rejecting states of M. More specifically, we let $M' = (k, Q, \phi, \Sigma, \Gamma, \delta, q_0, R, F)$. Obviously, it follows that $T(M') = \overline{T(M)}$, and thus $T(M) = \overline{T(M')}$. Therefore, $\mathscr{L}[\text{U}k\text{-HTM}(L(n))] \subseteq \text{co-}\mathscr{L}[\text{N}k\text{-HTM}(L(n))]$. Similarly, we can easily show that co-$\mathscr{L}[\text{N}k\text{-HTM}(L(n))] \subseteq \mathscr{L}[\text{U}k\text{-HTM}(L(n))]$. This completes the proof of the theorem. Q.E.D.

[Corollary 1] When restricted to a single-letter alphabet, $\mathscr{L}[\text{U}k\text{-HFA}]$ $(k \geq 1)$ contains only regular sets.

(Proof) In Ref. (2), it is shown that when restricted to a single-letter alphabet, $\mathscr{L}[\text{N}k\text{-HFA}]$ $(k \geq 1)$ contains only regular sets. On the other hand, it is well known that the class of regular sets is closed under complementation. From these facts and Theorem 1 above we can see that the corollary holds. Q.E.D.

[Corollary 2] Even when restricted to a single-letter alphabet

$$\mathscr{L}[\text{N}k\text{-HFA}] \cup \mathscr{L}[\text{U}k\text{-HFA}] \subsetneq \mathscr{L}[\text{A}k\text{-HFA}] \ (k \geq 2)$$

and $\bigcup_{1 \leq k < \infty} \mathscr{L}[\text{N}k\text{-HFA}] \cup \bigcup_{1 \leq k < \infty} \mathscr{L}[\text{U}k\text{-HFA}] \subsetneq \bigcup_{1 \leq k < \infty} \mathscr{L}[\text{A}k\text{-HFA}]$.

(Proof) It is easily seen that $\mathscr{L}[\text{N}k\text{-HFA}] \cup \mathscr{L}[\text{U}k\text{-HFA}] \subseteq \mathscr{L}[\text{A}k\text{-HFA}]$ $(k \geq 1)$ and $\bigcup_{1 \leq k < \infty} \mathscr{L}[\text{N}k\text{-HFA}] \cup \bigcup_{1 \leq k < \infty} \mathscr{L}[\text{U}k\text{-HFA}] \subseteq \bigcup_{1 \leq k < \infty} \mathscr{L}[\text{A}k\text{-HFA}]$. Let $L_1 = \{0^{2^n} | n \geq 1\}$. It is shown in the proof of Theorem 5 in Ref. (10) that $L_1 \in \mathscr{L}[\text{A2-HFA}]$. L_1 is a nonregular set. From Corollary 1, $L_1 \notin \mathscr{L}[\text{N}k\text{-HFA}] \cup \mathscr{L}[\text{U}k\text{-HFA}]$ for any $k \geq 1$. This completes the proof of the corollary. Q.E.D.

The following is the main theorem in this section.

[Theorem 2] There exists a set in $\mathscr{L}[\text{N2-HFA}]$, but not in $\bigcup_{1 \leq k < \infty} \mathscr{L}[\text{U}k\text{-HTM}(L(n))]$ for any $L:N \to R$ such that $\lim_{n \to \infty} [L(n)/n] = 0$.

(Proof) Let

$$T_1 = \{ w_1 \# w_2 \# \cdots \# w_{2b} \mid b \geq 1 \ \& \ \forall i \ (1 \leq i \leq 2b) [w_i \in \{0,1\}^* 2 \{0,1\}^*] \ \& \ \exists i, \exists j \ (1 \leq i, j \leq 2b) \cdot [(w_i = x2y) \ \& \ (w_j = x2z) \ \& \ (y \neq z)] \}.$$

It is easily seen that $T_1 \in \mathscr{L}[\text{N2-HFA}]$. We only show that $T_1 \notin \bigcup_{1 \leq k < \infty} \mathscr{L}[\text{U}k\text{-HTM}(L(n))]$ for any $L:N \to R$ such that $\lim_{n \to \infty} [L(n)/n] = 0$. On the contrary, suppose that $T_1 \in \bigcup_{1 \leq k < \infty} \mathscr{L}[\text{U}k\text{-HTM}(L(n))]$, where $\lim_{n \to \infty} [L(n)/n] = 0$. It follows that for some $k \geq 1$, T_1 is accepted by some Uk-HTM$(L(n))$ M. For a large n, let

$$V(n) = \{ w_1 \# w_2 \# \cdots \# w_{2b} \mid \forall i (1 \leq i \leq 2b) [w_i = w_{2b+1-i} = B^n(\min(i, 2b+1-i)) 2y \ \& \ y \in \{0,1\}^n] \}^\dagger$$

where $b = k(k+1)/2$. Note that for each word x in $V(n)$, $|x| = 4b(n+1) - 1$. Clearly, each word x in $V(n)$ is not in T_1, and so x is rejected by M. Therefore, there exists at least one rejecting computation path of M on x. (A computation path which leads to a rejecting ID is called "rejecting".)

Let s and r be the numbers of states and storage tape symbols of M, respectively. The type of a configuration $c = ((i_1, \ldots, i_k), q, \alpha, j)$, denoted by Type (c), is a k-tuple $([i_1/(n+1)], \ldots, [i_k/(n+1)])$. Note that the i-th element p_i of the type specifies that the i-th head of M is on $w_{p_i}\#$ ($w_{2b}\$$ if $p_i = 2b$) in this configuration when scanning a word in $V(n)$. Let $c_1(x), c_2(x), \ldots, c_{l_x}(x)$ be the sequence of configuration of M on an (arbitarily selected) rejecting computation path of a word x in $V(n)$. Here $c_{l_x}(x)$ is a rejecting configuration. Let $d_1(x), d_2(x), \ldots, d_{l'_x}(x)$ be the subsequence obtained by selecting $c_1(x)$ *and all subsequent* $c_i(x)$'s such that Type $(c_i(x)) \neq$ Type $(c_{i-1}(x))$. We call $d_1(x), d_2(x), \ldots, d_{l'_x}(x)$ the pattern of x.

Let $p(n)$ be the number of possible patterns of M on words in $V(n)$. Since each head of M can move only right, it is easily seen that $l'_x \leq k(2b-1)+1$. Therefore, we get the following inequality,

$$p(n) \leq [s(4b(n+1))^k L(g(n)) r^{L(g(n))}]^{k(2b-1)+1},$$

where $g(n) = 4b(n+1) - 1$.

Then we classify the words in $V(n)$ according to their patterns. Clearly, there is a set $S(n)$ $(\subseteq V(n))$ such that $|S(n)| \geq 2^{nb}/p(n)$, whose each element has the pattern $\widehat{d}_1, \widehat{d}_2, \ldots, \widehat{d}_{\widehat{l}}$.

Since $b = k(k+1)/2 > \binom{k}{2}$, it follows from Rosenberg's observation [1] that for each word in $V(n)$ there must be an i such that M cannot read $w_i\#$ and $w_{2b+1-i}\#$ ($w_{2b}\$$ if $i = 1$) simultaneously. The possible values for i are determined entirely by the pattern of the computation. Let i_0 be such a value of i for the pattern $\widehat{d}_1, \widehat{d}_2, \ldots, \widehat{d}_{\widehat{l}}$.

We now define a binary relation E on words in $S(n)$ as follows. Let

$$u = u_1 \# u_2 \# \cdots \# u_{i_0} \# \cdots \# u_{2b+1-i_0} \# \cdots \# u_{2b}, \quad \text{and}$$
$$v = v_1 \# v_2 \# \cdots \# v_{i_0} \# \cdots \# v_{2b+1-i_0} \# \cdots \# v_{2b}.$$

Then,

$$u \, E \, v \iff \forall i (\notin \{i_0, 2b+1-i_0\}) [u_i = v_i].$$

Obviously the relation E is an equivalence relation, and there are at most $q(n) = 2^{n(b-1)}$ E-equivalence classes of words in $S(n)$. Since $\lim_{n \to \infty} [L(n)/n] = 0$, it follows that $|S(n)| > q(n)$ for a large n. Therefore, there exist two different words

$$x = x_1 \# x_2 \# \cdots \# x_{i_0} \# \cdots \# x_{2b+1-i_0} \# \cdots \# x_{2b}, \quad \text{and}$$
$$y = y_1 \# y_2 \# \cdots \# y_{i_0} \# \cdots \# y_{2b+1-i_0} \# \cdots \# y_{2b},$$

in $S(n)$ which belong to the same equivalence class. Let

$$z = x_1 \# x_2 \# \cdots \# x_{i_0} \# \cdots \# x_{2b-i_0} \# y_{2b+1-i_0} \# x_{2b+2-i_0} \# \cdots \# x_{2b},$$

be the word obtained from x by replacing x_{2b+1-i_0} with y_{2b+1-i_0}. By an argument similar to that in the proof of Theorem 1 in Ref. (3), it can be shown that there is a reject-

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$\dagger B^n(i)$ denotes the binary representation of i with leading zeros such that $|B^n(i)| = n$.

ing computation path of M on z. Consequently, z must be rejected by M. This contradicts the fact that z is in T_1. This concludes that $T_1 \in \bigcup_{1\leq k<\infty} \mathscr{L}$[U$k$-HTM$(L(n))$] for any $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$. Q.E.D.

The following is another main theorem in this section.

[Theorem 3] There exists a set in $\mathscr{L}$[U2-HFA], but not in $\bigcup_{1\leq k<\infty}\mathscr{L}$[N$k$-HTM$(L(n))$], and thus not in $\bigcup_{1\leq r<\infty}\mathscr{L}$[D$k$-HTM$(L(n))$] for any $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$.

(Proof) Let T_1 be the set described in the proof of Theorem 2. T_1 is in $\mathscr{L}$[N2-HFA]. From this fact and Theorem 1, it follows that $\overline{T_1}\in\mathscr{L}$[U2-HFA]. On the other hand, it is shown in the proof of Theorem 2 that T_1 is not in $\bigcup_{1\leq k<\infty}\mathscr{L}$[U$k$-HTM$(L(n))$] for any $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$. From this fact and Theorem 1, we can easily see that $\overline{T_1}\notin\bigcup_{1\leq k<\infty}\mathscr{L}$[N$k$-HTM$(L(n))$], and thus $\overline{T_1}\notin\bigcup_{1\leq k<\infty}\mathscr{L}$[D$k$-HTM$(L(n))$] for any $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$. Q.E.D.

From Theorems 2 and 3, we can get the following result.

[Corollary 3] For each $k\geq 2$ and each function $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$,

(1) $\mathscr{L}$[Dk-HTM$(L(n))$] $\subsetneq\mathscr{L}$[Uk-HTM$(L(n))$] $\subsetneq\mathscr{L}$[Ak-HTM $(L(n))$],

(2) $\bigcup_{1\leq r<\infty}\mathscr{L}$[D$r$- HTM $(L(n))$] $\subsetneq\bigcup_{1\leq r<\infty}\mathscr{L}$[U$r$-HTM $(L(n))$] $\subsetneq\bigcup_{1\leq r<\infty}\mathscr{L}$[A$r$-HTM$(L(n))$],

(3) $\mathscr{L}$[Uk-HTM$(L(n))$] is incomparable with $\mathscr{L}$[Nk-HTM$(L(n))$], and

(4) $\bigcup_{1\leq r<\infty}\mathscr{L}$[U$r$-HTM$(L(n))$] is incomparable with $\bigcup_{1\leq r<\infty}\mathscr{L}$[N$r$-HTM$(L(n))$].

4. Basic Properties of Space Bounded Uk-HTM's

In this section, we first give a hierarchie of the accepting powers of Uk-HTM$(L(n))$'s, based on the number of input heads, for each $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$.

[Theorem 4] For each $k\geq 1$ and each function $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$,

$\mathscr{L}$[Uk-HTM$(L(n))$] $\subsetneq\mathscr{L}$[U$(k+1)$–HTM$(L(n))$].

(Proof) For each $b\geq 1$, let

$$T_2(b)=\{w_1\#w_2\#\cdots\#w_{2b}\mid\forall i(1\leq i\leq 2b)[(w_i=\{0,1\}^*)\&(w_i=w_{2b+1-i})]\}.$$

By using the same technique as in the latter part of Theorem 1 in Ref. (3), we can show that $T_2\ (k(k+1)/2)\notin\mathscr{L}$[N$k$-HTM$(L(n))$] for $k\geq 1$ and each $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$. (The proof is given in Ref. (7).) From this fact and from Theorem 1, it follows that $\overline{T_2(k(k+1)/2)}\notin\mathscr{L}$[U$k$-HTM$(L(n))$] for each $k\geq 1$ and each $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$. On the other hand, by using the same technique as in the former part of the proof of Theorem 1 in Ref. (3), we can easily show that $\overline{T_2(k(k+1)/2)}\in\mathscr{L}$[D$(k+1)$-HFA], and thus $\overline{T_2(k(k+1)/2)}\in\mathscr{L}$[U$(k+1)$-HFA]. This completes the proof of the theorem. Q.E.D.

In Ref. (10), the problem of whether $\mathscr{L}$[Ak-HFA] $\subsetneq\mathscr{L}$[A$(k+1)$-HFA] $(k\geq 1)$ or not is proposed as an open problem. The following corollary gives a partial solution for this problem.

[Corollary 4] For each $k\geq 1$,

$\mathscr{L}$[Uk-HFA] $\subsetneq\mathscr{L}$ [U$(k+1)$-HFA].

We then investigate closure properties of the classes of languages accepted by space bounded UMHTM's under each of the Boolean operations. The proof of the following theorem is omitted since it is easy to prove.

[Theorem 5] For each $k\geq 1$ and each function $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$, $\mathscr{L}$[Uk-HTM$(L(n))$] and $\bigcup_{1\leq r<\infty}\mathscr{L}$[U$r$-HTM$(L(n))$] are closed under intersection.

[Theorem 6] For each $k\geq 2$ and each function $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$, $\mathscr{L}$[Uk-HTM$(L(n))$] and $\bigcup_{1\leq r<\infty}\mathscr{L}$[U$r$-HTM$(L(n))$] are not closed under complementation.

(Proof) Let T_1 be the set described in the proof of Theorem 2. Then, $\overline{T_1}\in\mathscr{L}$[U2-HFA]. On the other hand, it is shown in the proof of Theorem 2 that $T_1\notin\bigcup_{1\leq r<\infty}\mathscr{L}$[U$r$-HTM$(L(n))$] for any function $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$. This completes the proof of the theorem. Q.E.D.

[Remark 1] It has been proved [12] that $\mathscr{L}$[U1-HTM $(L(n))$] is not closed under complementation for each function $L:N\to R$ such that $L(n)\geq\log n$ and $\lim_{n\to\infty}[L(n)/n]=0$.

[Theorem 7] For each $k\geq 2$ and each function $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$, $\mathscr{L}$[Uk-HTM$(L(n))$] is not closed under union.

(Proof) For each $b\geq 1$, let $T_2(b)$ be the set described in the proof of Theorem 4, and for each $1\leq i\leq b$, let

$$\overline{T(b,i)}=\{w_1\#w_2\#\cdots\#w_{2b}\mid\forall j\ (1\leq j\leq 2b)[w_j\in\{0,1\}^*]\&(w_i\neq w_{2b+1-i})\}.$$

It is easy to see that for each $b\geq 1$,

$$\overline{T_2(b)}=T(b,1)\cup T(b,2)\cup\cdots\cup T(b,b)\cup T'$$

where $T'=\{w\in\{0,1,\#\}^*\mid w$ is not of the form $w_1\#w_2\#\ldots\#w_{2b}$, where for each i, $w_i\in\{0,1\}^*\}$. It is easily seen that each $T(b,i)$ $(1\leq i\leq b)$ and T' are in $\mathscr{L}$[D2-HFA], and thus in $\mathscr{L}$[U2-HFA]. The theorem follows from these facts and from the fact (in the proof of Theorem 4) that $\overline{T_2(k(k+1)/2)}\notin\mathscr{L}$[U$k$-HTM$(L(n))$] for each $k\geq 2$ and each $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$. Q.E.D.

5. $\mathscr{L}$[Nk-HTM$(L(n))$] $\cap$ co-$\mathscr{L}$[Nk-HTM$(L(n))$] $\neq\mathscr{L}$[Dk-HTM$(L(n))$] for small $L(n)$

Whether or not NP $\cap$ co-NP = P is a very important problem in the computational complexity theory(15). We cannot solve this problem, but we here show that $\mathscr{L}$[Nk-HTM$(L(n))$] $\cap$ co-$\mathscr{L}$[Nk-HTM$(L(n))$] is not equal to $\mathscr{L}$[Dk-HTM$(L(n))$] for each $k\geq 2$ and any $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$.

[Theorem 8] For each $k\geq 2$ and each function $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$, there is a set in $\mathscr{L}$[Nk-HFA] and in $\mathscr{L}$[Uk-HFA] = co-$\mathscr{L}$[Nk-HFA], but not in $\mathscr{L}$[Dk-HTM$(L(n))$].

(Proof) For each $b\geq 2$, let

$$C(b)=\{u\#w_1\#w_2\#\cdots\#w_{2b}\mid\forall i(1\leq i\leq 2b)[(u,w_i\in\{0,1\}^*)\&(w_i=w_{2b+1-i})]\},$$

$$D(b)=\{u\#w_1\#\cdots\#w_{2b}du\mid\forall i\ (1\leq i\leq 2b)[u,w_i\in\{0,1\}^*]\&d\notin\{0,1,\#\}\},$$ and

$E(b)=C(b)\cup D(b)$.

In order to prove the theorem, it is sufficient to show that for each $k\geq 2$,

(1) $E(\binom{k}{2})\in\mathscr{L}$[U2-HFA],

(2) $E(\binom{k}{2})\in\mathscr{L}$[N$k$-HFA], and

(3) $E(\binom{k}{2})\notin\mathscr{L}$[D$k$-HTM$(L(n))$] for any $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$.

(1): The set $E(\binom{k}{2})$ is accepted by a U2-HFA M which acts as follows. Let $t=\binom{k}{2}$. Given an input x, M enters a universal state to choose one of two further actions.

① In one action, M checks that the input x is either of the form

$$u\#w_1\#w_2\#\cdots\#w_{2t} \qquad \text{(A)}$$

or of the form

$$u\#w_1\#w_2\#\cdots\#w_{2t}\,du' \qquad \text{(B)}$$

(where u, u' and each w_i are in $\{0, 1\}^*$), and M enters an accepting state only if the input is of the form (A) or the input is of the form (B) and $u=u'$.

② In the other action, M makes t universal branches, and M acts in the i-th branch $(1\leq i\leq t)$ as follows. M checks that the input x is either of the form (A) above or of the form (B) above, and M enters an accepting state only if (i) the input x is of the form (A) and $w_i=w_{2t+1-i}$, or (ii) the input x is of the form (B).

It will be obvious that M accepts $E(t)=E(\binom{k}{2})$.

(2): It can be shown that for each $k\geq 2$, both $C(\binom{k}{2})$ and $D(\binom{k}{2})$ are in $\mathscr{L}$[Nk-HFA]. (By using the same technique as in the former part of the proof of Theorem 1 in Ref. (3), we can easily show that $C(\binom{k}{2})\in\mathscr{L}$[D$k$-HFA].) From this fact and from the obvious fact that $\mathscr{L}$[Nk-HFA] is closed under union, part (2) follows.

(3): The proof of (3) is essentially the same as the proof of Theorem 1 in Ref. (5).

Suppose that there is a Dk-HTM$(L(n))$ M accepting $E(\binom{k}{2})$ for some function $L:N\to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$. Let s and r be the numbers of states and storage tape symbols of M, respectively. Let $t=\binom{k}{2}$, and for each $n\geq 1$, let

$$V_C(n)=\{u\#w_1\#w_2\#\cdots\#w_{2t}\mid \forall i\,(1\leq i\leq 2t)\,[(u,w_i\in\{0,1\}^+)\&(|u|=|w_i|=n)\&(w_i=w_{2t+1-i})]\},$$

$$V_D(n)=\{u\#w_1\#\cdots\#w_{2t}du\mid \forall i\,(1\leq i\leq 2t)\,[(u,w_i\in\{0,1\}^+)\&(|u|=|w_i|=n)\,\&\,(w_i=w_{2t+1-i})]\,\&\,d\notin\{0,1,\#\}\},\ \text{and}$$

$$V(n)=V_C(n)\cup V_D(n).$$

Note that every word in $V(n)$ is in $E(t)$, and so it is accepted by M. Specifically, we show that if M accepts every word in $V(n)$ then M accepts some word not in $E(t)$. This is a contradiction.

A prominent configuration is a configuration in the accepting computation on a word x in $V(n)$, in which the machine M moves one of its heads on the symbols #, d or \$.

A subsequence of prominent configurations of the accepting computation on the word x is called a pattern of x.

We shall consider the initial part of the accepting computation on a word x in $V(n)$, that begins in the initial configuration and ends in the prominent configuration, in which one of the heads has read the whole subword $x_1=u\#w_1\#w_2\#\ldots\#w_t\ \#w_t\#\ldots\#w_2\#w_1$ of the input word x. (Note that for each word x_1 in $V_C(n)$ and each $x_1\ du$ in $V_D(n)$, the initial parts of the accepting computations on the words x_1 and $x_1\ du$ are the same.)

Let $p(n)$ be the number of all patterns of the initial parts of the accepting computations on words x in $V(n)$. If we note that $|x_1|=2t(n+1)+n$, we can easily see that the number of all configurations on words x_1 is bounded by

$$s[(2t+1)(n+1)]^k L(g(n))\, r^{L(g(n))},$$

where $g(n)=(2t+1)(n+1)-1$. And so we obtain the following inequality

$$p(n)\leq[s[(2t+1)(n+1)^k L(g(n))r^{L(g(n))}]^{k(2t+1)},$$

because no pattern of the initial part of the computation can consist of more than $k(2t+1)$ prominent configurations.

Since the number of all words x_1 from $V_C(n)$ is

$$2^{(t+1)n},$$

there exists a pattern σ of the initial part of the computation such that at least

$$2^{(t+1)n}/p(n)$$

different words x_1 in $V_C(n)$ have the same pattern σ.

Now we distinguish two following cases according to the last prominent configuration $((i_1,\cdots,i_k),q,\alpha,j)$ of the pattern σ.

(i) $i_j>n$ for all j in $\{1,\cdots,k\}$, i.e., all heads have read the initial subword $u\in\{0,1\}^+$ of $x_1=u\ \#w_1\#w_2\#\cdots\#w_t\#w_t\#\cdots\#w_2\#w_1$.

(ii) There exists some j in $\{1,2,\cdots,k\}$ such that $i_j\leq n$, i.e., at least one head has not read the initial subword $u\in\{0,1\}^+$.

We shall below show that both (i) and (ii) lead to a contradiction.

(i) In this case we shall consider input words x_1du in $V_D(n)$ where x_1 has the pattern σ.

Noting that there exist at least

$$2^{(t+1)n}/p(n)$$

different words x_1 with the pattern σ, we see that there exist at least

$$m=\frac{2^{(t+1)n}}{p(n)}\cdot\frac{1}{2^{tn}}=\frac{2^n}{p(n)}$$

different words x_1 with the pattern σ which differ from each other only in the initial subword u, i.e., there exist at least m words $x_1=u\#y$, where $y=w_1\#w_2\#\cdots\#w_t\#w_t\#\cdots\#w_2\#w_1$ is fixed, with the pattern σ.

Since $\lim_{n\to\infty}[L(n)/n]=0$, it follows that $m=2^n/p(n)$ is greater than 2 for a sufficient large n. It means that for a sufficient large n there exist two words in $V_C(n)$, $v_1 = u_1\#y$ and $v_2 = u_2\#y$ (where $u_1 \neq u_2$), with the same pattern σ of the initial part of the computation. Since the machine M accepts the word $x = v_1 du_1 = u_1\#ydu_1$ in $V_D(n)$ and M is deterministic, it follows that M must also accept the word $x' = u_2\#ydu_1$ which clearly does not belong to $E(t)$.

(ii) In this case we shall consider input words x in $V_C(n)$. Let us consider the whole accepting computations on all

$$2^{(t+1)n}/p(n)$$

different words $x = x_1$ in $V_C(n)$ having the same pattern σ of the initial part of the computation.

Let $p'(n)$ be the number of all possible patterns of accepting computations on words x in $V_C(n)$. We obtain the following inequality,

$$p'(n) \leq [s[(2t+1)(n+1)]^k L(g(n)) r^{L(g(n))}]^{k(2t+1)},$$

where $g(n) = (2t+1)(n+1)-1$.

From this fact it follows that there exist at least

$$2^{(t+1)n}/p'(n)$$

different words x in $V_C(n)$ with the same pattern σ' containing the pattern σ as an initial subsequence.

In case (ii), it follows from Rosenberg's observation Ref. (1) that in the computations on the words x with the pattern σ' there exists a number $i_0 \in \{1, 2, \ldots, t\}$ such that both subwords w_{i_0} of the words $x = u\#w_1\# \cdots \#w_{i_0}\# \cdots \#w_t\#w_t\# \cdots \#w_{i_0}\# \cdots \#w_1$ are never read by any couple of heads at the same time.

It means that there exist at least

$$m = \frac{2^{(t+1)n}}{p'(n)} \cdot \frac{1}{2^{(t-1)n} \cdot 2^n} = \frac{2^n}{p'(n)}$$

different words x in $V_C(n)$ having the same pattern σ', which differ from each other in the subword w_{i_0} only.

Since $\lim_{n\to\infty} [L(n)/n] = 0$, it follows that $m \geq 2$ for a sufficiently large n and so there exist two words in $V_C(n)$

$$v_1 = u\#w_1\# \cdots \#w_{i_0}\# \cdots \#w_t\#w_t\# \cdots \#w_{i_0}\# \cdots \#w_1,$$

and

$$v_2 = u\#w_1\# \cdots \#w'_{i_0}\# \cdots \#w_t\#w_t\# \cdots \#w'_{i_0}\# \cdots \#w_1,$$

with the same pattern σ' of the accepting computations, where $w_{i_0} \neq w'_{i_0}$.

By an argument similar to that in the proofs of Theorem 1 in Ref. (3) and Theorem 1 in Ref. (5), it can be shown that the machine M must also accept the word $x' = u\#w_1\# \cdots \#w_{i_0}\# \cdots \#w_t\#w_t\# \cdots \#w'_{i_0}\# \cdots \#w_1$, which does not belong to $E(t)$. This is a contradiction. Q.E.D.

6. Conclusion

In this paper, we considered $L(n)$ space bounded one-way multihead alternating Turing machines such that $\lim_{n\to\infty} [L(n)/n] = 0$. It is easily seen that for any function L such that $L(n) \geq n$, $L(n)$ space bounded one-way one-head alternating Turing machines are equivalent to $L(n)$ space bounded two-way multihead alternating Turing machines.

We conclude this paper by stating a few open problems left in this paper.

For each function $L: N \to R$ such that $\lim_{n\to\infty} [L(n)/n] = 0$,

(1) is $\mathscr{L}[Ak\text{-HTM}(L(n))]$ properly contained in $\mathscr{L}[A(k+1)\text{-HTM}(L(n))]$ $(k \geq 1)$?;

(2) are $\mathscr{L}[U1\text{-HTM}(L(n))]$ and $\bigcup_{1\leq k<\infty} \mathscr{L}[Uk\text{-HTM}(L(n))]$ closed under union?;

(3) is there a set in $\mathscr{L}[U2\text{-HFA}] \cap \mathscr{L}[N2\text{-HFA}]$, but not in $\bigcup_{1\leq k<\infty} \mathscr{L}[Dk\text{-HTM}(L(n))]$?.

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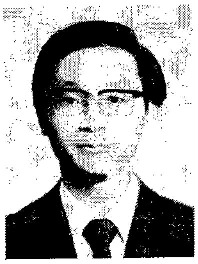

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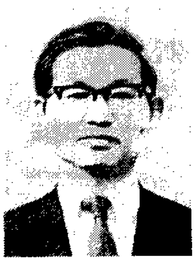

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