ENERGY EXTRACTION UTILIZING THE PITCHING MOTION OF A FLOATING VESSEL

Pallav KOIRALA[1], Kesayoshi HADANO[2] and Keisuke TANEURA[3]

[1]Student Member of JSCE, M. Eng., Graduate Student, Yamaguchi University(2-16-1 Tokiwadai, Ube, Yamaguchi, Japan)
[2]Member of JSCE, Dr. of Eng., Professor, Dept. of Hydraulic Engineering and Hydroscience, Yamaguchi University (2-16-1 Tokiwadai, Ube, Yamaguchi, Japan)
[3] Member of JSCE, M. Eng., Assistant Professor, Dept. of Hydraulic Engineering and Hydroscience, Yamaguchi University (2-16-1 Tokiwadai, Ube, Yamaguchi, Japan)

本研究では、潮流やその他の流れから大型浮体のピッチング運動をつくりだし、この運動を利用してエネルギーを獲得するシステムを提案している。浮体の揺動を作り出すために、浮体の底部に幅方向に配置した複数の板を流れの中で一定速度で回転する。回転板の回転はスピードコントローラーと流速計センサーを用いて板の回転速度と回転のオン・オフにより制御される。浮体に設置した空気室とタービンによりエネルギー変換することができる。本論文ではピッチング運動の初歩のモデルを提示している。その中で浮体の傾きが板に作用する流体力と浮体の喫水状態に及ぼす効果を考慮している。また、流れの中で板を回転させるのに必要な仕事率などを見積もっている。

Key Words : *Pitching motion, floating vessel, rotating plates, vertical water columns, resonance*

1. OBJECTIVE

A rising demand for energy coupled with the problem of environmental pollution, has led to investigations into potential new energy sources. Tidal current energy represents one of the most dependable and predictable sources of renewable energy available, which is free from the variations present in wind or solar energy. Although, energy extraction from tidal flow is practical only in a few areas where the current velocity is high[1)], it will still contribute significantly to the ever-rising demand of energy. In this paper, we propose a method for extracting energy from the tidal flow through the pitching motion of a floating vessel. A number of flat plates located laterally at the base of the vessel are rotated using batteries to obtain the pitching motion as shown in **Fig. 1**. The flow force acting on the plates varies periodically due to their rotation. The moment of this force causes the vessel to pitch about its transverse axis. It is evident that the frequency of the excitation force and hence the pitching motion are both functions of the frequency of the rotation of the plates. Therefore, resonant pitching motion, the desirable condition for energy extraction, can be obtained by setting the time period of plates' rotation equal to the natural period of pitching of the vessel by using in tandem a speed changer and a velocity sensor.

The relative water level in the vertical water columns located on either sides of the vessel will rise and fall as a result of the pitching. If the openings at the top of these columns are kept narrow, air flows in and out at a high velocity which can be utilized to drive a pair of Wells turbines. In places where the wind velocity is high, the plates may be placed at the top of the vessel as shown in **Fig. 2**, which forms the wind energy extracting system.

In this paper, the dynamics model of the pitching motion caused by tidal flow is discussed and calculations are made on the time series of the pitching angle at different periods of the rotation of the plates for a particular vessel. The model has been developed considering the effects of vessel's inclination on the flow forces experienced by the plates and the submerged area of the vessel. The equations of the work and torque to rotate the plates

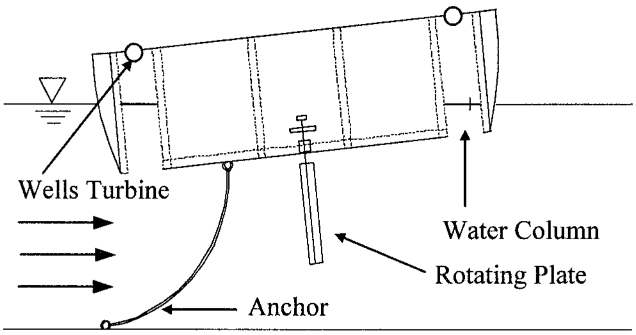


Fig.1 Schematic diagram of tidal energy extracting system

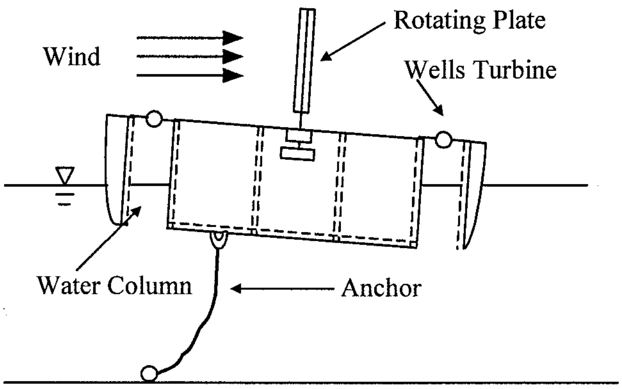


Fig.2 Schematic diagram of wind energy extracting system

in the flow have been formulated.

2. DYNAMICS OF PITCHING MOTION

The moments of the periodically varying flow force on the plates and the submerged area of the vessel cause the vessel to oscillate about its transverse axis. These periodic forces depend on the time period of the plates' rotation. In this section, a mathematical model of the pitching is presented along with its numerical calculation.

(1) Equation of the pitching motion

As shown in **Fig. 3**, a floating vessel is located on the surface of a flow with velocity u by mooring. For simplicity, the vessel is assumed to be rectangular in cross section with a length, width and height of a, b and c respectively. The mass moment of inertia of the vessel, I, is given by

$$I = \sigma bca^3/12 \tag{1}$$

where σ is the density of the vessel. The moment of inertia of the cross sectional area of the vessel at the water line, I_y, is given by Eq.(2).

$$I_y = ba^3/12 \tag{2}$$

The equation for the pitching motion can be written as

$$\left(I\frac{d^2\theta}{dt^2} + \rho g I_y \theta \right) = M_1 + M_2 \tag{3}$$

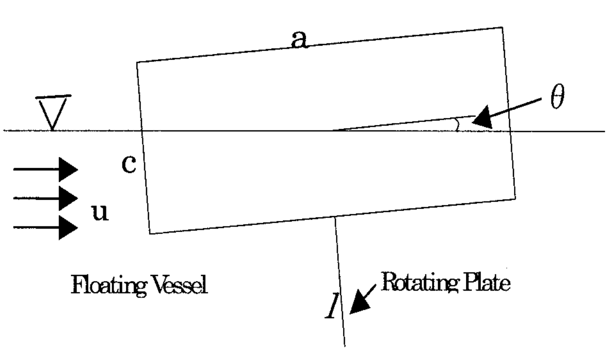


Fig.3 Dynamics of the pitching motion

where M_1 is the moment of the force exerted by the flow on the submerged area of the vessel.

$$M_1 = \frac{1}{2}\rho C_D b[\frac{1}{2}u^2\cos^2\theta(\sigma c/\rho)^2 - \frac{2}{3}u\cos\theta\frac{d\theta}{dt}(\sigma c/\rho)^3 + \frac{1}{4}\left(\frac{d\theta}{dt}\right)^2(\sigma c/\rho)^4] \tag{4}$$

M_2 is the moment of the force exerted by the flow on the rotating plates.

$$M_2 = n\frac{1}{2}\rho C_D\frac{b}{m}|\cos(\omega t)|\cdot\left[\frac{1}{2}u^2\cos^2\theta\{(\sigma c/\rho)^2 - (\sigma c/\rho)^2\}\right] - \frac{2}{3}u\cos\theta\frac{d\theta}{dt}\{(\sigma c/\rho + l)^3 - (\sigma c/\rho)^3\} + \frac{1}{4}\left(\frac{d\theta}{dt}\right)^2\{(\sigma c/\rho + l)^4 - l^4\} \tag{5}$$

Using Eq.(4) and Eq.(5) in Eq.(3), we obtain

$$\frac{d^2\theta}{dt^2} + \frac{4C_D}{a^3}u\cos\theta\left[(\sigma c/\rho)^2 + \frac{n\rho}{m\sigma c}|\cos(\omega t)|\{(\sigma c/\rho+l)^3 - (\sigma c/\rho)^3\}\right]\left(\frac{d\theta}{dt}\right) - \frac{3C_D}{2a^3}\left[(\sigma c/\rho)^3 + \frac{n\rho}{m\sigma c}|\cos(\omega t)|\{(\sigma c/\rho + l)^4 - (\sigma c/\rho)^4\}\right]\left(\frac{d\theta}{dt}\right)^2 + \frac{\rho g}{\sigma c}\theta = \frac{3C_D}{a^3}\left[(\sigma c/\rho) + \frac{n\rho}{m\rho c}|\cos\omega t|\{(\sigma c/\rho+l)^2 - (\rho c/\rho)^2\}\right]u^2\cos^2\theta \tag{6}$$

Eq.(6) describes the pitching motion taking into account the inclination of the vessel on the forces exerted on the plates and the vessel. From Eq.(6), the natural period of pitching becomes

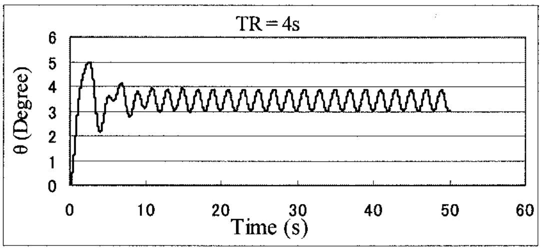
$$T_n = 2\pi(\sigma c/\rho g)^{1/2} \tag{7}$$

(2) Time series of the pitching angle

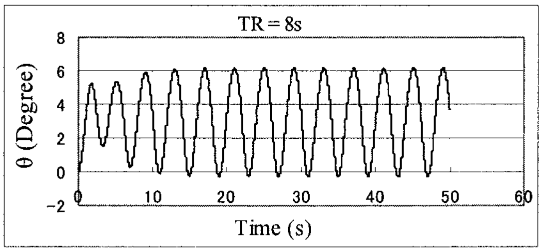
Eq.(6) has been solved numerically using the Runge-Kutta method for a particular vessel whose dimensions and other physical quantities are given in **Table 1**. The value C_D=1 is adopted because the Reynolds number is of the order of 10^5, the velocity of the rotating plate is one order smaller than the flow velocity and that the C_D value primarily depends on the flow velocity. **Fig. 4** shows the results of the calculation for different periods of the rotation of the plates(TR). Initial conditions of the vessel are given by Eq.(8).

Table 1 Calculation conditions for the vessel and plates

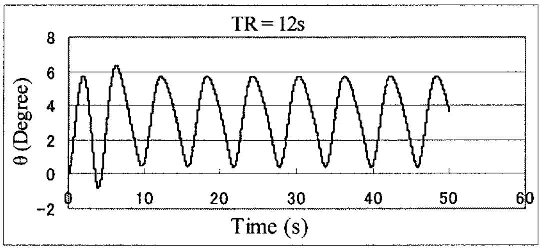
Physical Quantities		Value
Density of water	ρ (kg/m^3)	1000
Density of vessel	σ (kg/m^3)	700
Flow velocity	u (m/s)	2
Drag coefficient	C_D	1
Length of vessel	a (m)	10
Width of vessel	b (m)	7
Height of vessel	c (m)	6
Length of plates	l (m)	5
Number of plates	n	6



(a) TR = 4s



(b) TR = 8s



(c) TR = 12s

Fig.4 Time series of the pitching angle for various values of TR

$$\theta(0) = 0, \frac{d\theta(0)}{dt} = 0 \quad (8)$$

Fig. 4(a) shows that at short periods of the plate rotation, the magnitude of the pitching is small. This is because the frequency of the force acting on the plates oscillates too rapidly to be able to overcome the inertial force, or the vessel becomes too sluggish to respond to the high frequency oscillating force.[2] θ increases gradually with increasing TR and attains a maximum at around 8s, which is near the resonance period, as shown in **Fig.4(b)**. With a further increase of TR, the magnitude of pitching decreases as shown in **Fig.4(c)**, although it is still greater than for periods shorter than 8s. Occurrence of the highest maximum amplitude at TR = 8s can be attributed to the resonance phenomenon when the natural frequency of oscillation of the vessel matches the frequency of the pitching motion.

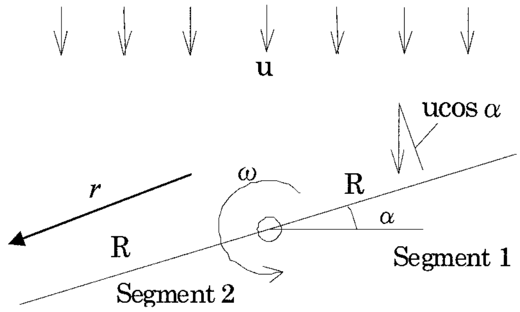


FIG. 5 Plan view of the plate at any instant during rotation

3. DYNAMICS OF THE PLATE ROTATION

In this section, the equations for the work and torque required to rotate the plates are presented. Calculation of the work is important especially because external power source is used.

Fig. 5 shows the plan view of an isolated plate with a width of *2R* and length of *l* rotating at an angular velocity ω. For simplicity the flow is assumed turbulent everywhere around the plate and the effect of the inclination of the vessel on the relative velocity between the flow and the plate is ignored. The relative velocity at any point on the plate located at a distance *r* from its center can be written as

$$V_\alpha = u \cos \alpha + r\omega \quad (9)$$

$$V_\alpha = u \cos \alpha - r\omega \quad (10)$$

where u is the flow velocity and α is the angle of rotation of the plate taken anticlockwise from the position perpendicular to the flow. Eq.(9) is applicable in segment 1 of the plate and Eq.(10) in segment 2 as shown in **Fig. 5**.

(1) Equations for the work and torque needed to rotate segment 1 of the plate from $t=0$ to $t=\pi/2\omega$

The required work can be written as follows

$$P_1 = \sum_i F_i V_i = \sum (\frac{1}{2}\rho C_D V_\alpha^2 l.dr) * (r\omega) \quad (11)$$

where F :drag force; V: relative velocity, ρ is the water density and C_D is the drag coefficient. Substituting V_α in Eq.(11) from Eq.(9), we obtain the following equation.

$$P_1 = \frac{1}{2}\rho C_D l\omega \int_0^R (u\cos\alpha + r\omega)^2 r dr \quad (12)$$

Finally, after integration Eq.(12) can be written as

$$P_1 = \frac{1}{2}\rho C_D l\omega\left(\frac{1}{2}u^2\cos^2\alpha R^2 + \frac{2}{3}u\cos\alpha\omega R^3 + \frac{1}{4}\omega^2 R^4\right) \quad (13)$$

The torque is given by

$$M_1 = \frac{1}{2}\rho C_D l\left(\frac{1}{2}u^2\cos^2\alpha R^2 + \frac{2}{3}u\cos\alpha\omega R^3 + \frac{1}{4}\omega^2 R^4\right) \quad (14)$$

(2) Equations for the work and torque required to rotate segment 2 of the plate from $t=0$ to $t=\pi/2\omega$

Since the direction of the component of flow velocity acting on the plate and the tangential component of the plate rotation, ωr, is same in this case, a point R_l can be defined along the plate where the relative velocity between the flow and the plate rotation is zero (i.e. $u\cos\alpha = \omega R_1$). R_1 lies either on the plate or outside depending on the position of the plate and ω during rotation. From $r=0$ to $r=R_l$, V_α will be positive and the plate rotates with the aid of the force from the flow, while in the remaining part which is from $r=R_l$ to $r=R$, V_α will be negative and the plate rotates relatively against the flow. It should be noticed that the positive or negative value of V_α does not correspond to whether the flow is against or toward the plate motion. Consequently, we have two different cases of calculation for the required work and the torque in segment 2. If R_l lies just at the edge of the plate (i.e. at $r = R$) when $t = t_l$, then we can write

$$u\cos\omega t_1 = \omega R_1 = \omega R \quad (15)$$

which gives

$$t_1 = \cos^{-1}(R\omega/u)/\omega \quad (16)$$

a) From $t=0$ to $t=\cos^{-1}(R\omega/u)/\omega$

The required work is given by

$$P_2 = \sum_i F_i V_i = \sum\left(\frac{1}{2}\rho C_D V_\alpha^2 l dr\right)(-r\omega) \quad (17)$$

Using Eq.(10) in Eq.(17), we obtain

$$P_2 = -\frac{1}{2}\rho C_D l\omega \int_0^R (u\cos\alpha - r\omega)^2 r dr \quad (18)$$

The torque in this case is given by

$$M_2 = -\frac{1}{2}\rho C_D l \int_0^R (u\cos\alpha - r\omega)^2 r dr \quad (19)$$

b) From $t=\cos^{-1}(R\omega/u)/\omega$ to $t=\pi/2\omega$

The work is given by

$$P_3 = \left[\sum_i F_i V_i\right]_{r=0\,to\,R_1} + \left[\sum_i F_i V_i\right]_{r=R_1\,to\,R} \quad (20)$$

Eq.(20) can be expressed as Eq.(21) after substituting V from Eq.(10).

$$P_3 = -\frac{1}{2}\rho C_D l\omega \int_0^{R_1} (u\cos\alpha - r\omega)^2 r dr + \frac{1}{2}\rho C_D l\omega \int_{R_1}^{R} (u\cos\alpha - r\omega) r dr \quad (21)$$

The torque is given by the following equation.

$$M_3 = -\frac{1}{2}\rho C_D l \int_0^{R_1} (u\cos\alpha - r\omega)^2 r dr + \frac{1}{2}\rho C_D l \int_{R_1}^{R} (u\cos\alpha - r\omega) r dr \quad (22)$$

(3)Total work and torque for the whole plate

a) From $t=0$ to $t=\cos^{-1}(R\omega/u)/\omega$

The total work required to rotate the plate in this time interval can be obtained by combining the Eq.(12) of segment 1 and Eq.(18) of segment 2 as given below.

$$P_{total\ (R<R_1)} = \frac{2}{3}\rho C_D l\omega^2 R^3 u\cos\alpha \quad (23)$$

And the total torque is obtained by combining Eq.(14) and Eq.(19) as follows.

$$M_{total\ (R<R_1)} = \frac{2}{3}\rho C_D l\omega R^3 u\cos\alpha \quad (24)$$

b) From $t=\cos^{-1}(R\omega/u)/\omega$ to $t=\pi/2\omega$

Combining Eq.(12) and Eq.(21) gives Eq.(25) as the total required work for this time interval.

$$P_{total\ (R>R_1)} = \frac{1}{2}\rho C_D l\omega\left(u^2\cos^2\alpha\cdot R^2 + \frac{1}{2}\omega^2 R^4\right) - \frac{1}{2}\rho C_D l\omega\left(u^2\cos^2\alpha\cdot R_1^2 - \frac{4}{3}u\cos\cdot\omega R_1^3 + \frac{1}{2}R_1^4\right) \quad (25)$$

The total torque is obtained by combining Eq.(14) and Eq.(22) as follows.

$$M_{total\ (R>R_1)} = \frac{1}{2}\rho C_D l\left(u^2\cos^2\alpha\cdot R^2 + \frac{1}{2}\omega^2 R^4\right) - \frac{1}{2}\rho C_D l\left(u^2\cos^2\alpha\cdot R_1^2 - \frac{4}{3}u\cos\cdot\omega R_1^3 + \frac{1}{2}R_1^4\right) \quad (26)$$

c) From $t=\pi/2\omega$ to $t=\pi/\omega - 1/\omega(\cos^{-1}(R\omega/u)$

The work and the torque have been calculated here from analyses similar to that in the previous sections.

$$P=\frac{1}{2}\rho C_D l\omega\left[u^2R^2\cos^2(\pi-\omega t)+\frac{1}{2}\omega^2R^4\right]-\frac{1}{2}\rho C_D l\omega$$
$$\left[u^2R_1^2\cos^2(\pi-\omega t)-\frac{4}{3}u\omega R_1^3\cos(\pi-\omega t)+\frac{1}{2}\omega^2R_1^4\right] \quad (27)$$
$$M=\frac{1}{2}\rho C_D l\left[u^2R^2\cos^2(\pi-\omega t)+\frac{1}{2}\omega^2R^4\right]-\frac{1}{2}\rho C_D l$$
$$\left[u^2R_1^2\cos^2(\pi-\omega t)-\frac{4}{3}uR_1^3\cos(\pi-\omega t)+\frac{1}{2}\omega R_1^4\right] \quad (28)$$

d) From $t=\pi/\omega-1/\omega(\cos^{-1}(R\omega/u)$ to $t=\pi/\omega$

$$P=\frac{2}{3}\rho C_D l\omega^2uR^3\cos(\pi-\omega t) \quad (29)$$
$$M=\frac{2}{3}\rho C_D l\omega uR^3\cos(\pi-\omega t) \quad (30)$$

The summation of Eqs.(23), (25), (27) and (29) gives the total work required to rotate the plate for one half rotation. The total work required for one complete rotation is twice this amount. It is found that the equations of the work and torque contain terms proportional to the second, third and fourth power of the width of the plate $2R$. Therefore, for the given width of the vessel, it will be more economical to use many plates with small width than a few plates with large width.[3)]

(4) Calculation of the required work

The work required to rotate the plates at a time period TR =8 seconds is calculated here for a 7m wide vessel using Eqs.(23), (25), (27) and (29) for the conditions given in **Table 1**. **Fig. 6** shows the result of the calculation for R = 0.25 m and 0.5 m. For the given 7m wide vessel, we can locate 14 plates and 7 plates at its base with R = 0.25m and 0.5m respectively. The total work required when the narrow plates are used is found to be much less than when the wider plates are used. The nonlinear relationship between the width of the plates and the work needed to rotate them is shown in **Fig. 7**.

4. ESTIMATION OF THE POWER OUTPUT

We take the case of a vessel similar in size as that in section 2. The difference is that this vessel has an air chamber running throughout the length of the vessel, partitioned at the center, and the pipe which connects both sides of air chamber and air turbine inside as shown in **Fig. 8.** The maximum potential power output is calculated using the maximum amplitude of pitching at resonance (about 6 degrees) as given in **Fig. 4b**. The work is calculated as the volume change in the air chambers due to the vessel's pitching motion. Furthermore, the process is assumed to be a constant temperature for simplicity. Therefore, the equation for work done by the air inside the chambers can be written as follows

$$W=mRT\ln(v2/v1) \quad (31)$$

Where, m is the mass of air inside the chambers, R is the universal gas constant for air (287 J/kg.K), T is the ambient temperature (303 K for calculation), $v1$ is the initial volume in the chamber when the

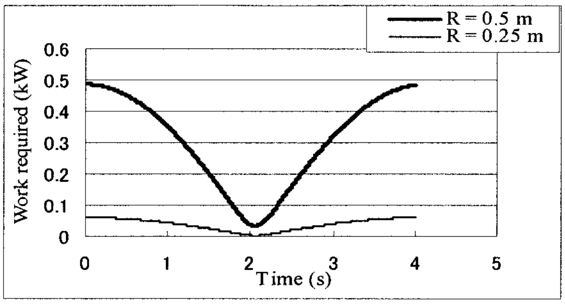


Fig.6 Work required to rotate a plate with a period of 8s

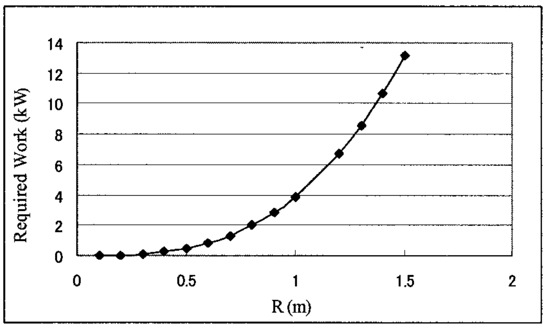


Fig.7 Average work required for different widths of the plate

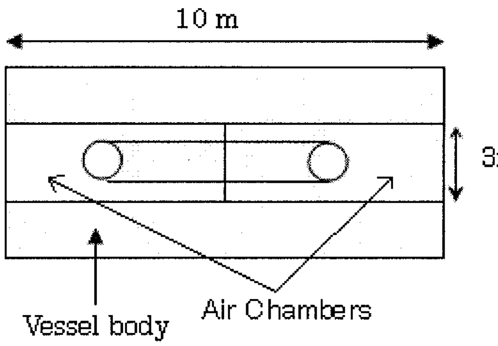


Fig.8 Plan view of the vessel with a different orientation of air chambers

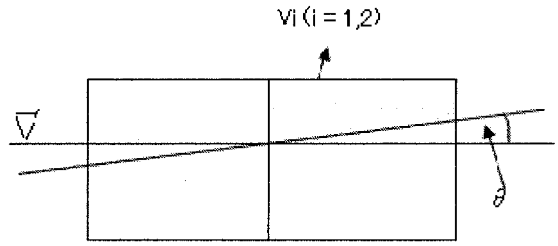


Fig.9 Side view of the vessel and air chamber at inclined state

vessel is horizontal and *v2* is the volume in the chamber at maximum inclination of the vessel during pitching. Mass of air inside the air chambers, m, is 28.82kg.For the given vessel, the change in the volume is found to be 3.94 m^3 and the corresponding power is 49.38kW.

5. CONCLUSIONS

So far we have proposed a floating vessel which can extract the energy from wind or water current through the resonant pitching motion. The formulation for the time series of the work and torque needed to rotate the flat plates in order to induce the pitching motion has been given. The expressions for the work and torque contain terms which are proportional to the second, third, and fourth power of the width of the plates. Therefore, it is more economical to use many plates with smaller widths. Results of the numerical simulation of the pitching motion showed that the magnitude of the pitching angle is largest during resonance where the natural frequency of the pitching motion of the vessel matches the excitation frequency. The model for the pitching motion presented was developed taking into account the effects of the inclination of the vessel on the forces exerted by the flow.

REFERENCES

1) Pontes, M.T., Falcao, A.: Ocean Energies: Resources and Utilization, *World Energy Council, 18th Congress*, October 2001.
2) Hooft, J. P.: Advanced Dynamics of Marine Structures, John Wiley & Sons, 1982.
3) Hadano, K., Koirala, P., Watanabe, M.: A proposal for energy extracting float, *Proceedings of the Fifteenth International Offshore and Polar Engineering Conference*, 2005-TS-07, 2005.