Simulation Analysis of Rill Patterns that develop on Hillslopes

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Abstract

The concepts of fractal dimension and spectrum analysis have been used for the numerization of the very complex patterns of the natural world. In this paper, a development model of rill patterns was made with the aid of a random pattern model, which is used in the field of fractal geometry. The rill patterns, that develop on hillslopes, were simulated by a personal computer based on this model. Though the development of the actual rill depends considerably on many factors, such as soil, hydraulics, rainfall, and the topographic conditions. The fractal dimensions and the entropy which is characteristics of a power spectrum, are all within range of the values of an actual rill.

1. Introduction

In recent years, soil erosion has rapidly increased in developing countries and the damage to irrigation from the accumulation of sediment, floods caused by the deposits of sediment on river beds, and water pollution in rivers have become serious problems. Usually soil erosion advances slowly under natural conditions and soil loss is compensated by the regeneration of new soil from an underlying soil. But the above problems are caused mainly by the indiscriminate development of natural resources by advanced nations. For instance, overcutting woodland is the largest cause of soil erosion.

The process of soil erosion by rainfall can be classified into two processes. One is the detachment and splash of soil particles by the impact of rainfall. This impact also compacts the soil surface and make perviousness worse. The second process is the transportation of soil particles by the tractive force of surface flow. As surface flow is concentrated in distinct small channels (rills), the velocity of the flow in the rills increases rapidly. Soil particles transported in the rills increase as well (rill erosion). It is said that the amount of soil loss which is carried away by the rills is roughly 80 percent of all of the soil loss. Therefore, it is very important to reveal the mechanism of the generation of rills and of the development of rill patterns.

In this study, we consider the process of the development of rill patterns, and simulate the growing patterns of the rills. As rill patterns are highly complex, classical simulation models, such as the random walk model, which can explain the patterns of river networks, could not explain rill patterns well. The concept of fractal

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geometry is an effective method of the quantification of natural patterns, so rill patterns can be simulated by methods based on the fractal geometry.

The Method of Simulation

In the natural world, there exists a lot random patterns that are not at all systematic at first sight. For example, a mountain range, coastlines, rivers, cracks in glass, surface of the ground, and also rill networks. It has been very difficult to describe these patterns quantitatively because they are random and complex.

The mechanism of the generation and the development of rills on slopes is still unclear. Therefore, it seems that the simulation of the development of rill patterns with time is very difficult. The patterns of rills developed on hillslopes have fractal characteristics as described by Fujiwara et al.[1], so in this study, we apply a new geometrical concept of fractal which was originated by Mandelbrot[2] to simulate the geometrical characteristic of rill patterns.

In general, the mechanism of the formation of natural patterns is not a simple process but is often intricate involving several factors. As a first step of the simulation, we made an ideal model which was based on the basic physical mechanism (section 2.1). Secondly, the geometrical properties of the patterns, that are simulated under such ideal conditions were compared to the patterns of the experimental or observed patterns of rills. We examined as geometrical properties ; the forms, fractal dimensions and entropy of the rill patterns. Small channels observed on hillslopes or in the experiment by simulated rainfall, expanded not only on the ground surface, but the volume also expanded in a vertical direction. In this study, the simulation is

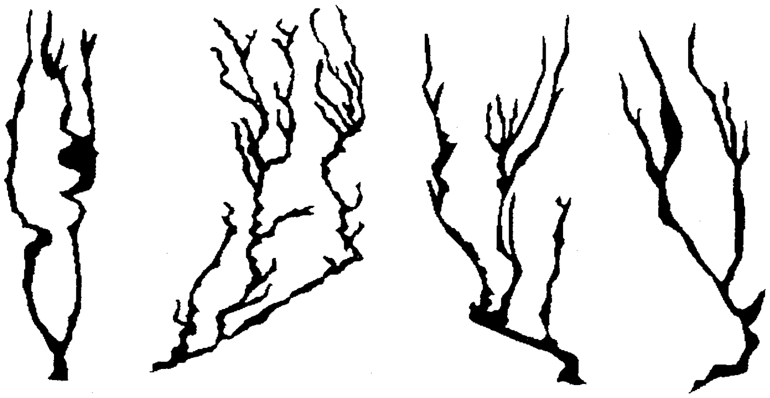


Fig. 1 The actual rills (at Shiraishi in UBE city)

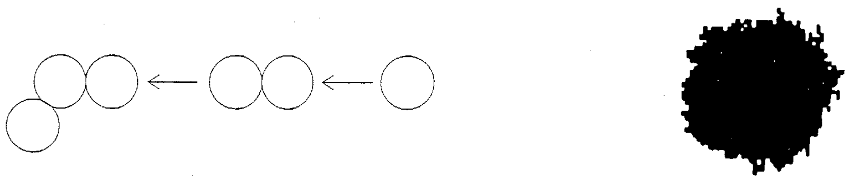
restricted to the plane, that is, to two dimensions.

1. Growing Rondom Pattern Model

In fractal geometry, the central concept is the statistically self-similarity or scale -invariance of patterns. In other words, if we pick a part of a complicated pattern and magnify it to the same degree as the whole pattern, it has the same pattern as the whole. Recently, this fractal characteristic has been used in the field of computer simulation. In the field of material physics, several models have been proposed to explain the various types of growing random phenomena, they are, (1) EDEN model[3], (2) Percolation model[4] (3) Kinetic-Cluster-Aggregation model or KCA model[5], (4) Diffusion-Limited-Aggregation model or DLA model[6], and so on. I would like to mention simply the content of the these model as follows.

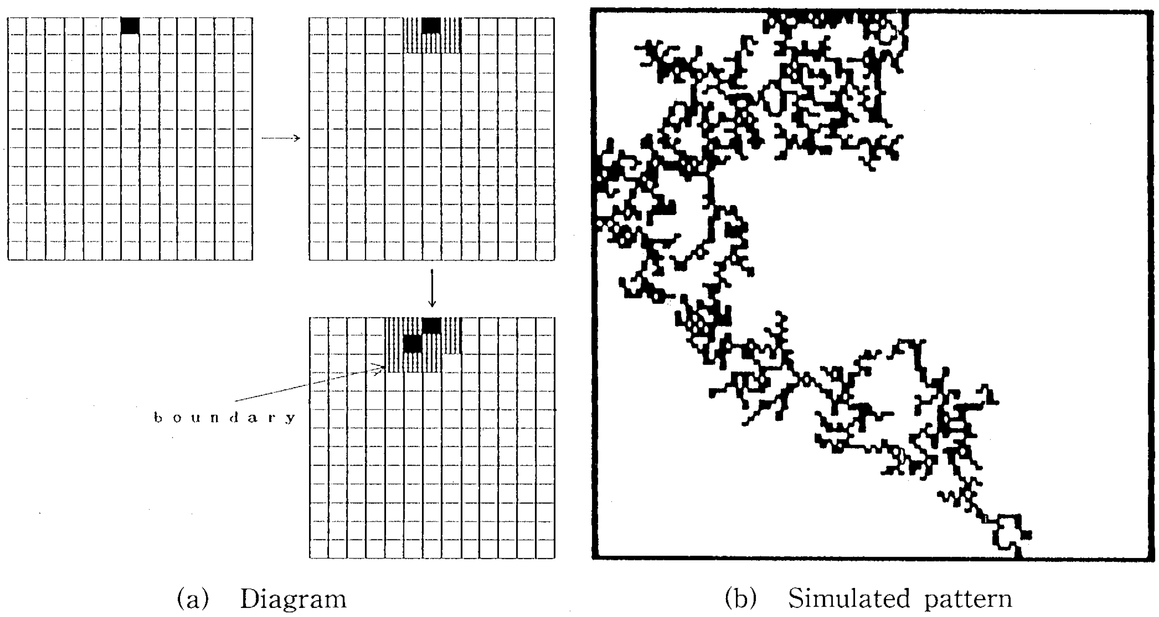
(1) *EDEN model*

This is considered as a model of the increase of cancer, This method of computer simulation is after putting a particle in the origin of a two dimensional plane selects, at random and at even probability, a point on the border of the particle, the pattern then becomes a two particle pattern. Secondly, selecting another point at random on the border of these two particles, repeats the above operation, the pattern grows continuously. Fig.2 is an example of the pattern made by the EDEN model by repeating this operation more than one thousand times.



(a) Diagram (b) Simulated pattern

Fig. 2 EDEN model



(a) Diagram (b) Simulated pattern

Fig. 3 Percolation model

(2) *Percolation type model*

This is the model that describes the phenomena of pouring and penetration of immiscible liquids into another liquid contained in a substance, for example, the penetration of water into petroleum in sand.

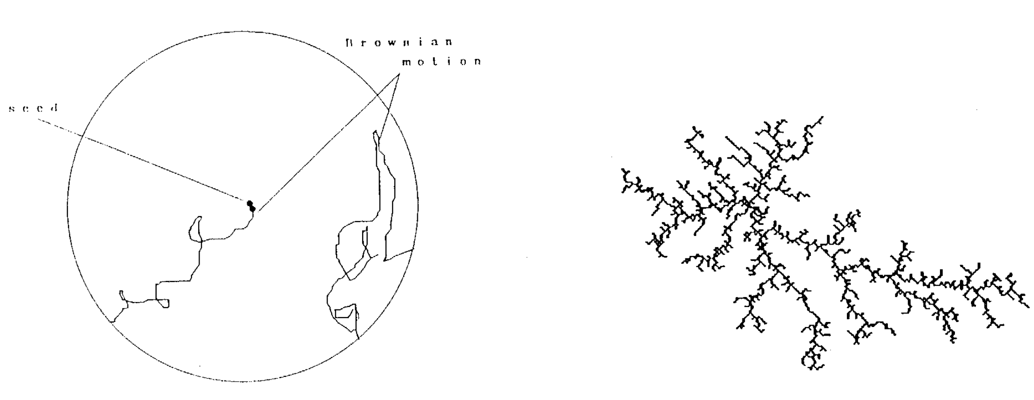
The method of simulation is, first, we assign random numbers at all points of the lattice in the plane. Secondly, looking for the point that has the least random numbers in the border of the pattern, which starts growing from a point (this point is considered to be set up initially). If we pour water at the first point, this second point is considered to be the weakest one against the force of percolation of water. Repeating the same process continuously, the weakest point on the border of the pattern becomes the seed of growth.

(3) *Kinetic-Cluster-Aggregation (KCA) model*

When scattering particles having Brownian movement are close to each other, they link together and become a cluster. This cluster also has Brownian movement according to its size. And when clusters are close to each other, they become a bigger cluster. The pattern grows by repeating this process.

(4) *Diffusion-Limited-Aggregation (DLA) model*

The general aggregation model is such that by putting a seed at the origin of the xy -plane, and selecting another point at random on the circumference of a circle whose center is at the origin, releases a particle from this point. This particle has Brownian movement (random movement). When this particle arrives close to the seed, the two particles join together to form a bigger particle. If the released particle passes far from the seed, the two particles do not join. In such cases, again select another point at random on the circumference of a circle, releasing a particle from there. Only if the particle arrives close to the point of the cluster does the released particle become a member of it. Repeating this operation, a DLA pattern grows around the origin. This is a random pattern that has large and small branches, and has the characteristics of statistical self-similarity. In such patterns, the protruded branches prevent the released particles from entering inside the branches. so the branches grow toward the



(a) Diagram (b) Simulated pattern

Fig. 4 Diffusion-Limited-Aggregation (DLA) pattern outside more often.

Fig.2-Fig.4 show the examples of simulated patterns on the basis of (1), (2), (3) models. As a result of this simulation, it seems that the Diffusion-Limited-Aggregation (DLA) model is the most effective model to describe the formation of rills. Accordingly, the simulation model of rills is restricted to the DLA model in this study.

2. The Model of Rill Development

Fig.4 is the general DLA model pattern. But this pattern does not look like rill patterns developed on hillslopes. Accordingly, it is necessary to build a rill model on the basis of the DLA model. When we observe the formation of a rill on the slope, we can observe that small channels are generated at the bottom or middle part of the slope, and they begin to progress toward the upper part with repeating head down, and with repeating tributaries or joining other channels. It seems that such a behavior of rill patterns was promoted by the impact of raindrops on the ground surface, that is, soil particles on the ground be come easier to move by the accelerated flow. Accordingly, the model of rill development based on the physical ground was improved as follows as follows(Fig.5). At first, putting a seed on the bottom of a squared slope, and selecting a point at random in this frame. This point releases a particle (this particle is regarded as a raindrop) which strats to move in a random direction. But this particle moves toward the bottom of the slope (direction of the x -axis) more frequently than in other directions. and it is considered that the movement up the slope is zero. If a particle attach the old pattern, then the pattern grows continuously upwards. After repeating this process, the rill pattern grows.

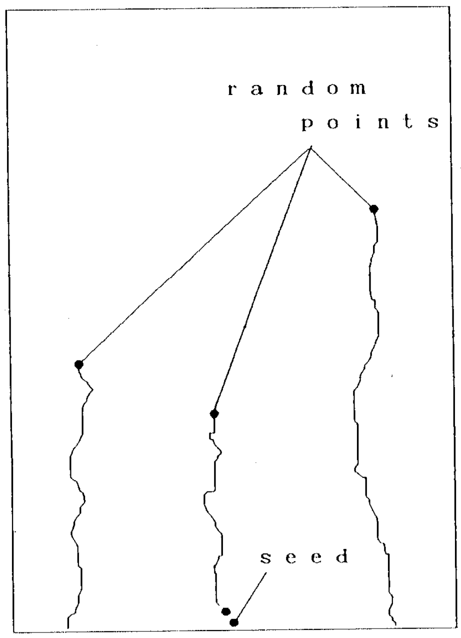


Fig. 5 An illustration of rill development model

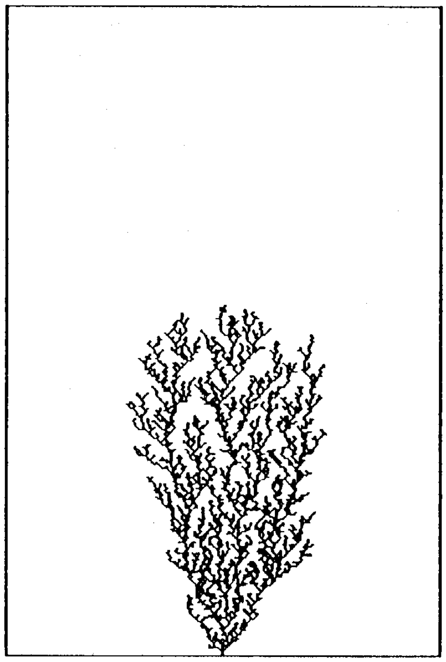


Fig. 6 The pattern of rill development model

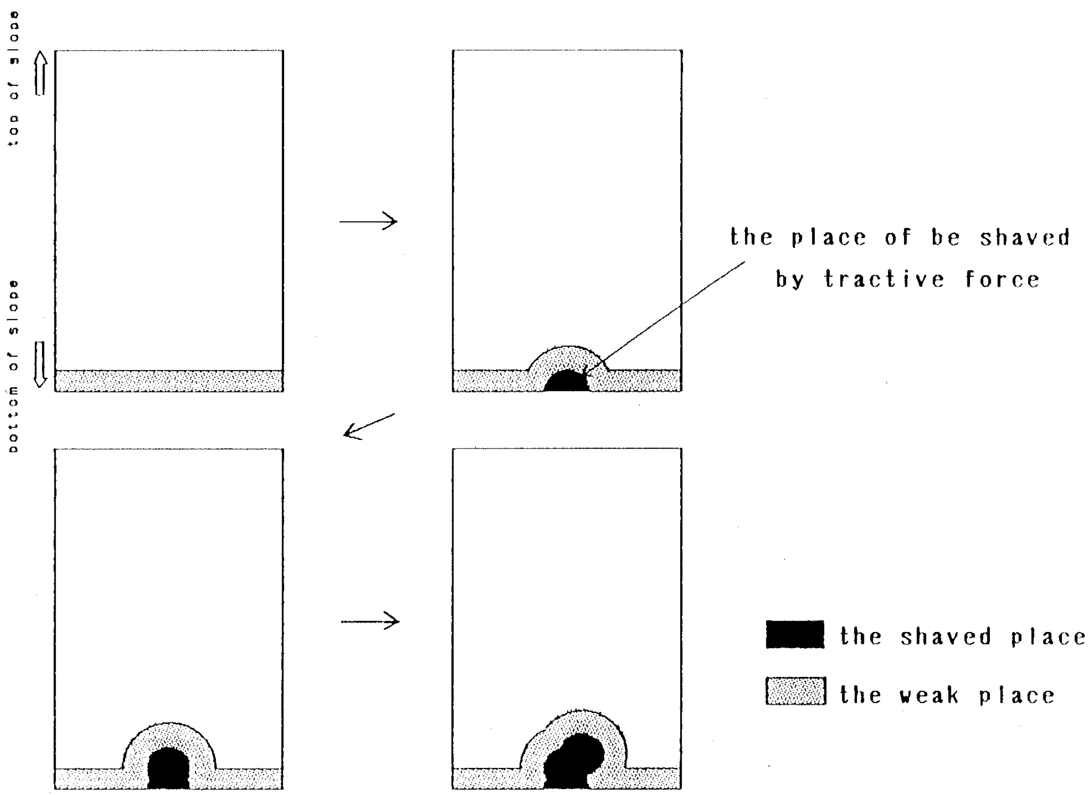


Fig. 7 A development process of rill patterns

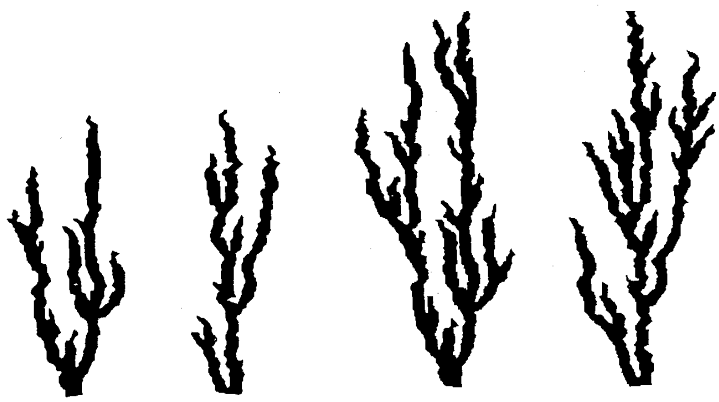


Fig. 8 Simulated patterns by the development model

Fig.6 shows the simulated rill pattern obtained by the method mentioned in 2.2. Comparing this pattern with the actual rill pattern developed on hillslopes, there still remains some problems in this model, namely ; (1) The number of bifurcations is too many. (2) The width of the channel is not expressed. (3) The destruction of the ground surface caused by raindrop impact is not considered. Thus, the model was modified ; A released particle is easier to attach to the tip of rill patterns than at other points of the rill pattern. The edge of the top of the channel is lowered more sharply than other parts of the edge of the channel, so, the soil near the tip of the rill is easier to collapse. This procedure, as showed in Fig.7, leads to a decrease in the number of bifurcations and the quantity of water flow that passes any point is assumed to be in proportion to the number of raindrops and the width of the channel is given proportionally to the amount of water flow. In addition, we suppose that when the width spreads to a certain length, the erosion spreads in a vertical direction, so, the plan patterns of the rill cease to change. Fig.8 show the examples of the simulated pattern of rills by this model.

The Numerization of Simulated Patterns

Through this simulation seems impossible to make identical patterns of actual rills, they are apparently similar patterns. But some quantitative comparisons of these patterns are necessary. So, we described the degree of complication of the actual and simulated rill patterns by applying the concept of fractal and spectrum.

1. The concept of Fractal

A fractal pattern means, in a broad sense, that it looks random ; but in a narrow sense, it has self-similarity ; moreover, in the narrowest sense. It means that the set of Hausdorff dimension is larger than the topological dimension. In the shape of fractal, there are a curved line, a curve surface and a discontinuous pattern. The

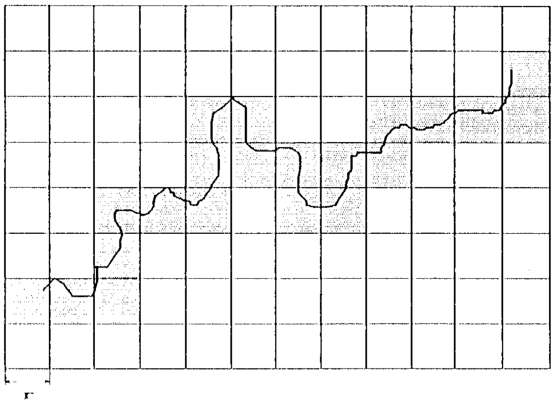


Fig. 9 Square method

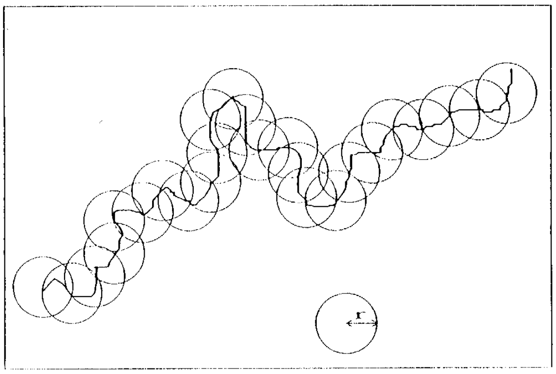


Fig. 10 Compass method

shape of fractal is characterized by a noninteger number, called a fractal dimension. In this section, as an index of complexity, the fractal dimension of patterns (D) was obtained. It is well known that a coastline is referred to as an example of a fractal. After taking a picture of the rill pattern developed on a slope, the fractal dimension of the pattern was obtained by the use of a computer program, and the fractal dimension (D) of the simulated one was also obtained by the same method. Though several methods have been used to obtain the fractal dimension, as is shown in Fig.9 and Fig.10 ; we obtained the fractal dimension by the compass method and the square method. These methods are used to obtain the D of the flow line of the rill and the periphery pattern of the rill, respectively. It is considered that the dimension (D) shows the degree of branching, the meandering of the channel and the area of rill channel. The fractal dimension can be obtained by the relation $D=\log N/\log (1/r)$, where N is the number of segments with size r, or the number of squares with a size r length.

In actual rill patterns, the range of the fractal dimension was from 1.03 to 1.23 (the slope is from degrees 12 to 38). On the other hand, Table 1 shows the results of Calculating the fractal dimensions D of simulated patterns. The fractal dimensions of the simulated patterns were within the range of values obtained in the actual rill.

Table 1 Farctal dimensions of simulated patterns

	square method		compass method
	periphery	whole	
A	1.51116	1.58411	1.11949
B	1.59483	1.60385	1.12862
C	1.64332	1.65020	1.16063
D	1.55102	1.55903	1.16296

Table 2 Entropy of simulation pattern

	Entropy
A	6.28356
B	6.50876
C	6.94575
D	7.12619

2. Power Spectrum and Entropy of Simulated Rill Patterns

As an analytical method for describing the degree of spatial fluctuation of figures, spectrum analysis has been used. In spectrum analysis, the characteristic of the power spectrum is described by entropy. After inputting the coordinates of the circumference of the actual rill by an input device(digitizer) into a presonal computer, the spectrum analysis was done and the power spectrum and the entropy of the rill patterns were obtained. The result of the analysis shows that the range of entropy is 4.0 to 7.5. On the other hand, Table.2 shows the values of entropy that were obtained from simulated patterns by the same method. These values are within the range of values calculated for the actual rill patterns.

The Result of Simulation

The change of rill patterns with time was simulated with the use of the concept of fractal geometry, and the quantification of the patterns was investigated by the use of fractal dimensions and entropy which were defined on the basis of spectrum analysis. Though this simulation was performed disregarding the physical mechanism of rill development, hydraulics and soil conditions, the simulated patterns are not only similar to the actual patterns, but also the fractal dimension and entropy of the simulated patterns are within that of the actual rill patterns. Accordingly, the development model of rill patterns that was used in this study could express the characteristics of rill patterns that develop on actual slopes.

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