A Note on Multihead On-Line Turing Machines

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Abstract

This paper introduces a tape-bounded multihead on-line Turing machine which can be considered as a multihead version of a tape-bounded 1-head on-line Turing machine.

We first investigate hierarchies based on the number of input heads. We then investigate the difference between the accepting powers of tape-bounded nondeterministic multihead on-line Turing machines and deterministic ones. Finally, closure properties of the classes of languages accepted by tape-bounded multihead on-line Turing machines are investigated.

1. Introduction

During the past ten years, many investigations about multihead one-way finite automata (MHFAs) have been made [1–5]. On the other hand, some properties on tape-bounded 1-head on-line Turing machines (1-HONTMs) are investigated in [6]. Here, the 1-head on-line Turing machine is the 1-head one-way finite automaton which has an infinite storage tape and a storage head.

This paper investigates some properties of an $L(n)$ tape-bounded multihead on-line Turing machine (MHONTM($L(n)$)), which can be considered as a multihead version of an $L(n)$ tape-bounded 1-head on-line Turing machine (1-HONTM($L(n)$)), where n is the length of an input word.

Section 2 gives terminologies and notations necessary for this paper. In Section 3, we give several properties of MHONTM($L(n)$)s for some $L(n)$ such that $\lim_{n\to\infty} [L(n)/n]=0$. In Section 3.1, we show that there exist hierarchies based on the number of input heads. Section 3.2 investigates the difference between the accepting powers of nondeterministic MHONTM($L(n)$)s and deterministic ones. Furthermore, in Section 3.3, we examine some closure properties of the classes of languages accepted by MHONTM($L(n)$)s.

It has been shown in [2] that there are hierarchies based on the number of input heads among MHFAs and that the class of languages accepted by deterministic MHFAs is included in that of nondeterministic ones. Furthermore, it has been shown in [5] that for each $k\geq 2$, the classes of languages accepted by deterministic k-head one-way finite automata are not closed under concatenation, reversal, and Kleene closure operations. Our main results are an extension of the results of these two papers.

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2. Preliminaries

We formally define a *nondeterministic k-head on-line Turing machine* (Nk-HONTM) M ($k \geq 1$) to be a 7-tuple $M=(k, Q, \Gamma, \Sigma, q_0, F, \delta)$, where

k is the number of input heads;
Q is a finite set of states;
Γ is a finite storage tape alphabet ($B \in \Gamma$ is the blank symbol);
Σ is a finite input alphabet ($\$ \notin \Sigma$ is the right endmarker);
$q_0 \in Q$ is the initial state;
$F \subseteq Q$ is the set of accepting states;
δ is the next move relation which maps from $Q \times (\Sigma \cup \{\$\})^k \times \Gamma$ into subsets of $Q \times \Gamma \times (\{\text{no move, right}\})^k \times \{\text{left, no move, right}\}$.

The machine M consists of a finite control, a read-only input tape with the right endmarker \$, a semi-infinite storage tape fulled blank initially, k read-only input heads which can move only right, and a read-write storage head which can move left or right. Initially, the k input heads and the storage head are positioned in the leftmost cell on an input tape and a storage tape, respectively.

A *step* of M consists of reading each symbol from each head on each tape, writing a symbol on the storage tape, moving the input heads and the storage head in specified directions, and entering a new state in accordance with the next move relation δ. Note also that the machine usually cannot write the blank symbol B.

A *sensing* Nk–HONTM is an Nk–HONTM whose input heads are allowed to sense the presence of other heads on the same input position. We denote a sensing Nk–HONTM by NSNk–HONTM.

Deterministic versions of multihead on-line Turing machines are defined as usual. In order to represent 'deterministic', we use 'D' in place of 'N', which is used to represent 'nondeterministic'. Thus, for example, Dk–HONTM (resp. DSNk–HONTM) denotes a deterministic k-head on-line Turing machine (resp. a deterministic sensing k-head on-line Turing machine).

Let $L:N \rightarrow R$ be a function with one variable n, where N and R denote the set of all positive integers and the set of all non-negative real numbers, respectively. We say that M is *$L(n)$ tape-bounded* if for each input word of length n (excluding the right endmarker \$), M uses at most $L(n)$ cells on its storage tape until M enters an accepting state.

By Nk–HONTM($L(n)$) (resp. NSNk–HONTM($L(n)$), Dk–HONTM($L(n)$), DSNk–HONTM ($L(n)$)) denote an $L(n)$ tape-bounded Nk–HONTM (resp. NSNk–HONTM, Dk–HONTM, DSNk–HONTM).

A nondeterministic multihead one-way finite automaton [3, 4] is a nondeterministic multihead on-line Turing machine whose storage tape has no space to be used, and various versions (e.g. deterministic or sensing versions) are defined similarly. We use 'FA' in place of 'ONTM' to represent 'one-way finite automaton'. For example, Nk–HFA (resp. DSNk–HFA) denotes a nondeterministic k-head one-way finite automaton (resp. a deterministic sensing k-head one-way finite automaton).

For a given above automaton M, let $T(M)$ be the set of input words accepted by M.

For the Nk–HONTM ($k \geq 1$), we denote the class of sets of words accepted by Nk–HONTMs by

$$\mathscr{L}[\mathrm{N}k\text{–HONTM}] = \{T \mid T = T(M) \text{ for some N}k\text{-HONTM } M\}.$$

For various versions of multihead on-line Turing machines and multihead one-way finite automata, those classes are defined similarly. For example, $\mathscr{L}$[Dk–HONTM $L(n)$)] (resp. $\mathscr{L}$[NSNk–HFA]) denotes the class of sets of words accepted by Dk–HONTM($L(n)$)s (resp. NSNk–HFAs).

3. Results

3.1 *Hierarchies based on the number of input heads*

This subsection shows that for each $X \in \{D, N\}$ and each $k \geq 1$, X(k+1)–HONTM($L(n)$)s (resp. XSN(k+1)–HONTM($L(n)$)s) are more powerful than Xk–HONTM($L(n)$)s (resp. XSNk–HONTM($L(n)$)s), if $\lim_{n\to\infty}[L(n)/n]=0$.

Lemma 1 For each $b \geq 1$, let

$$T_1(b) = \{w_1 * w_2 * \cdots * w_{2b} \mid (w_i \in \{0, 1\}^*) \ \& \ \forall i \ (1 \leq i \leq b) \ [w_i = w_{2b+1-i}]\}.$$

Then, for each $k \geq 1$ and each function $L: N \to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$,

(1) $T_1(k(k+1)/2) \in \mathscr{L}[\mathrm{D}(k+1)\text{–HFA}] = \mathscr{L}[\mathrm{D}(k+1)\text{–HONTM}(0)]$, and

(2) $T_1(k(k+1)/2) \notin \mathscr{L}[\mathrm{NSN}k\text{–HONTM}(L(n))]$.

Proof. (1): The proof is given in the proof of Theorem 1 in [2].
(2): Suppose that there is an NSNk–HONTM($L(n)$) M accepting $T_1(k(k+1)/2)$ for some $k \geq 1$ and some $L(n)$ such that $\lim_{n\to\infty}[L(n)/n]=0$. Let s and r be the number of states (of the finite control) and storage tape symbols of M, respectively.

For each $n \geq 1$, let

$$V(n) = \{w_1 * w_2 \cdots * w_{k(k+1)} \mid \forall i \ (1 \leq i \leq k(k+1)) \\ [(w_i \in \{0, 1\}^*) \ \& \ (|w_i|^{\ddagger} = n) \ \& \ (w_i = w_{k(k+1)+1-i})]\}.$$

Note that for each word x in $V(n)$, $|x| = (n+1)k(k+1) - 1$. Clearly, each word x in $V(n)$ is in $T_1(k(k+1)/2)$, and so x is accepted by M.

A configuration of M is a (k+3)-tuple $(i_1, i_2, \ldots, i_k, q, \alpha, j)$ where i_l ($1 \leq l \leq k$) is the l-th input head position, q is the state (of finite control), α is the non-empty contents

‡ For any word w, $|w|$ denotes the length of w.

of the storage tape, and j is the storage head position.

The *type* of a configuration $C=(i_1,..., i_k, q, \alpha, j)$ denoted by Type(C) is a k-tuple $(\lceil i_1/(n+1) \rceil,..., \lceil i_k/(n+1) \rceil)$‡. Note that the ith element h_i of the type specifies that the ith head of M is on $w_{h_i}*$ ($w_{k(k+1)}\$$ if $h_i=k(k+1)$).

Let $C_1(x)$, $C_2(x)$,..., $C_{l_x}(x)$ be the sequence of configurations of M during an (arbitrary selected) accepting computation of a word x in $V(n)$. Here l_x is the length of this computation. Let $d_1(x)$, $d_2(x)$,..., $d_{l'_x}(x)$ be the subsequence obtained by selecting $C_1(x)$ and all subsequent $C_i(x)$s such that Type $(C_i(x)) \neq$ Type $(C_{i+1}(x))$. We call $d_1(x)$, $d_2(x)$,..., $d_{l'_x}(x)$ the *pattern* of x.

Let $P(n)$ be the number of possible patterns of M on x in $V(n)$. Since $l'_x \leq k(k(k+1)-1)+1$, we get the following inequality,

$$P(n) \leq (s(N+1)^k L(N) r^{L(N)})^{k(k(k+1)-1)+1},$$

where $N=(n+1)k(k+1)-1$.

Then we classify the words in $V(n)$ according to their patterns. Naturally, there is a set $S(n)$ ($\subseteq V(n)$) such as $|S(n)| \geq 2^{nk(k+1)/2}/P(n)$.‡‡ We assume that each word x in $S(n)$ has the pattern $\hat{d}_1$, $\hat{d}_2$,..., $\hat{d}_{\hat{l}}$.

On the other hand, as is shown in the proof of Theorem 3 in [1], for each word in $V(n)$ there must be an i, such that M cannot read w_i* and $w_{k(k+1)+1-i}*$($w_{k(k+1)}\$$ if $i=1$) simultaneously, which is decided by the pattern of the computation. Thus, let i_0 be such a value of i for the pattern $\hat{d}_1$,..., $\hat{d}_{\hat{l}}$.

We now define a binary relation E on words in $S(n)$ as follows. Let

$$u=u_1*u_2*\cdots*u_{i_0}*\cdots*u_{k(k+1)+1-i_0}*\cdots*u_{k(k+1)}, \text{ and}$$

$$v=v_1*v_2*\cdots*v_{i_0}*\cdots*v_{k(k+1)+1-i_0}*\cdots*v_{k(k+1)}.$$

Then,

$$uEv \Leftrightarrow {}^{\forall}i(\notin \{i_0, k(k+1)+1-i_0\})[u_i=v_i].$$

Obviously the relation E is an equivalence relation, and there are at most $q(n)=2^{n(k(k+1)-1)/2}$ E-equivalence classes of words in $S(n)$.

Since $\lim_{n\to\infty}[L(n)/n]=0$, it follows for large n that $|S(n)|>q(n)$. Therefore, there exist two different words in $S(n)$ which belong to the same equivalence class.

Let

$$x=x_1*x_2*\cdots*x_{i_0}*\cdots*x_{k(k+1)+1-i_0}*\cdots*x_{k(k+1)}, \text{ and}$$

$$y=y_1*y_2*\cdots*y_{i_0}*\cdots*y_{k(k+1)+1-i_0}*\cdots*y_{k(k+1)},$$

be such words in $S(n)$. Since x and y are in $S(n)$, recall that for each $i \notin \{i_0, k(k+1)+1-i_0\}$, $x_i=y_i$. And let

$$z=z_1*z_2*\cdots*z_{k(k+1)},$$

‡ For any real number r, $\lceil r \rceil$ denotes the smallest integer greater than or equal to r.

‡‡ For any set S, $|S|$ denotes the number of elements of S.

$$=x_1*\cdots*x_{i_0}*\cdots*x_{k(k+1)-i_0}*y_{k(k+1)+1-i_0}*x_{k(k+1)+2-i_0}*\cdots*x_{k(k+1)}.$$

By a similar argument to that in the proof of Theorem 1 in [2], it can be shown that there is an accepting computation of M on z. Consequently, z must be accepted by M. This contradicts the fact that z is not in $T_1(k(k+1)/2)$. ■

From Lemma 1, we can get immediately the following theorem.

Theorem 1 For each $k \geq 1$, each $X \in \{D, N\}$, and each function L: $N \to R$ such that $\lim_{n\to\infty} [L(n)/n]=0$,

(1) $\mathscr{L}$[Xk–HONTM($L(n)$)] $\subsetneq$ $\mathscr{L}$[X($k+1$)–HONTM($L(n)$)], and

(2) $\mathscr{L}$[XSNk–HONTM($L(n)$)] $\subsetneq$ $\mathscr{L}$[XSN($k+1$)–HONTM($L(n)$)].

3.2 *Determinism and nondeterminism*

This subsection shows that for each (sensing) MHONTM($L(n)$), nondeterministic version is more powerful than that of deterministic version, if $\lim_{n\to\infty} [L(n)/n]=0$.

Lemma 2 Let

$$T_2=\{w_1*w_2*\cdots*w_{2b} \mid \exists b \geq 1[\forall i\ (1 \leq i \leq 2b)[w_i \in \{\{0, 1\}^* ¢\ \{0, 1\}^*\}]\ \&$$
$$\exists j, k[(w_j=x ¢ y)\ \&\ (w_k=x ¢ z)\ \&\ (y \neq z)]]\}.$$

Then for each function L: $N \to R$ such that $\lim_{n\to\infty} [L(n)/n]=0$,

(1) $T_2 \in \mathscr{L}$[N2–HFA] $=\mathscr{L}$[N2–HONTM(0)], and

(2) $T_2 \notin \bigcup_{1 \leq k < \infty} \mathscr{L}$[DSN$k$–HONTM($L(n)$)].

Proof. (1): The proof is given in the proof of Theorem 4 in [2].

(2): By using similar techniques to those in the proof of Lemma 1, we can easily show that T_2 is not in $\bigcup_{1 \leq k < \infty} \mathscr{L}$[DSN$k$–HONTM($L(n)$)]. The details are omitted here. ■

From Lemma 2, we can get the following theorem.

Theorem 2 For each $k \geq 2$, and each function L: $N \to R$ such that $\lim_{n\to\infty} [L(n)/n]=0$,

(1) $\mathscr{L}$[Dk–HONTM($L(n)$)] $\subsetneq$ $\mathscr{L}$[Nk–HONTM($L(n)$)],

(2) $\mathscr{L}$[DSNk–HONTM($L(n)$)] $\subsetneq$ $\mathscr{L}$[NSNk–HONTM($L(n)$)],

(3) $\bigcup_{1 \leq r < \infty} \mathscr{L}$[D$r$–HONTM($L(n)$)] $\subsetneq$ $\bigcup_{1 \leq r < \infty} \mathscr{L}$[N$r$–HONTM($L(n)$)], and

(4) $\bigcup_{1 \leq r < \infty} \mathscr{L}$[DSN$r$–HONTM($L(n)$)] $\subsetneq$ $\bigcup_{1 \leq r < \infty} \mathscr{L}$[NSN$r$–HONTM($L(n)$)].

3.3 *Closure properties*

In this subsection, we will investigate several closure properties of classes of languages accepted by deterministic and nondeterministic (sensing) MHONTM($L(n)$).

We first examine closure properties for the deterministic case.

Lemma 3 For each $b \geq 1$ and each $n \geq 1$, let

$C_b(n) = \{ucw_1cw_2c\cdots cw_bcw_bc\cdots cw_2cw_1 \mid {}^{\forall}i(1 \leq i \leq b)\ [u, w_i \in \{0, 1\}^n]\}$,

$D_b(n) = \{v_1dv_1d \mid v_1 \in C_b(n)\}$,

$E_b(n) = \{ucw_1cw_2c\cdots cw_bcw_{b+1}c\cdots cw_{2b-1}cw_{2b} \mid {}^{\forall}i(1 \leq i \leq 2b)$

$[u, w_i \in \{0, 1\}^n]\ \&\ {}^{\exists}j(1 \leq j \leq b)\ [w_j \neq w_{2b+1-j}]\}$, and

$F_b(n) = \{u_1cw_1cw_2c\cdots cw_bcw_bc\cdots cw_2cw_1du_2cw_1cw_2c\cdots cw_bcw_bc\cdots cw_2cw_1d \mid {}^{\forall}i\ (1 \leq i \leq b)$

$[u_1, u_2, w_i \in \{0, 1\}^n]\ \&\ (u_1 \neq u_2)\}$.

In addition, let $C_b = \bigcup_{1 \leq n < \infty} C_b(n)$, $D_b = \bigcup_{1 \leq n < \infty} D_b(n)$, $E_b = \bigcup_{1 \leq n < \infty} E_b(n)$, and $F_b = \bigcup_{1 \leq n < \infty} F_b(n)$, for each $b \geq 1$.

Furthermore, for each $k \geq 2$, let $b(k) = \binom{k}{2}$ and let $A_1(k)$ be a language satisfying the following conditions:

(1) $A_1(k) \supseteq C_{b(k)} \cup D_{b(k)}$; and

(2) $A_1(k) \cap (E_{b(k)} \cup F_{b(k)}) = \phi$.

Then, for each $k \geq 2$ and each function $L: N \to R$ such that $\lim_{n \to \infty} [L(n)/n] = 0$,

$$A_1(k) \notin \mathscr{L}[\text{DSN}k\text{–HONTM}(L(n))].$$

Proof. Suppose that there is a DSNk–HONTM$(L(n))$ M accepting $A_1(k)$ for some $k \geq 2$ and some $L(n)$ such that $\lim_{n \to \infty} [L(n)/n] = 0$. We now show that if M accepts all words in $C_{b(k)} \cup D_{b(k)}$, where $b(k) = \binom{k}{2}$, M must also accept some word in $E_{b(k)} \cup F_{b(k)}$.

Let s and r be the number of the states and the storage tape symbols of M, respectively. Similar to the proof of Lemma 1, we again define the configuration, the type, and the pattern of M. Let y be a word in $C_{b(k)}(n) \cup D_{b(k)}(n)$ and let y have an initial subword v_1 in $C_{b(k)}(n)$. Clearly, $|C_{b(k)}(n)| = 2^{[b(k)+1]n}$.

We shall consider the initial computation of the word y, which begins in the initial configuration and ends in the configuration in which one of the heads reads through the whole word v_1. Let $P(n)$ be the number of possible patterns of initial computation. Then we get the following inequality,

$$P(n) \leq (s(N+1)^k L(N') r^{L(N')})^{2kb(k)+1},$$

where $N = (k(k-1)+1)(n+1)-1$, and $N' = 2(N+1)$. Thus, for some pattern q, there is at least $2^{|C_{b(k)}(n)|}/P(n)$ different words with the pattern q in $C_{b(k)}(n)$.

Let $(t_1, t_2, \ldots, t_k)$ $({}^{\forall}j(1 \leq j \leq k)\ [1 \leq t_j \leq b(k)+1])$ be the type of the last configuration of M. Then we consider the following two cases:

(1) $\forall j\ (1 \leq j \leq k)$: $t_j > 1$; and

(2) $\exists j\ (1 \leq j \leq k)$: $t_j = 1$.

In case (1), we consider words in $D_{b(k)}(n)$. Since $\lim_{n\to\infty}[L(n)/n]=0$, there exists two different words y_1 and y_2 with the pattern q in $D_{b(k)}(n)$ for large n. And let

$$y_1 = u_1cxdu_1cxd, \text{ and}$$

$$y_2 = u_2cxdu_2cxd,$$

where u_1cx, $u_2cx \in C_{b(k)}(n)$. Since $\forall j\ (1 \leq j \leq k)$: $t_j > 1$, M cannot read the both subwords u_1 (resp. u_2) in y_1 (resp. y_2) with the pattern q simultaneously. Then M accepts a word $y' = u_1cxdu_2cxd$ which is not in $D_{b(k)}(n)$ but in $F_{b(k)}(n)$.

In case (2), we consider words in $C_{b(k)}(n)$. Similar to the proof of Lemma 1, let y_1 and y_2 be two different words with the pattern q in $C_{b(k)}(n)$ for large n as follows,

$$y_1 = ucw_1c\cdots cw_{i_0}c\cdots cw_{b(k)}cw_{b(k)}c\cdots cw_{i_0}c\cdots cw_1, \quad \text{and}$$

$$y_2 = ucw_1c\cdots cw'_{i_0}c\cdots cw_{b(k)}cw_{b(k)}c\cdots cw'_{i_0}c\cdots cw_1.$$

Then, some subword w_{i_0} and w'_{i_0} cannot be read simultaneously by M.

Furthermore, let

$$y' = ucw_1c\cdots cw_{i_0}c\cdots cw_{b(k)}cw_{b(k)}c\cdots cw'_{i_0}c\cdots cw_1.$$

Clearly, M accepts y' which is not in $C_{b(k)}(n)$ but in $E_{b(k)}(n)$.

This completes the proof of the lemma. ■

Lemma 4 For each $k \geq 2$, let $b(k) = \binom{k}{2}$. And let $A_2(k)$ be a language satisfying the following conditions,

(3) $A_2(k) \supseteq \{e\} \cdot C_{b(k)} \cup \{e\} \cdot D_{b(k)}$, and

(4) $A_2(k) \cap (\{e\} \cdot E_{b(k)} \cup \{e\} \cdot F_{b(k)}) = \phi$,

where $C_{b(k)}$, $D_{b(k)}$, $E_{b(k)}$, and $F_{b(k)}$ are the languages given in Lemma 3. Then, for each $k \geq 2$ and each function L: $N \to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$,

$$A_2(k) \notin \mathscr{L}[\text{DSN}k\text{–HONTM}(L(n))].$$

Proof. It is a matter of easy technical considerations to show that if there exists a language $A_2(k)$ satisfying the conditions of Lemma 1 such that $A_2(k) \in \mathscr{L}$[DSNk-HONTM($L(n)$)] for some $k \geq 2$ and some $L(n)$ such that $\lim_{n\to\infty}[L(n)/n]=0$, then there would exist a language $A_1(k)$ fulfilling the conditions of Lemma 3 such that $A_1(k) \in \mathscr{L}$[DSNk-HONTM($L(n)$)]. This contradicts Lemma 3. ■

Theorem 3 For each $k \geq 2$ and each function L: $N \to R$ such that $\lim_{n\to\infty}[L(n)/n]=0$, neither $\mathscr{L}$[DSNk–HONTM($L(n)$)] nor $\mathscr{L}$[Dk–HONTM($L(n)$)] are closed under the following operations: (1) concatenation "·", (2) reversal "R", and (3) Kleene closure "*".

Proof (1): This proof is similar to that of Theorem 2 in [5]. Let

$$L_1=\{0,1\}^*c\cup\{\varepsilon\}^{\ddagger}, \quad \text{and}$$

$$L_2=\{udud \mid u\in\{0,1,c\}^*\}\cup\{\varepsilon\}.$$

And for each $b\geq 1$, let

$$G_b=\{w_1cw_2c\cdots cw_bcw_bc\cdots cw_2cw_1 \mid {}^{\forall}i\ (1\leq i\leq b)[w_i\in\{0,1\}^*]\}\cup\{\varepsilon\}.$$

Clearly,

$$L_1\in\mathscr{L}[\text{D1–HFA}]=\mathscr{L}[\text{D1–HONTM}(0)], \quad \text{and}$$

$$L_2\in\mathscr{L}[\text{D2–HFA}]=\mathscr{L}[\text{D2–HONTM}(0)].$$

It is also clear for each $k\geq 2$ that

$$G_{\binom{k}{2}}\in\mathscr{L}[\text{D}k\text{–HFA}]=\mathscr{L}[\text{D}k\text{–HONTM}(0)].$$

Furthermore, let

$$L_3(k)=L_1\cdot L_2\cdot G_{\binom{k}{2}},$$

then it is easily seen that $L_3(k)\supseteq C_{b(k)}\cup D_{b(k)}$ and $L_3(k)\cap(E_{b(k)}\cup F_{b(k)})=\phi$, where $C_{b(k)}$, $D_{b(k)}$, $E_{b(k)}$, and $F_{b(k)}$ are the languages given in Lemma 3.

Therefore, from Lemma 3, we can get

$$L_3(k)\notin\mathscr{L}[\text{DSN}k\text{–HONTM}(L(n))].$$

(2): This proof is similar to that of Theorem 3 in [5].

For each $k\geq 2$, let

$$L_4(k)=L_2\cup\{0,1\}^*cG_{\binom{k}{2}},$$

then it is easily seen that $L_4(k)\supseteq C_{b(k)}\cup D_{b(k)}$ and $L_4(k)\cap(E_{b(k)}\cup F_{b(k)})=\phi$, where $C_{b(k)}$, $D_{b(k)}$, $E_{b(k)}$, and $F_{b(k)}$ are the languages given in Lemma 3.

Therefore, from Lemma 3, we can get

$$L_4(k)\notin\mathscr{L}[\text{DSN}k\text{–HONTM}(L(n))].$$

On the other hand, as is easily seen that $L_4(k)^R=L_2^R\cup G_{\binom{k}{2}}c\{0,1\}^*=\mathscr{L}[\text{D}k\text{–HFA}]=\mathscr{L}[\text{D}k\text{–HONTM}(0)]$.

(3): This proof is similar to that of Theorem 4 in [5].

For each $k\geq 2$, let

$$L_5(k)=\{e\}\cdot L_2\cup\{0,1\}^*cG_{\binom{k}{2}}\cup\{e\}.$$

‡ ε denotes the empty string.

Then clearly

$$L_5(k) \in \mathscr{L}[\text{D}k\text{–HFA}] = \mathscr{L}[\text{D}k\text{–HONTM}(0)].$$

On the other hand, it is also clear that $L_5(k)^* \supseteq \{e\} \cdot C_{b(k)} \cup \{e\} \cdot D_{b(k)}$ and $L_5(k)^* \cap (\{e\} \cdot E_{b(k)} \cup \{e\} \cdot F_{b(k)}) = \phi$, where $C_{b(k)}$, $D_{b(k)}$, $E_{b(k)}$, and $F_{b(k)}$ are the languages given in Lemma 3.

Therefore, from Lemma 4, we can get

$$L_5(k)^* \notin \mathscr{L}[\text{DSN}k\text{–HONTM}(L(n))].$$

This completes the proof of the theorem. ■

We next examine closure properties for the nondeterministic case.

Theorem 4 For each $k \geq 2$ and each function $L: N \to R$ such that $\lim_{n\to\infty} [L(n)/n] = 0$, neither $\mathscr{L}[\text{NSN}k\text{–HONTM}(L(n))]$ nor $\mathscr{L}[\text{N}k\text{–HONTM}(L(n))]$ are closed under the following operations: (1) intersection, and (2) complementation.

Proof. (1): For each $k \geq 2$, and each $r \left(1 \leq r \leq \binom{k+1}{2}\right)$, let

$$T_3(k, r) = \{w_1 * w_2 * \cdots * w_{2\binom{k+1}{2}} \mid \forall i \left(1 \leq i \leq 2\binom{k+1}{2}\right)[w_i \in \{0, 1\}^*] \ \& \ (w_r = w_{2\binom{k+1}{2}+1-r}\}.$$

For an arbitrary fixed k and each $r \left(1 \leq r \leq \binom{k+1}{2}\right)$, it is obvious that $T_3(k, r)$ is accepted by D2–HFA (=D2–HONTM(0)).

From Lemma 1, however, the following language is not accepted by any NSNk–HONTM($L(n)$):

$$T_3(k, 1) \cap T_3(k, 2) \cap \cdots \cap T_3\left(k, \binom{k+1}{2}\right) = T_1\left(\binom{k+1}{2}\right).$$

(2): From Lemma 1, for each $k \geq 1$,

$$T_1(k(k+2)/2) \notin \mathscr{L}[\text{NSN}k\text{–HONTM}(L(n))],$$

where $T_1(k(k+2)/2)$ is the language given in Lemma 1.

From Theorem 2 in [2], however, we can get

$$\overline{T}_1(k(k+2)/2)^{\ddagger} \in \mathscr{L}[\text{N2–HFA}] = \mathscr{L}[\text{N2–HONTM}(0)].$$

This completes the proof of the theorem. ■

4. Conclusion

In addition to the above results, we have got several properties about the classes of

‡ For some language L, $\bar{L}$ denotes the complementation of L.

the languages accepted by tape-bounded simple multihead on-line Turing machines (SPMHONTM($L(n)$)s). An SPMHONTM($L(n)$) is an MHONTM($L(n)$) whose only one input head is able to distinguish the symbols in the input alphabet, and whose other input heads can only detect whether they are on the right endmarker \$ or on a symbol in the input alphabet.

We conclude this paper by stating a following open problem left in this paper.

For each $k \geq 2$, each $X \in \{N, D\}$, and each $L: N \to R$ such that $\lim_{n \to \infty} [L(n)/\log n] = 0$,

$$\mathscr{L}[Xk\text{–HONTM}(L(n))] = \mathscr{L}[Xk\text{–HONTM}(0)]\,?$$

References

1) A .L. Rosenberg: "On multihead finite automata", IBM J. Res. and Dev. **10**, 388–394 (1966).
2) A. C. Yao and R. L. Rivest: "$k+1$ heads are better than k", J. ACM. **25**, No. 2, 337–340 (1978).
3) I. H. Sudborough: "One-way multihead writing finite automata", Information and Control, **30**, 1–20 (1976).
4) O. H. Ibarra, S. K. Sahni and C. E. Kim: "Finite automata with multiplication", Theort. Comput. **2**, 271–294 (1976).
5) J. Hromkovic: "One-way multihead deterministic finite automata", Acta Inform. **19**, 377–384. (1983).
6) J. E. Hopcroft and J. D. Ullman: "Some results on tape-bounded Turing machines", J. ACM. **16**, No. 1, 168–177 (1969).