Multiple Correlation of a Two-dimensional Turbulent Wake

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Abstract

Measurements of higher order products, that is multiple correlation, were made in the fully developed two-dimensional wake produced by a circular cylinder. It was found that the profile of the triple and fourth order moments of fluctuating velocity components were attained in the self-preservation beyond $x/d=211.0$. The skewness and flatness factors across the section were also presented. The turbulent kinetic energy budget for the present measurement doesn't correspond so well with Townsend's work.

1. Introduction

The two-dimensional turbulent wake have been considered to be one of the simple self-preserving flows. The turbulent structure of the wake flow was investigated by Townsend[1] in 1949. None of the works followed were sufficient compared with Townsend's work, because they were defined to measurement of the mean flow structure and weren't connected with the turbulent structure. Up to date, therefore, all the results of wakes reviewed by Hinze[2] and Tennekes & Lumley[3] and so on have been based on Townsend's experiment. The monograph by Tani[4], however, points out that Townsend's measurements lead to a strong suspicion because of the unbalance of the turbulent kinetic energy budget.

There exists our great interest how long distance the two-dimensional turbulent wake reaches equilibrium. As to this, Tennekes & Lumly state that mean velocity profiles beyond $x=80d$ and turbulent intensities and Reynolds shear stress beyond $x=200d$ exhibit the self-preservation, and furthermore the higher the order of velocity moment is, the longer it takes to reach self-preservation. On the other hand, Townsend explains based on his experiment that the turbulent wake flow reaches equilibrium at about 500 cylinder diameters.

As seen from the mentioned above, it is not known at all how long distance the high order velocity moments reach self-preservation except for the second order moments. In addition, there exist few experimental data with reliability which have been reported with respect to the energy budget[5] in detail. These are due to the failure of the experimental accuracy as to the diffusion and dissipation terms. For instance, measurement of the pressure-velocity correlation in the diffusion term is extremely difficult and the value of time derivative associated with the dissipation

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term depends on the cut-off frequency of a low pass filter.

Firstly, this work aims to measure the higher order correlation and to reveal the process approaching the self-preserving state and the fully developed profiles. Next, it is tried to confirm the turbulent kinetic energy balance without the assumption as much as possible.

Nomenclatures used in this paper are the same as previous paper[6].

2. Description of experiment

Under the constant Reynolds number ($Rd=4000$), the experiment was carried out by use of x-type sensor with the hot wire anemometer. Refer to our previous paper[6] with respect to another experimental details. In order to obtain the triple and fourth velocity correlations, the voltage signal e_1, e_2 and e_3 related to the velocity fluctuation u', v' and w' respectively are led to the multiplier circuit as shown in Figure 1. All the conventional average of the output signals from the multiplier circuit is done by means of the integrator. The derivative signal of the each velocity fluctuation component is obtained by using the differentiating circuit which has a low pass filter with the cut-off frequency of about 4 KHz.

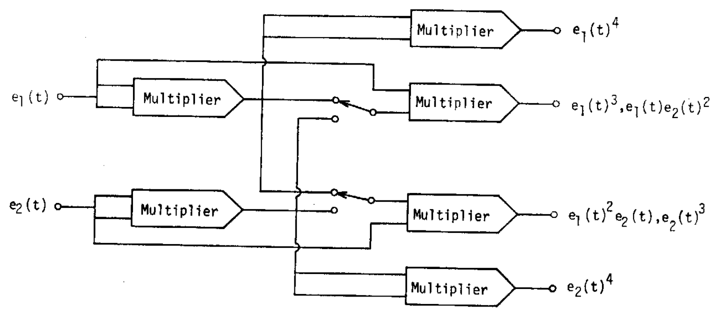

Fig. 1 Principle diagram of multiplier circuit.

3. Results and discussion

3.1 Fourth velocity correlation

In Figures 2–4 are shown the fourth velocity correlations, which should emphasize the intensity of the velocity fluctuation. All the fourth order velocity products, which

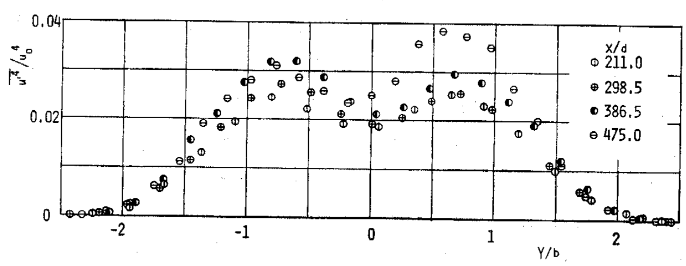

Fig. 2 The longitudinal fourth velocity correlation.

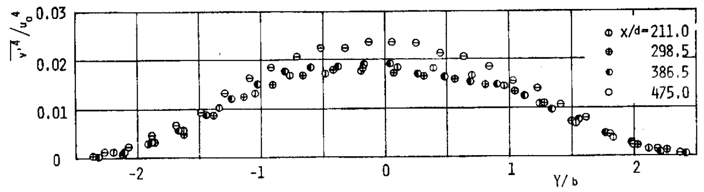

Fig. 3 The lateral fourth velocity correlation.

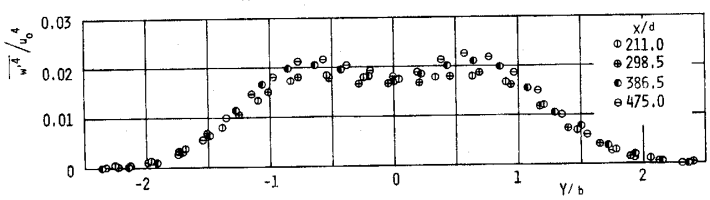

Fig. 4 The transverse fourth velocity correlation.

are normalized by use of the maximum velocity defect u_0 and half width b, are almost similar in the range $211 \leqq x/d \leqq 475$ within admissible scatter. The profiles of $\overline{u'^4}/u_0^4$ and $\overline{w'^4}/u_0^4$ shows the peak at about $y/b=0.7$ as the turbulent intensity does, and the value of $\overline{u'^4}/u_0^4$ is considerably large than others across the cross section of wake. The magnitude of $\overline{v'^4}/u_0^4$ and $\overline{w'^4}/u_0^4$ are closer compared with that of $\overline{u'^4}/u_0^4$ in spite of the difference of the profile shape.

Figures 5–7 show the flatness factors of the velocity fluctuation defined as follow respectively:

$$\overline{u'^4}/(\overline{u'^2})^2, \quad \overline{v'^4}/(\overline{v'^2})^2, \quad \overline{w'^4}/(\overline{w'^2})^2 .$$

It is well known that the flatness factor represents the extent of the skirt of a probability density profile of the fluctuation, and that its value becomes 3 if a probability density follows the Gaussian profile. The results obtained indicates that all the flatness factors are approximately similar in the range $211 \leqq x/d \leqq 475$ and much close to 3 in the range $-1 < y/b < 1$ within the experimental accuracy.

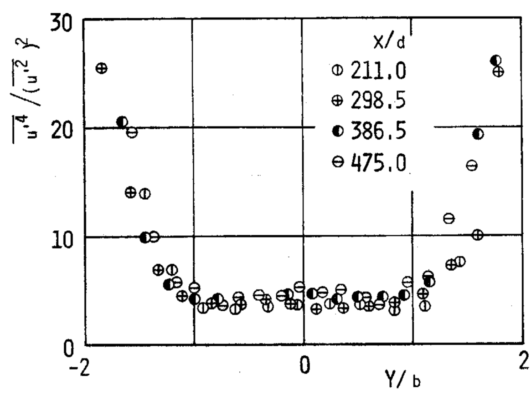

Fig. 5 The flatness factor of longitudinal velocity u'.

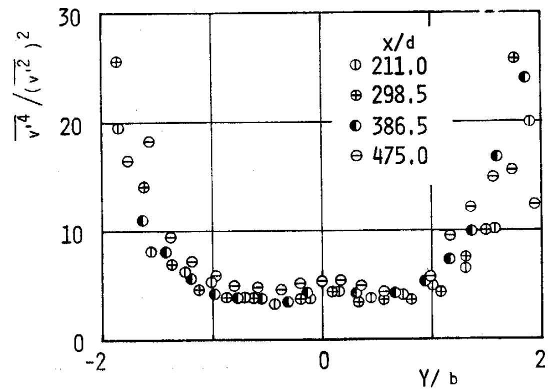

Fig. 6 The flatness factor of lateral velocity v'.

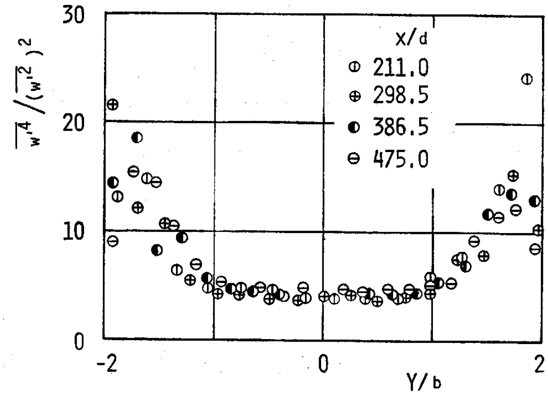

Fig. 7 The flatness factor of transverse velocity w'.

3.2 Triple velocity correlation

The triple velocity correlation which is the third order moment of the fluctuation is useful to estimate the diffusion term in the turbulent kinetic energy equation. The principal triple velocity correlations

$$\frac{\overline{u'^3}}{u_0^3},\quad \frac{\overline{v'^2u'}}{u_0^3},\quad \frac{\overline{w'^2u'}}{u_0^3},\quad \frac{\overline{u'^2v'}}{u_0^3},\quad \frac{\overline{v'^3}}{u_0^3},\quad \frac{\overline{u'^2w'}}{u_0^3}\quad \text{and}\quad \frac{\overline{w'^3}}{u_0^3}$$

are shown in Figures 8–14 respectively. The results were consistently repeatable. All of the seven profiles indicate to be similar beyond $x/d=211$.

With regard to the longitudinal transport rate by the fluctuation component u' of the turbulent energy $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{w'^2}$, it is seen that these profiles are axisymmetric by their definition respectively as shown in Figures 8–10. They exhibit the same tenden-

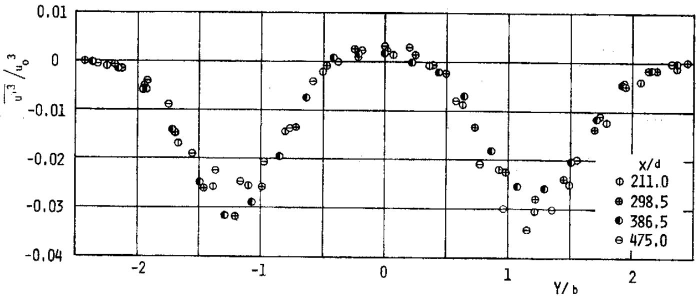

Fig. 8 The longitudinal transport rate of turbulent energy $\overline{u'^2}$.

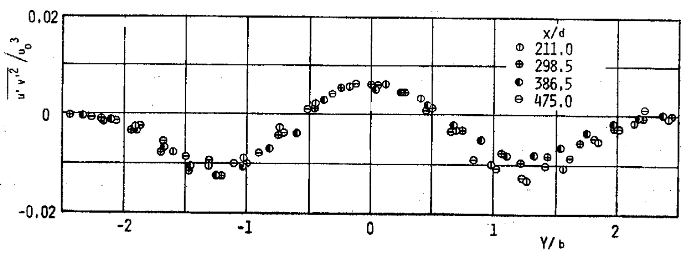

Fig. 9 The longitudinal transport rate of turbulent energy $\overline{v'^2}$.

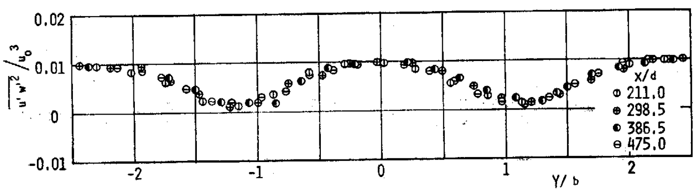

Fig. 10 The longitudinal transport rate of turbulent energy $\overline{w'^2}$.

cies, that is, having the maximum value at the center of the wake and the minimum at the location about $y/b=1.2$. If discussed each term in detail, the value of $\overline{u'^3}/u_0^3$ is slightly positive near the center region of the wake and negative in the other region and has the minimum negative value at the location $y/b=1.2$. The profile of $\overline{v'^2u'}/u_0^3$ is positive in the region smaller than about $y/b=0.5$ and have negative values which are not so large compared with $\overline{u'^3}/u_0^3$ in the other region. The value of $\overline{w'^2u'}/u_0^3$ is positive entirely across the wake and becomes maximum both at the wake center and at the free stream region.

As to the lateral transport rates by the fluctuation v' of $\overline{u'^2}$ and $\overline{v'^2}$, it is seen from the Figures 11–12 that the profile of $\overline{u'^2v'}/u_0^3$ has negative part in the region smaller than about $y/b=0.6$, which means an inward transport to the center of the wake and that the value of $\overline{v'^3}/u_0^3$ is positive all over the cross section, which means the outward diffusion of the turbulent energy out of the wake center. Since the contribution to the rate of change of the turbulent intensity at a given point is represented by means of the y-derivative of that quantities, the profile of $\overline{u'^2v'}/u_0^3$ and $\overline{v'^3}/u_0^3$ should be closer to the gradient of the fourth order $\overline{u'^4}$ and $\overline{v'^4}$ profiles shown in Figures 2–4 corresponding to the intensity profiles. Therefore the profile curves in Figures 11–12 accords almost with the course of the fourth order profile gradient. The regions of inward transport

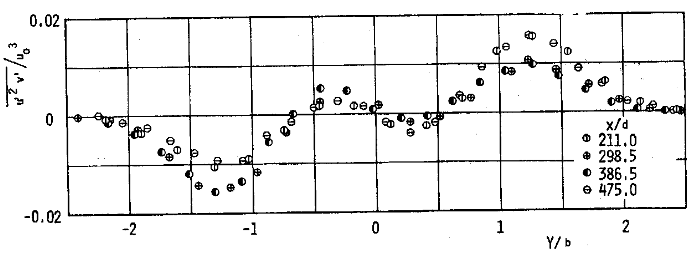

Fig. 11 The lateral transport rate of turbulent energy $\overline{u'^2}$.

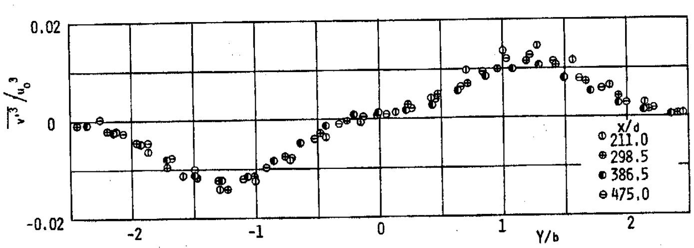

Fig. 12 The lateral transport rate of turbulent energy $\overline{v'^2}$.

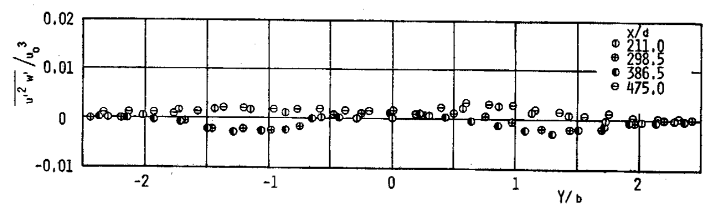

Fig. 13 The transverse transport rate of turbulent energy $\overline{u'^2}$.

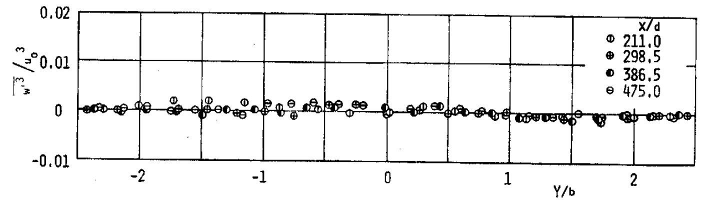

Fig. 14 The transverse transport rate of turbulent energy $\overline{w'^2}$.

and of positive intensity gradients near the center, however, do not entirely overlap as Hinze[2] illustrates.

With the transverse transport rates by the fluctuation w' of the $\overline{u'^2}$ and $\overline{w'^2}$, it is seen from the Figures 13–14 that in the two-dimensional wake there does not exist the transverse transport by w' of turbulent energy.

In the following, in Figures 15–17 are shown the skewness factors $\overline{u'^3}/(\overline{u'^2})^{3/2}$, $\overline{v'^3}/(\overline{v'^2})^{3/2}$ and $\overline{w'^3}/(\overline{w'^2})^{3/2}$ respectively, which mean the degree of symmetry of the turbulent fluctuating signal. The skewness factors of u', v' and w' show the inherent tendency respectively, and they seem to accord with the course of the third-order moments $\overline{u'^3}/u_0^3$, $\overline{v'^3}/u_0^3$ and $\overline{w'^3}/u_0^3$ in spite of with scatter in the outer edge of wake.

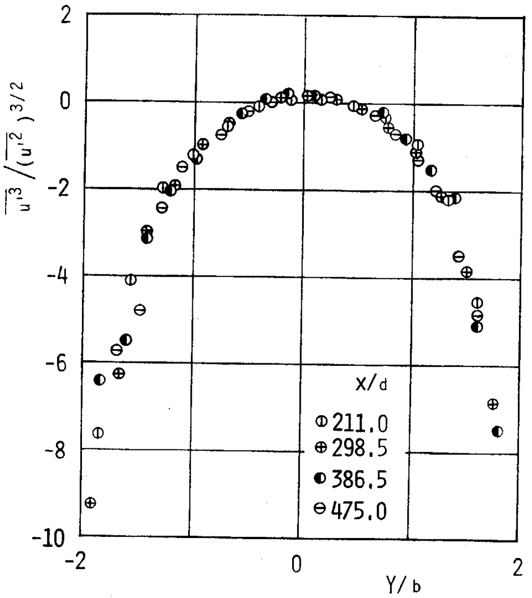

Fig. 15 The skewness factor of longitudinal velocity u'.

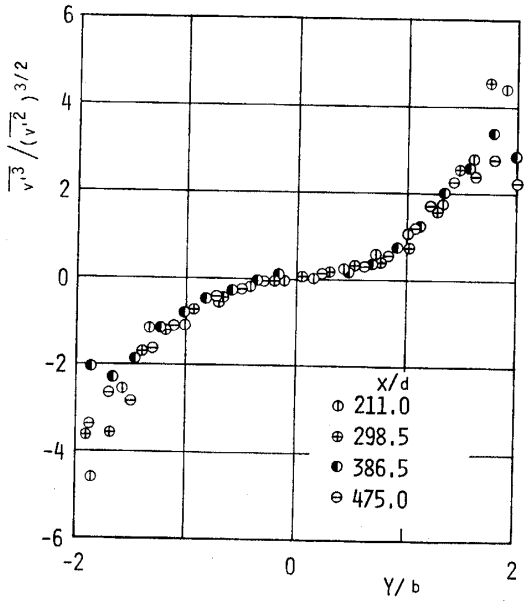

Fig. 16 The skewness factor of lateral velocity v'.

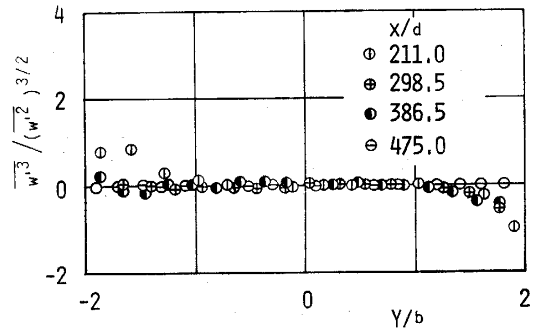

Fig. 17 The skewness factor of transverse velocity w'.

The value of $\overline{u'^3}/(\overline{u'^2})^{3/2}$ decreases rapidly toward the outer edge from the slightly positive value in the center, while the value of $\overline{v'^3}/(\overline{v'^2})^{3/2}$ increases comparatively slowly toward the outer edge. On the other hand, the skewness factor $\overline{w'^3}/(\overline{w'^2})^{3/2}$ obtained indicates no distortion of a probability density of the w' signal.

3.3 Behaviour of the individual components of dissipation term

Three out of the nine derivatives composing the dissipation term are estimated by means of following method. A fluctuating signal is led to a low pass filter of 4 KHz, and followed to a differentiator. The time-derivatives are transformed to the spatial-derivatives by making use of Taylor's hypothesis. As the influences of the low pass filter for measured value have not been ascertained, any correction is not made.

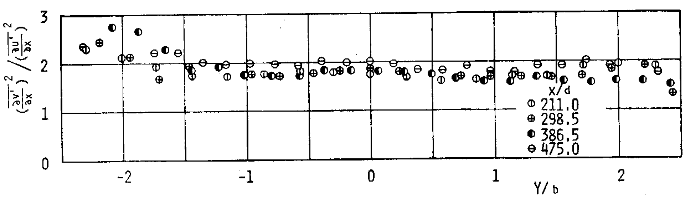

Fig. 18 The ratio between $\overline{\left(\frac{\partial u'}{\partial x}\right)^2}$ and $\overline{\left(\frac{\partial v'}{\partial x}\right)^2}$.

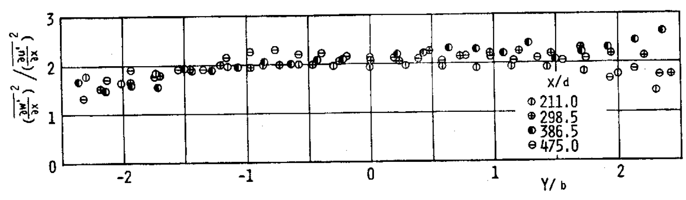

Fig. 19 The ratio between $\overline{\left(\frac{\partial u'}{\partial x}\right)^2}$ and $\overline{\left(\frac{\partial w'}{\partial x}\right)^2}$.

In the central region of wake, Townsend showed the following relations, valid for isotropic turbulence,

$$\overline{\left(\frac{\partial u'}{\partial x}\right)^2}=\frac{1}{2}\overline{\left(\frac{\partial v'}{\partial x}\right)^2}=\frac{1}{2}\overline{\left(\frac{\partial w'}{\partial x}\right)^2}.$$

Authors also attempted to confirm this matter experimentally. Present results shown in Figures 18–19 support the relation mentioned above across the cross section.

In Figure 20 is shown the dissipation rate estimated by

$$\varepsilon = 3\nu\left[\overline{\left(\frac{\partial u'}{\partial x}\right)^2} + \overline{\left(\frac{\partial v'}{\partial x}\right)^2} + \overline{\left(\frac{\partial w'}{\partial x}\right)^2}\right].$$

Though the dissipation rate $b\varepsilon/u_0^3$ obtained in the present experiment is about 1.4 times as large as that in Townsend's measurement, the budget of turbulent kinetic energy (see Figure 21) in the central region of the wake is quite well.

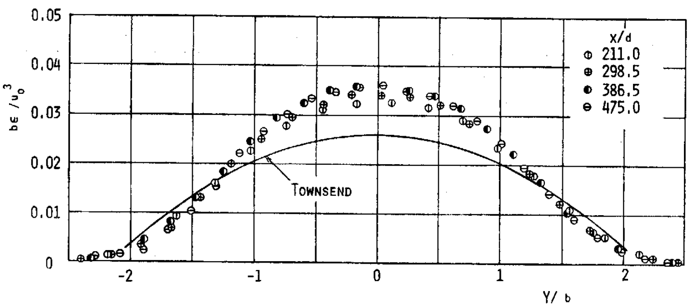

Fig. 20 The dissipation rate distribution.

3.4 Energy balance in self-preserving region

The turbulent kinetic energy equation can be written in following non-dimensional form

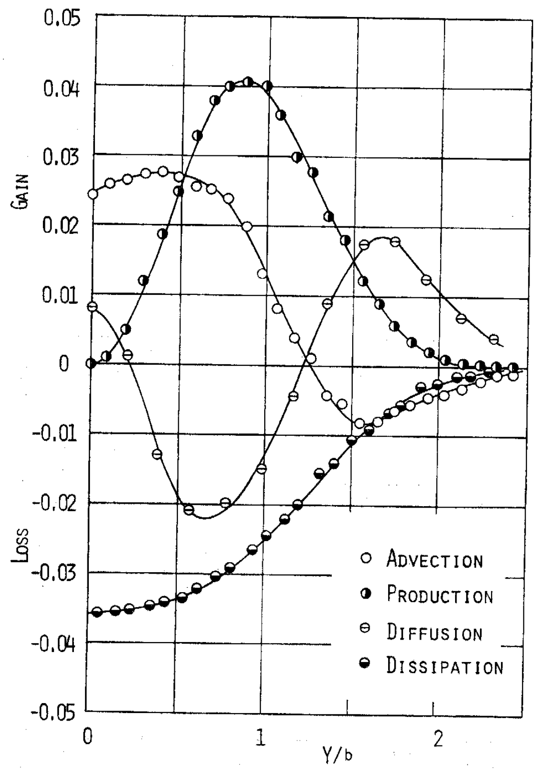

Fig. 21 The budget of turbulent kinetic energy.

$$\frac{b}{u_0^3}\left[-U\frac{\partial}{\partial x}\left(\frac{1}{2}\overline{q^2}\right)-V\frac{\partial}{\partial y}\left(\frac{1}{2}\overline{q^2}\right)-\overline{u'v'}\frac{\partial U}{\partial y}-\frac{\partial}{\partial y}\overline{v'\left(\frac{1}{2}q^2+\frac{p'}{\rho}\right)}-\varepsilon\right]=0.$$

The budget of the turbulent kinetic energy in the situation $x/d=475$ where the wake already becomes a self-preserving flow is examined, and each term on turbulent energy equation is plotted in Figure 21. But the pressure transport term $\overline{v'p'}$ is not measured.

The sum of the diffusion term across the wake is approximately zero. Nevertheless, the balance of the turbulent energy budget is not satisfied entirely across the flows, particularly the unbalance at the position of half width $y/b=1$ is too great. The magnitude of the advection and production terms are different from Townsend's measurements, that is, the advection term is much smaller and the production term is much larger. The magnitude of the advection term is in good agreement with the result of Bragg and Seshagiri[5], but that of the production term isn't. Unfortunately, the measurements for each term in the turbulent kinetic energy equation are not still uniform in the literature.

4. Conclusions

Turbulent measurements on two-dimensional wake beyond about 200 times cylinder diameters were made in detail. The following have been revealed.

(1) Each of the triple and fourth velocity correlations of the velocity fluctuation is almost similar beyond $x/d=211$ within the experimental scatter.

(2) All the flatness factor profiles with respect to the velocity fluctuation components become almost similar shape. The skewness factor profiles, however, exhibit the characteristics in the different region respectively.

(3) The profiles of the terms on the turbulent kinetic energy equation are not in agreement with Townsend's measurements.

Acknowledgement

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