# Study on the Coefficient of Decrease of Complex Modulus

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#### **Abstract**

Study on fatigue property of asphalt mixture is very important to design rational pavement. Author gave an account of the facts about the number of cycle to failure of several asphalt mixtures in previous reports<sup>1)~3)</sup>, but it seems probable that the process came to be fatigue failure is also important. So, author made the process came to be fatigue failure clear by use of the coefficient of decrease of complex modulus. From experimental results it became clear that there is a near connection between coefficient of decrease of complex modulus and strain, temperature, frequency etc. And these relation could be easily represented by following equations.

C.D.C.M.= $10^{a_1}e^{a_2}$ C.D.C.M.= $10^{(T-b_2)/b_1}$ C.D.C.M.= $10^{c_1}f^{c_2}$ 

where, C.D.C.M.: coefficient of decrease of complex modulus

ε: strain

T: temperature

f: frequency

 $a_1, a_2, \sim c_1, c_2$ : some constants

#### 1. Introduction

At the time of investigation in connection with fatigue property of asphalt mixture number of cycle to failure is important, but the process came to be fatigue failure will be also important to design rational pavement. In order to represent mechanical property came to be fatigue failure C. L. Monismith<sup>4</sup>) practiced static bending test about a test piece which was given repeated load of some number, and he had a discussion about the variation of broken nature in repeated loading test. K. D. Raithby<sup>5</sup>) has also discussed about them in his paper, but very detailed consideration has never been practiced. So author intend to explain variation of dynamic property by means of coefficient of decrease of complex modulus (here, coefficient of decrease of complex modulus will be simply represented as C. D. C. M.). That is to say, complex moduluses which have been decided at the time of initial load application, in the middle of load application and at the time when test piece was failured, were measured about several asphalt mixtures which have different proportion or binder property. In general the dynamic method applies an alternating load or deformation to a rod of the material

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under test, which is suspended freely or is held fixed on one end. The deformation varies sinusoidally with time; the strain at any moment can be expressed by

$$\varepsilon = \varepsilon_0 e^{i\omega t} \qquad \cdots \cdots (1)$$

The amplitude of the load also varies sinusoidally with the same frequency and can, at any moment, be represented by

$$\sigma = \sigma_0^* e^{i\omega t} \qquad \cdots \cdots (2)$$

where  $\omega$ =angular frequency, and  $\sigma_0^*$  become a complex number. Complex modulus is defined by

$$E^* = \sigma_0^* / \varepsilon_0 \qquad \cdots (3)$$

and

$$E^* = E' + iE'' \qquad \cdots (4)$$

Absolute value of  $E^*$  is

$$|E^*| = \sqrt{(E')^2 + (E'')^2}$$
 .....(5)

and it seems probable that  $|E^*|$  is almost the same kinds of parameter as stiffness. Tan  $\delta$  (where,  $\delta$  is the difference in phase between strain and stress) is generally called loss tangent and it's value is defined by  $\tan \delta = E''/E'$ . E' is real part of complex modulus and is called dynamic modulus. E'' is imaginally part of complex modulus and it is called loss modulus. And their magnitude is defined as the followings

$$E' = |E^*| \cos \delta$$
,  $E'' = |E^*| \sin \delta$  .....(6)

Complex modulus varies with load application as in Fig. 1. Initial complex modulus, complex modulus at failure and number of cycle to failure are decided as being shown in it's figure, and inclination of straight line  $l_1$  is generally called C. D. C. M.<sup>6)</sup> Vector representation of complex modulus become as Fig. 2 shows. In it's figure  $E_1^*$  is initial complex modulus and  $E_2^*$  is complex modulus after load application. Difference in phase between strain and stress of  $E_2^*$  is generally larger than that of  $E_1^*$  and coefficient of decrease of loss modulus is smaller than that of dynamic modulus.

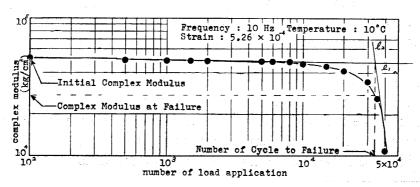


Fig. 1. Variation of complex modulus due to repeated load.

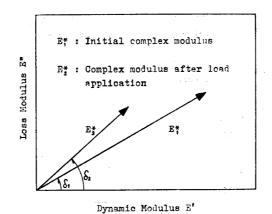


Fig. 2. Vector representation of complex modulus.

## 2. Experimental procedure

Proportions of asphalt mixture and binder properties are shown in Table 1. Three

Sieve	San. As.	Den.G. As.	Gussas.	
opening	(%)	(%)	(%)	Binder
(mm)	passing	passing	passing	
20.0		100.0	100.0	St. as.
13.0		97.8	86.9	60/80
10.0	100.0	86.7	73.8	Pen. 72
5.0	84.1	58.0	65.7	Tr&b 47.5
2.5	70.7	34.7	51.7	P.I. −1.3
0.6	49.5	19.5	41.2	20/40
0.3	41.0	14.8	32.1	Pen. 35
0.15	35.0	11.2	28.6	Tr&b 58.2
0.074	29.3	8.5	24.8	P.I0.2
As.	8%	6%	8%	

Table 1. Proportion in weight of test-piece and binder property.

kinds of proportion and two kinds of binder were adopted in this study. The combinations of them are the followings,

	Proportion	Binder
a.	Sand asphalt	 Straight asphalt (60/80)
b.	Dense graded asphalt concrete	 Straight asphalt (60/80)
c.	Guss asphalt	 Straight asphalt (20/40)

Manufacturing method of test piece is the same as previous report.<sup>2)</sup> Size of specimen is  $4.0 \times 4.0 \times 35$  cm, and the four point bending in a span length of 30 cm was employed. Distribution of apparent density is shown in Fig. 3. This distribution is looked on as normal distribution. This is the distribution of apparent density in dense graded asphalt concrete, but this tendency is almost the same as other mixtures. In dense graded asphalt concrete average of apparent density is 2.380 and standard deviation

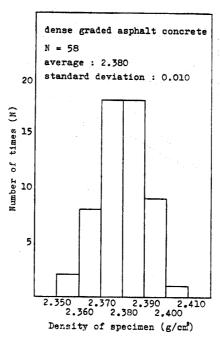


Fig. 3. Variance of apparent density.

is 0.010. Minimum apparent density is 2.353 and maximum apparent density is 2.408 as it's figure show. Temperature of 15°C and frequency of 10 Hz were adopted as standard experimental conditions at every asphalt mixtures, but especially in dense graded asphalt concrete the experiment was performed at the temperature being  $10^{\circ}$ C,  $5^{\circ}$ C,  $0^{\circ}$ C and  $-5^{\circ}$ C, furthermore the frequency being 20, 5 and 2.5 Hz.

# 3. Experimental results

Relation between strain and coefficient of decrease of complex modulus at different asphalt mixtures are shown in Fig. 4. C.D.C.M. become large with the increase of strain, and this coefficient of increase is almost the same value about three kinds of asphalt mixtures. In case of strain being the same C.D.C.M. of sand asphalt is the smallest among them and that of guss is the largest, and this difference between the former and the latter is about  $7 \times 10^{-2}$ . Coefficients of decrease of complex modulus

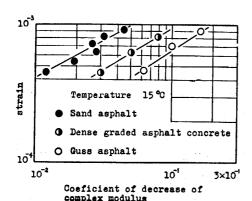


Fig. 4. Relation between strain and coefficient of decrease of complex modulus at different asphalt mixtures.

as a function of number of cycle to failure for three kinds of asphalt mixtures are shown in Fig. 5. From this figure C. D. C. M. become small with the increase of the number of cycle to failure. In case of strain being small C. D. C. M. is also small and number of cycle to failure become large<sup>2</sup>, from this reason it stands to reason that C. D. C. M. become small with the increase of the number of cycle to failure. Especially at dense graded asphalt concrete practical value of C. D. C. M. is about the order of 10<sup>-3</sup>, so number of cycle to failure will be the value in the range between 10<sup>6</sup> and 10<sup>7</sup>. Relations between temperature and coefficient of decrease of complex modulus are shown in Fig. 6. This figure represents the relation above mentioned at constant strain (5.3 × 10<sup>-4</sup>) and frequency (10 Hz). It is a matter of course for C. D. C. M. to increase with the rise of temperature, but this coefficients of increase are the same value for both mixtures (sand asphalt and dense graded asphalt concrete). And in case of the rise of temperature being 20°C C. D. C. M. is increasing to about ten times as larger as initial value. This tendency is the same as guss asphalt. Relations between coefficient of decrease of complex modulus and frequency are represented in Fig. 7. This is a figure

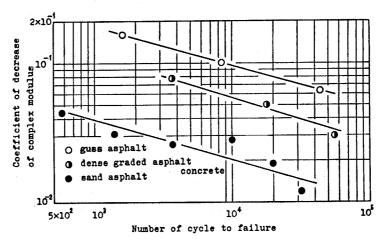


Fig. 5. Coefficient of decrease of complex modulus as a function of number of cycle to failure for three kinds of asphalt mixtures.

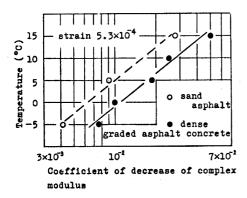


Fig. 6. Relation between temperature and coefficient of decrease of complex modulus.

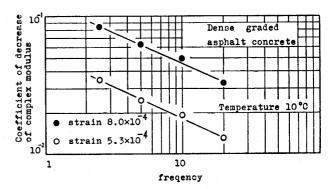


Fig. 7. Coefficient of decrease of complex modulus as a function of frequency.

of dense graded asphalt concrete and temperature being 10°C. C.D.C.M. decrease with increase of frequency, and this coefficient of decrease is almost the same value about two kinds of strain levels  $(8.0 \times 10^{-4})$  and  $5.3 \times 10^{-4}$ ). It is generally said that reduced time of 10 Hz is almost equivalent to reduced time which is measured at the traffic speed being 70 (Km/hr)7), and the strain which occur in the lowest part of surface course is about the value of 10<sup>-5</sup> 8), so in practical problem C.D.C.M. will be immediately small (namely, it's value will be about  $10^{-4} \sim 10^{-3}$ ). Relations between initial complex modulus and coefficient of decrease of complex modulus at constant frequency (frequency is 10 Hz) are represented in Fig. 8. In this figure variation of temperature is from 15°C to -5°C and in case of temperature being lowest one (namely,  $-5^{\circ}$ C) initial complex modulus is the largest, so in that case C. D. C. M. is smaller than other cases. Furthermore relations between initial complex modulus and coefficient of decrease of complex modulus at constant temperature of 10°C is shown in Fig. 9. These initial complex modulus of Fig. 9 is initial value in each frequency, but in the narrow sense, in case of frequency being 2.5 and 5.0 Hz initial complex modulus is the value which is measured at load application being 10 times, and in case of frequency being 10 and 20 Hz it is the value at one hundred times. Complex modulus change scarcely it's value until load application become one hundred times at the frequency being 10 and 20 Hz. This tendency is the same in the frequency being 2.5 and 5 Hz.

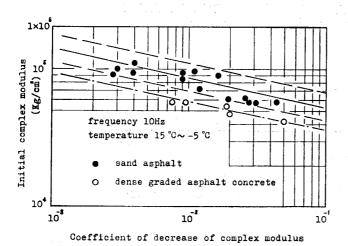


Fig. 8. Relation between initial complex modulus and coefficient of decrease of complex modulus at constant frequency.

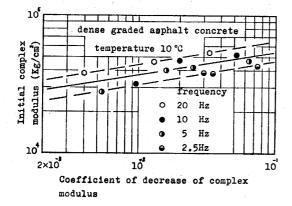


Fig. 9. Relation between initial complex modulus and coefficient of decrease of complex modulus at constant temperature of 10°C.

It will be easily understood at the sight of Fig. 9 that initial complex modulus increase with increase of C. D. C. M. at constant temperature. Relations between stress at failure and number of cycle to failure and also the relations between initial complex modulus and number of cycle to failure are shown in Fig. 10. The former is Fig. 10-a and the latter is Fig. 10-b. Stress at failure and initial complex modulus are decrease with increase of the number of cycle to failure at constant temperature. For the last time number of load application dependency of Loss modulus or Dynamic modulus are shown in Fig. 11. Coefficient of decrease of Dynamic modulus is almost the same value as that of complex modulus, but in Loss modulus this tendency is immediately different from Dynamic modulus. Loss modulus increase with increase of the number of load application until the number of load application become some values, and after that it begin decrease and finally the destruction occurs. The difference in phase between strain and stress increase with increase of load application, and this is the

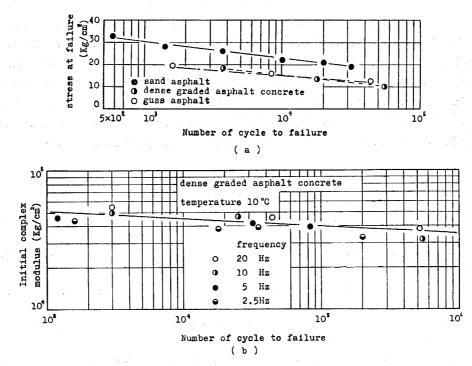


Fig. 10. Stress at failure and initial complex modulus as a function of number of cycle to failure.

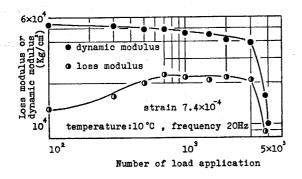


Fig. 11. Loss modulus and dynamic modulus as a function of number of load application.

reason why Loss modulus increase with increase of load application. By the reason above mentioned most part of destruction is occupied by the decrease of dynamic modulus, so elastic term will largely give influence to the property of destruction and viscourse term will scarcely give influence.

### 4. Summary

From experimental results above mentioned it became clear that there is a near connection between coefficient of decrease of complex modulus and strain, temperature, frequency etc. And these relation can be represented by the equations of straight line in log-log or semi-log diagram. C. D. C. M. become large with the increase of strain, the rise of temperature and decrease of frequency. It is generally said that reduced time of 10 Hz is almost equivalent to reduced time which was measured at the traffic speed being 70 (Km/hr) and the strain which occur to the lowest part of surface course is about the value of  $10^{-5}$ , so in these practical problem C. D. C. M. will be immediately small (namely, it's value is about  $10^{-4} \sim 10^{-3}$ ). In connection with Loss modulus and Dynamic modulus the difference in phase between strain and stress increase with increase of load application, so Loss modulus become large with increase of load application, but Dynamic modulus decrease and the coefficient of decrease of dynamic modulus is almost the same as that of complex modulus. From the reason above mentioned most part of destruction will be occupied by the decrease of dynamic modulus.

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