Noise Reduction by Means of Enclosure

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Abstract

The invention of silent machine is a social demand. Notwithstanding, it is an important but difficult problem to manufacture a special machine which is inherentry quiet.

The most effective way to substantially reduce the noise of uproarious machine is to completely enclose the machine itself.

This paper describes the theoretical investigation of noise reduction by enclosure and the analysis performed regarding the sound transmission loss of various types of wall.

1. Investigation of power level of the output of sound radiated from solid sound sources

It is not easy to analyze theoretically the noise which is radiated from a machine in operation. So, the author assume here that a spherical breathing apparatus with a volume velocity of U and a surface area of S_s (a radius of a) is a fairly good approximation for a simulation. With regard to a low-frequency sound, the radiation impedance Z is expressed as follows:[1]

$$Z=\frac{\rho c}{S_s}\frac{(ka)^2+jka}{1+(ka)^2} \qquad \cdots\cdots\cdots(1)$$

where

$$\left.\begin{aligned} k&=2\pi f/c=2\pi/\lambda \\ j&=\sqrt{-1} \end{aligned}\right\} \qquad \cdots\cdots\cdots(2)$$

and f is the frequency of sound, c the sound velocity, λ the wavelength and ρ the density of a medium.

In free space, radiated acoustic power W is obtained by multiplaying the square of volume velocity U by the real part of the radiation impdeance Z, and hence

$$W=U^2\frac{\rho c}{S_s}\frac{(ka)^2}{1+(ka)^2} \qquad \cdots\cdots\cdots(3)$$

Since volume velocity U is a product of the vibration velocity u and the surface area S_s, W is expressed by substituting $U=uS_s$ into the above equation as follows:

$$W=u^2\rho cS_s\frac{(ka)^2}{1+(ka)^2} \qquad \cdots\cdots\cdots(4)$$

The power level of a sound source PLW is expressed as the following;

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$$PWL=10\log_{10}W+120 \qquad (dB) \qquad \cdots\cdots\cdots(5)$$

where W is expressed by *Watt* (W) and 10^{-12} W is taken to be 0 dB. According to *Eq*. (3), PWL is expressed as the following:

$$PWL=20\log_{10}U+10\log_{10}\frac{(ka)^2}{1+(ka)^2}-\log_{10}S_s+10\log_{10}\rho c+120 \qquad (dB) \qquad \cdots\cdots\cdots(6)$$

By substituting the numerical values of ρ and c and assuming the sound source to be a sphere of radius a and vibration velocity u, the following are obtained, that is, by substituting the values of ρ ($=1.2$ kg/m^3) and c ($=340$ m/s) into *Eq*. (6),

$$PWL=20\log_{10}U+10\log_{10}\frac{(ka)^2}{1+(ka)^2}-10\log_{10}S_s+146 \qquad (dB) \qquad \cdots\cdots\cdots(7)$$

by substituting the relation $S_s=4\pi a^2$ into *Eq*. (7),

$$PWL=20\log_{10}U+20\log_{10}k-10\log_{10}\{1+(ka)^2\}+135 \qquad (dB) \qquad \cdots\cdots\cdots(8)$$

and by substituting the relation $S_s=4\pi a^2 u$ into *Eq*. (8),

$$PWL=20\log_{10}u+20\log_{10}k-10\log_{10}\{1+(ka)^2\}+40\log_{10}a+157 \qquad (dB) \qquad \cdots\cdots\cdots(9)$$

1.1 When the size of a sound source is small enough compared with the wavelength

According to *Eqs*. (3) and (4), radiated acoustic power W may be expressed under the condition that $ka \ll 1$ by the following:

$$W=U^2\frac{\rho c}{S_s}(ka)^2 \qquad (W) \qquad \cdots\cdots\cdots(10)$$

$$W=u^2\rho c S_s(ka)^2 \qquad (W) \qquad \cdots\cdots\cdots(11)$$

To express the radiated power by power level (PWL), *Eq*. (11) is induced to the following equation:

$$PWL=20\log_{10}U+20\log_{10}ka-10\log_{10}S_s+146$$
$$=20\log_{10}U+20\log_{10}f+100 \qquad (dB) \qquad \cdots\cdots\cdots(12)$$

Thus, it is deduced that when the size of a sound source is small enough as compared with the wavelength, whatever shape and intensity they may take, all sound sources are equal in power level to one another, as far as they are the same in volume velocity. At first sight, the PWL in the above equation seems to be independent of the size of a sound source. But it is due to that S_s is included in U.

1.2 When the size of a sound source is large enough compared with the wavelength

According to *Eqs*. (3) and (4), radiated acoustic power W may be given under the condition of $ka \gg 1$, by the following:

$$W = U^2 \frac{\rho c}{S_s} \quad (W) \qquad \cdots\cdots\cdots(13)$$

$$W = u^2 \rho c S_s \quad (W) \qquad \cdots\cdots\cdots(14)$$

To express the radiated power by power level (*PWL*), *Eq.* (14) is induced to the following equation:

$$PWL = 20\log_{10} U - 10\log_{10} S_s + 146$$

$$= 20\log_{10} U - 20\log_{10} a + 135 \quad (dB) \qquad \cdots\cdots\cdots(15)$$

2. Investigation of the sound pressure level in a package when a solid sound source is enclosed in a package

If a solid sound source (say, a machine) is enclosed within a package, the sound pressure in the package is built up and the increased acoustic energy will be radiated outward through the wall of the package. Consequently, it may be said that the use of a package for enclosing a noise source is not necessarily effective for reducing noise. Hereupon, this problem will be discussed for two cases.

2.1 When the space in the package is small enough compared with the wavelength

When the volume of the space between the sound source and the package is taken to be V and the compliance of the volume is to be C,

$$C = V/\rho c^2 \qquad \cdots\cdots\cdots(16)$$

Consequently, the absolute value p_p of the sound pressure produced in the package by a sound source with a volume velocity of U is expressed as follows:

$$p_p = \frac{U}{\omega C} = \frac{U}{\omega}\frac{\rho c^2}{V} = \frac{U}{2\pi f}\frac{\rho c^2}{V} \qquad \cdots\cdots\cdots(17)$$

This is the greatest value of the sound pressure in the package. When the package is not hard enough, that is, the value of C is greater than that given by *Eq.* (16), the value of p_p will be smaller than that given by *Eq.* (17). When there is any loss due to sound absorptive mechanism in the package or on the machine, the value of p_p will also be smaller than that given by *Eq.* (17).

When no package is provided, the sound pressure on the surface of the sound source is a product of the volume velocity and the radiation impedance. For the low frequency: $ka \ll 1$, the radiation impedance is well approximated by the imaginary part of *Eq.* (1) and the effective value of the sound pressure on the surface is expressed as follows:

$$p_s = U \frac{\rho c}{S_s} ka \qquad \cdots\cdots\cdots(18)$$

Then, p_p/p_s will represent the greatest value of the build up rate of the sound-pressure in the package, provided that the size of the package is small enough compared with the wavelength.

If this value is taken to be R_p, then

$$R_p=\left(\frac{U}{2\pi f}\frac{\rho c^2}{V}\right)\Big/\left(\frac{U\rho c}{4\pi a^2}ka\right)=\frac{4\pi a}{k^2V} \qquad \cdots\cdots\cdots(19)$$

If a sound source of a spherical breathing apparatus with radius 0.5 m is enclosed in a package with radius 0.6 m, the sound-pressure build up rate R_p in the package is computed as follows:

$$R_p=\frac{4\pi\times0.5}{k^2\times0.381}=\frac{16.5}{k^2} \qquad \cdots\cdots\cdots(20)$$

That is, $R_p=4.1$ for 108 Hz ($k=2$). In other words, a large sound-pressure build up rate of as much as 12 dB is estimated. And it can be said that the build up of the sound-pressure in the package is an important problem in packaging.

The above value is obtained by replacing the radiation in free space with the radiation in a lossless narrow space. In other words, it represents a theoretical uppermost bound concerning the build up of the sound-pressure in package and, accordingly, is not necessarily applicable to real package. It is obvious from the above discussion, that the build-up of the sound pressure in a package should be checked for the design of the package to reduce the radiated sound.

If the sound pressure of 2×10^{-5} Pa is taken to be 0 dB, the equation giving the sound pressure level in the package SPL_p is obtained from *Eq.* (17) as follows:

$$SPL_p=20\log_{10}U-20\log_{10}V-20\log_{10}f+181 \qquad (dB) \cdots\cdots\cdots(21)$$

Here, a sound source whose power level is PWL in the free field will be introduced. The vibration velocity on the surface of the sound source is assumed to be constant even when it is enclosed, i.e. the constant velocity sound source is assumed. By substituting *Eq.* (12) into *Eq.* (21), the sound pressure level in a package SPL_p is expressed by using PWL in the free field. That is:

$$SPL_p=PWL-40\log_{10}f-20\log_{10}V+81 \qquad (dB) \cdots\cdots\cdots(22)$$

If the sound is transmitted only through the wall of the package and the area of the surface is taken to be S_p, the incident acoustic power level PWL_i to that part of the package will be obtained from the following equation:

$$PWL_i=SPL_p+10\log_{10}S_p \qquad (dB) \qquad \cdots\cdots\cdots(23)$$

Then, the acoustic power level PWL_t to be radiated through the package will become as follows:

$$PWL_t=PWL-40\log_{10}f-20\log_{10}V+10\log_{10}S_p-TL+81 \qquad (dB) \cdots\cdots\cdots(24)$$

where, TL is the transmission loss of the surface. It can be deduced from the above equation that, in order to reduce PWL_t, the space V in the package should be made as large as possible, the surface area S_p of the package should be reduced and transmission loss TL should be increased.

2.2 When the space in the package is large enough compared with the wavelength

Here, the space in a package is assumed to be large enough to constitute a reverberant field in it.

If the acoustic power from a sound source in a package is nearly equal to that produced in a free field and the sound source can be considered to be a spherical breathing apparatus with a volume velocity U, the radiated acoustic power W given in *Eq.* (3) may hold with a fairly good approximation.

When the volume of the space in a package is taken to be V, the surface area of a machine (sound source), S_m, the floor area between the machine (sound source) and the package, S_f, and the area through which sound waves are transmitted to the outside of the package, S_p, the average acoustic energy density J produced in an enclosure with the average sound absorbing coefficient $\bar{\alpha}$ (inclusive of the machine and floor surface) will be obtained by the following equation, under the condition that the sound field is well diffused.

$$J=\frac{4W}{c}\frac{1-\bar{\alpha}}{-2.3(S_p+S_m+S_f)\log_{10}(1-\bar{\alpha})} \quad (J/m^3) \quad \cdots\cdots\cdots(25)$$

When $\bar{\alpha}$ is small, *Eq.* (25) will be approximated by the following equation:

$$J=\frac{4W(1-\bar{\alpha})}{c\bar{\alpha}(S_p+S_m+S_f)} \quad (J/m^3) \quad \cdots\cdots\cdots(26)$$

In the range of $\bar{\alpha}<0.7$, *Eq.* (26) is less than *Eq.* (25) by not more than 3 dB, and so *Eq.* (26) will be used hereafter, since it is simpler in form.

If the radiated acoustic power from a sound source W is constant, regardless of the presence of package, the incident acoustic power W_i on the wall of the package will be as follows:

$$W_i=\frac{W(1-\bar{\alpha})}{\bar{\alpha}}\frac{S_p}{S_p+S_m+S_f} \quad (W) \quad \cdots\cdots\cdots(27)$$

According to this equation, the acoustic power W_t to be radiated to the out of the package will be as follows, provided that the energy transmission coefficient is taken to be τ,

$$W_t=\frac{W\tau(1-\bar{\alpha})}{\bar{\alpha}}\frac{S_p}{S_p+S_m+S_f} \quad (W) \quad \cdots\cdots\cdots(28)$$

Then, the acoustic power level PWL_t to be radiated to the out of the package becomes as the following:

$$PWL_t=PWL-\log_{10}\frac{\bar{\alpha}}{1-\bar{\alpha}}-10\log_{10}\frac{S_p+S_m+S_f}{S_p}-TL \quad (dB)\cdots\cdots\cdots(29)$$

It can be deduced from the above equation that the material having a greater transmission loss should be used, the average sound absorbing coefficient $\bar{\alpha}$ in the package should be increased and the value of $(S_p+S_m+S_f)/S_p$ should be increased.

3. Investigation of the sound reducing effect of the package

When no package is provided, the sound pressure level SPL_r at a distant point r apart from the sound source in a free field is expressed by the following equation:

$$SPL_r = PWL - 20\log_{10} r - 8 \qquad (dB) \qquad \cdots\cdots\cdots(30)$$

where the sound propagation is assumed to be hemispherical. In a room with room constant R, the sound pressure level at the point r apart from the sound source is expressed by the following equation:

$$SPL_{r'} = PWL + 10\log\left(\frac{Q}{4\pi r^2} + \frac{4}{R}\right) \qquad (dB) \qquad \cdots\cdots\cdots(31)$$

where Q is a directional gain of the sound source. When this sound source is enclosed within a package, as in the preceding section, the sound pressure level at the same measuring point under the same conditions is expressed by the following equations:

$$SPL_{rt} = PWL_t - 20\log_{10} r - 8 \qquad (dB) \qquad \cdots\cdots\cdots(32)$$

and

$$SPL_{rt'} = PWL + 10\log_{10}\left(\frac{Q}{4\pi r^2} + \frac{4}{R}\right) \qquad (dB) \qquad \cdots\cdots\cdots(33)$$

Then, the difference between the sound pressure level without a package and that with a package at the same point becomes as follows:

$$Att = SPL_r - SPL_{rt} = SPL_{r'} - SPL_{rt'} = PWL - PWL_t \qquad (dB) \cdots\cdots\cdots(34)$$

By substituting PWL_t of *Eq.* (24) and *Eq.* (29) into *Eq.* (34), the following relations are established:

$$Att = 40\log_{10} f + 20\log_{10} V - 10\log_{10} S_p + TL - 81 \qquad (dB) \cdots\cdots\cdots(35)$$

and

$$Att = 10\log_{10}\frac{\bar{\alpha}}{1-\bar{\alpha}} + 10\log_{10}\frac{S_p+S_m+S_f}{S_p} + TL \qquad (dB) \cdots\cdots\cdots(36)$$

4. Investigation of sound insulation of the wall

4.1 Transmission loss of sound insulating material

Transmission loss of a sound insulating material of a single composition is obtained by the following formulae (37) and (38)[2)3)].

As for normal incident sould

$$TL=10\log_{10}\left\{1+\left(\frac{\omega m}{2\rho c}\right)^2\right\} \quad (dB) \qquad \cdots\cdots\cdots(37)$$

As for random incident sound

$$TL=10\log_{10}\left\{1+\left(\frac{\omega m}{2\rho c}\right)^2\right\}-10\log_{10}\left[2.3\log_{10}\left\{1+\left(\frac{\omega m}{2\rho c}\right)^2\right\}\right] \quad (dB) \qquad \cdots\cdots\cdots(38)$$

where, m is the surface density of the material in kg/m^2, ω the angular frequency, ρ the density of the air and c the sound velocity. For automobiles and others, however, lighter packages are required, which causes poor transmission loss from the above formulae. Therefore, the following investigation was carried out.

4.2 Transmission through sound absorbing material

We have sometimes experienced that a thick sound absorbing material insulates sound fairly well, but there are very few literatures relating to the sound insulation effect of sound absorbing materials. Meanwhile, sound absorption coefficients of various sound absorbing materials have been measured and published. So, it may be very useful to estimate the transmission loss of a sound absorbing material from its absorption coefficient.

Acoustic power transmitted through a bulk of the absorbing material of the thickness h will be δ^h times the incident sound power of the incident sound, if the power attenuats at a rate of δ per unit thickness and the reflection of sound on the surface of the material can be neglected. That is, the transmission coefficient τ of a material of thickness h is given by the following equation:

$$\tau=\delta^h \qquad \cdots\cdots\cdots(39)$$

A sound absorbing material with thickness h' and the absorption coefficient α is taken up here, where α is measured by attaching the material on the surface of a rigid body. Under the assumption that the incident sound is reflected only on the rigid surface, the absorption coefficient of the sound absorbing material of thickness h' is expressed by the following equation:

$$\alpha=1-\delta^{2h'} \qquad \cdots\cdots\cdots(40)$$

therefore

$$\delta=(1-\alpha)^{1/2h'} \qquad \cdots\cdots\cdots(41)$$

By substituting *Eq.* (41) into *Eq.* (39), the transmission coefficient of the sound absorbing material of thickness h is expressed by the following equation:

$$\tau=(1-\alpha)^{h/2h'} \qquad \cdots\cdots\cdots(42)$$

Hence, the transmission loss TL will be as follows:

$$TL = 10\log_{10}(1-\alpha)^{-h/2h'} \qquad \cdots\cdots\cdots(43)$$

It can be seen from the results that the value TL of the single sound absorbing material is so small that the material cannot be put to practical use.

4.3 Transmission through a layer of a sound insulating plate and sound absorbing material

It was stated in 2.2 that glass wool packed in a sound insulation package is very effective in reducing the high-frequency sound produced in the package. Transmission loss TL through such a laminated wall will then be investigated in the following.

Here, the transmission loss and the transmission coefficient of the sound absorbing material are taken to be TL_B and τ_B, respectively, and the transmission loss and the coefficient of the sound insulating plate are taken to be TL_C, and τ_C, respectively. The relations between them are as follows:

$$\left.\begin{aligned} \tau_B &= 10^{-TL_B/10} \\ \tau_C &= 10^{-TL_C/10} \end{aligned}\right\} \qquad \cdots\cdots\cdots(44)$$

Since the incident power into the sound insulating plate through the sound absorbing material of thickness h is $\tau_B W_i$, the transmission coefficient τ_T of the composite layer of absorbing and insulating materials will be induced from *Eqs.* (39) and (44) as follows:

$$\tau_T = \tau_B \tau_C = \delta^h \times 10^{-TL_C/10} \qquad \cdots\cdots\cdots(45)$$

Accordingly, transmission loss TL will be as follows:

$$TL = 10\log_{10}\delta^{-h} + TL_C \quad (dB) \qquad \cdots\cdots\cdots(46)$$

and

$$TL = 10\log_{10}(1-\alpha)^{-h/2h'} + TL_C \quad (dB) \qquad \cdots\cdots\cdots(47)$$

where, α is the sound absorption coefficient of the absorbing material of thickness h' laid upon a perfectly rigid surface.

4.4 Transmission through a double wall packed with a layer of sound absorbing material

With a double wall made of two single panels and packed with a layer of sound absorbing material between the plates, a greater TL can be expected.

The following model of the insulation wall is supposed here, that is, the wall is composed of three layers A, B and C, where A and C are the panels having the transmission coefficient τ and B is a sound absorbing material whose thickness is h and transmission coefficient δ^h. When the incident sound power to the panel A is W_i, τW_i is transmitted through the panel A and $(1-\tau)W_i$ is reflected by the panel. On the panel C, the rates of transmission and reflection are the same as on the panel A.

Then, the transmitted power with no reflecting process will become as follows:

$$\tau_1 W_i = \delta^h \tau^2 W_i \qquad \cdots\cdots\cdots(48)$$

The power transmitted through the absorbing material B will be reflected on the surface of the panel C. The reflected power becomes $\delta^h \tau(1-\tau)W_i$ according to the above assumption. This reflected power is attenuated by B to $\delta^{2h}\tau(1-\tau)W_i$ in the process of the transmission to the surface of A and is reflected by A. The reflected sound power $\delta^{2h}\tau(1-\tau)^2 W_i$ is attenuated by B and the incident power to C is $\delta^{3h}\tau(1-\tau)^2 W_i$. Then, the power $\tau_2 W_i$ transmitted through C becomes as follows:

$$\tau_2 W_i = \delta^{3h}\tau^2(1-\tau)^2 W_i \qquad \cdots\cdots\cdots(49)$$

The remainder is reflected in the direction of A and repeat this process is repeated. Consequently, the overall transmission coefficient τ_e is expressed by the following equation:

$$\tau_e = \delta^h\tau^2 + \delta^{3h}\tau^2(1-\tau)^2 + \cdots + \delta^h\tau^2\{\delta^{2h}(1-\tau)^2\}^n + \cdots$$

$$= \frac{\delta^h \tau^2}{1-\delta^{2h}(1-\tau)^2} \qquad \cdots\cdots\cdots(50)$$

Accordingly, TL is given by the following equation:

$$TL = 10\log_{10}\frac{1-\delta^{2h}(1-\tau)^2}{\delta^h\tau^2} \quad (dB) \qquad \cdots\cdots\cdots(51)$$

5. Conclusions

In this study, various theoretical analyses were carried out with regard to the sound insulation effects of packages on the assumption that a complete vibration isolation was provided, and a number of design formulae were induced.

The noticeable points in this report can be summarized as follows.

(1) By modelling a small-sized engine as a spherical breathing apparatus, the relations between the vibration velocity level the radiated power level and the sound pressure level are made clear for the conditions of small space, free field and diffused field with sound absorbing walls.

(2) A build-up of the sound pressure in the internal space of the package is formulated. It is necessary to give the loss of acoustic energy in the package which reduces the amount of the build-up.

(3) In order to lower the power level of the sound radiated from a package, TL and V should be increased for low-frequency sound and TL, α and $(S_p+S_m+S_f)/S_p$ should be increased for high-frequency sound.

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