Identification of the System Described as the Canonical Form Using Filtering Technique

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Abstract

The simultaneous parameter identification and state estimation algorithm in the linear discrete-time system described by the canonical form is developed in computational purpose.

The problem is formulated under the assumption that the system parameters are unknown constants and the noise statistics are given a priori as well as the known order of the system.

The procedure requires the Kalman filter which is applied for state estimation and parameter identification at each stage, and the smoother which is for state-smoothing. The smoothing calculation is utilized to get the observation matrix with respect to parameter vector. Two approaches where the parameter has fictitious noise inputs and nothing are shown.

The effective choice of fictitious noise has led the estimation better in the numerical example.

Introduction

In the past years a lot of different identification and parameter estimation methods for dynamic systems have been described in the literatures[1),2)].

For instance, the least squares, the maximum-likelihood method, the maximum a posteriori probability method and stochastic approximation method are well known.

The most important algorithms among all the identification algorithms are the ones that perform on-line by sequentially the parameter and state estimates from noisy measurement.

This paper considers the simultaneous estimation of the parameter and state in the linear discrete-time system described by the canonical form with single input-output observation system. For the on-line estimation of the parameter and the state, the augmented vector method using filtering technique is available in general, but the problem of observability has to be considered strictly and the divergence has often occurred.

The purpose of this paper is to represent the algorithm of identification of the canonical form system by the use of the Kalman filter and smoother.

It is assumed that the order of the system and the additional measurement noise statistics are known. The identification procedure is separated into two parts. The first one is the state estimation where the estimate of the unknown parameter is utilized instead of the true one. One the other hand, in the second part where the parameter estimation is required, we use the estimated state of the first part as the state necessary for calculation.

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Statement of the Problem

Consider the following system with canonical form,

$$\mathbf{X}_{k+1}=\mathbf{F}\mathbf{X}_k+\mathbf{G}W_k \tag{1}$$

$$Z_k=\mathbf{h}\mathbf{X}_k+V_k \tag{2}$$

where,

$\mathbf{X}$ $n\times 1$ state vector
Z scalar measurement
W scalar input signal
$\mathbf{F}$ $n\times n$ matrix
$\mathbf{G}$ $n\times 1$ matrix
$\mathbf{h}$ $1\times n$ matrix
V scalar measurement noise.

Parameter matrices **F**, **G** and **h** are represented as follows,

$$\mathbf{F}=\begin{bmatrix} \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} & \mathbf{I} \\ \cdots & \cdots \\ -a_1\ -a_2 & \cdots\ -a_n \end{bmatrix} \tag{3}$$

$$G=\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{4}$$

$$\mathbf{h}=[1\quad 0\quad 0] \tag{5}$$

where, **I** indicates unit matrix.

The reason why we consider the Eq. (3) and Eq. (4) is that the total number of unknown parameters reduces more markedly than in the case of the general dynamic equation. In the case of canonical form expression, the number of the parameters that we should estimate is $2n$, while we have to estimate n^2 unknown parameters in the general system whose order is n.

It is assumed that the measurement noise is the gaussian noise whose mean is zero and covariance is R.

The purpose of this paper is to obtain the estimates of unknown constant parameters $a_1, a_2,\ldots, a_n, b_1, b_2,\ldots, b_n$ on condition that the order of the system, n is known.

Computational Algorithm for Identification

The concatenated unknown parameter vector is defined as,

$$\mathbf{P}_k=[-a_1,\ -a_2,\dots,\ -a_n,\ b_1,\ b_2,\dots,\ b_n] \tag{6}$$

Eq. (2) may be written as Eq. (7) substituting Eq. (1) into Eq. (2).

$$Z_k=\mathbf{H}_k\mathbf{P}_k+V_k \tag{7}$$

$\mathbf{H}_k$ in Eq. (7) is a $1\times 2n$ matrix and described as,

$$\mathbf{H}_k=[\mathbf{X}_{k-n}^T,\ \mathbf{U}_k] \tag{8}$$

where,

$$\mathbf{U}_k=[W_{k-1},\ W_{k-2},\dots,\ W_{k-n}] \tag{9}$$

Eq. (7) may be considered to be the observation equation with respect to unknown parameter vector $\mathbf{P}_k$. On the assumption that unknown parameters are time-invariant, the transition equation associated with $\mathbf{P}_k$ is able to be denoted as,

$$\mathbf{P}_{k+1}=\mathbf{P}_k \tag{10}$$

If $\mathbf{H}_k$ is known exactly, then the estimate of $\mathbf{P}_k$ is obtained by the Kalman filter. However, $\mathbf{H}_k$ included unknown actual state $\mathbf{X}_k$, so we cannot apply the Kalman filter for Eq. (7) directly. We employ $\hat{\mathbf{H}}_{k/k}$ which is described as Eq. (11) instead of $\hat{\mathbf{H}}_k$.

$$\hat{\mathbf{H}}_{k/k}=[\hat{\mathbf{X}}_{k-n/k},\ \mathbf{U}_k] \tag{11}$$

In Eq. (11), $\hat{\mathbf{X}}_{k-n/k}$ is the smoothed value of $\mathbf{X}_{k-n}$ at time k.
Then the observation equation is represented approximately as,

$$Z_k=\hat{\mathbf{H}}_{k/k}\mathbf{X}_k+V_k \tag{12}$$

Since $\hat{\mathbf{X}}_{k-n/k}$ and $\mathbf{U}_k$ in $\hat{\mathbf{H}}_{k/k}$ are all known at time k, the Kalman filter is able to be applied in Eq. (10) and Eq. (12).

The identification algorithm is represented by the following recursive equations,

$$\hat{\mathbf{P}}_{k+1/k+1}=\hat{\mathbf{P}}_{k/k}+\mathbf{K}_{k+1}^p(Z_{k+1}-\hat{\mathbf{H}}_{k+1/k+1}\hat{\mathbf{P}}_{k/k}) \tag{13}$$

$$\mathbf{K}_{k+1}^p=\mathbf{S}_{k/k}^p\hat{\mathbf{H}}_{k+1/k+1}^T(\hat{\mathbf{H}}_{k+1/k+1}\mathbf{S}_{k/k}^p\hat{\mathbf{H}}_{k+1/k+1}^T+R)^{-1} \tag{14}$$

$$\mathbf{S}_{k+1/k+1}^p=(\mathbf{I}-\mathbf{K}_{k+1}^p\hat{\mathbf{H}}_{k+1/k+1})\mathbf{S}_{k/k}^p \tag{15}$$

where $\hat{\mathbf{X}}_{k+1/k+1}$, $\mathbf{S}_{k+1/k+1}^p$ and $\mathbf{K}_{k+1}^p$ represent respectively the filtered estimate, the error covariance of the filtered estimate and the filter gain at time $k+1$ concerning $\mathbf{P}$. On the other hand, $\hat{\mathbf{H}}_{k+1/k+1}$ is obtained from the following smoothing recursive equation[3],

$$\hat{\mathbf{X}}_{j/k+1}=\hat{\mathbf{X}}_{j/j}+\mathbf{K}_j^x(\hat{\mathbf{X}}_{j+1/k+1}-\hat{\mathbf{X}}_{j+1/j}) \tag{16}$$

$$\mathbf{K}_j^x=\mathbf{S}_{j/j}^x\mathbf{F}_j^T\mathbf{S}_{j+1/j}^{x\ -1} \tag{17}$$

$$\mathbf{S}_{j/k}^x=\mathbf{S}_{j/j}^x+\mathbf{K}_j^x(\mathbf{S}_{j+1/k+1}^x-\mathbf{S}_{j+1/j}^x)\mathbf{K}_j^{xT} \tag{18}$$

$$\hat{\mathbf{X}}_{k+1/k+1}=\hat{\mathbf{X}}_{k+1/k}+\mathbf{K}_{k+1}^x(Z_{k+1}-\mathbf{h}\hat{\mathbf{X}}_{k+1/k}) \tag{19}$$

$$\mathbf{K}^x_{k+1} = \mathbf{S}_{k+1/k}\mathbf{h}^T(\mathbf{h}\mathbf{S}^x_{k+1/k}\mathbf{h}^T + R)^{-1} \tag{20}$$

$$\mathbf{S}^x_{x+1/k+1} = (\mathbf{I} - \mathbf{K}^x_{k+1}\mathbf{h})\mathbf{S}^x_{k+1/k} \tag{21}$$

$$\hat{\mathbf{X}}_{k+1/k} = \mathbf{F}\hat{\mathbf{X}}_{k/k} + \mathbf{G}W_k \tag{22}$$

$$\mathbf{S}^x_{k+1/k} = \mathbf{F}\mathbf{S}^x_{k/k}\mathbf{F}^T \tag{23}$$

The filtered estimate is utilized to obtain the smoothed values of the state in Eq. (16)-Eq. (18).

When $\hat{\mathbf{X}}_{k+1/k+1}$ is calculated, the values of parameters **F** and **G** are necessary. However, their values have not known precisely, so we employ $\mathbf{P}_{k/k}$ which is the parameter estimate at time before one sampling time.

The overall flow chart for identification is shown in Fig. (1).

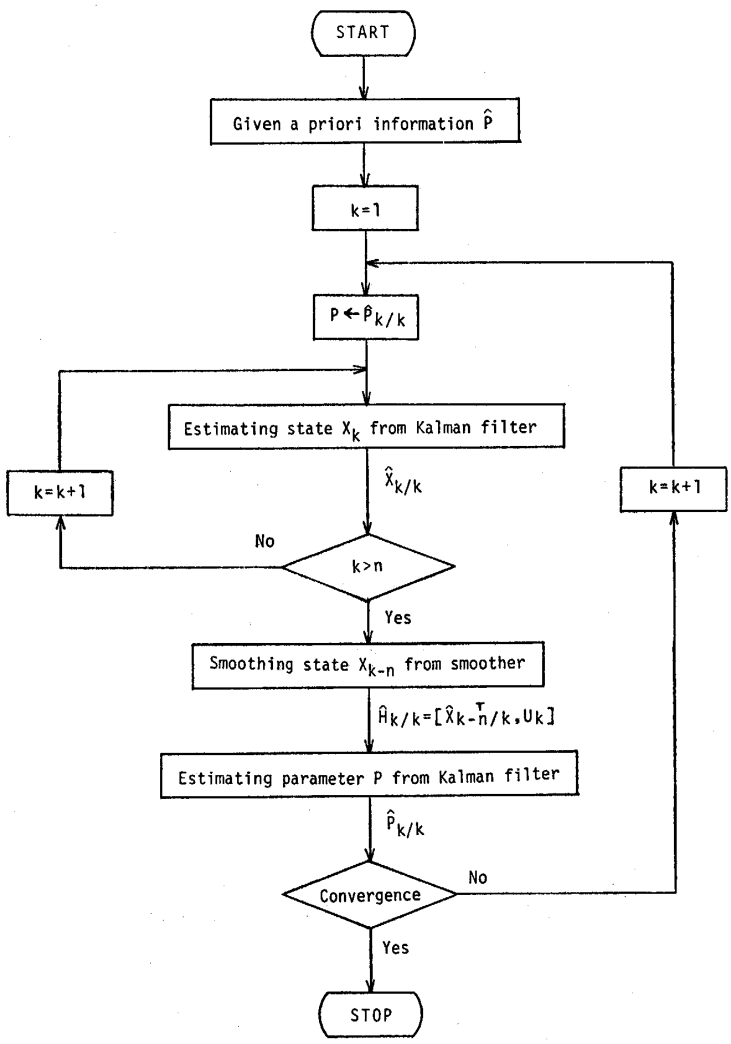

Fig. 1 The overall flow chart for identification.

Fictitious Noise Input to Parameter

It is known in Ref. 5) that the parameter estimates described in the previous section do not converge into the true values because the filter gain becomes increasingly small as the time increases. This is as same as the estimation problem with modeling error[4]. It is seen that in such a case, fictitious noise input, which is essentially cover model error, can be effective. We take this approach for error compensation.

It is assumed that the unknown parameter is described as,

$$\mathbf{P}_{k+1}=\mathbf{P}_k+\xi_k \tag{24}$$

where, ξ_k is gaussian noise which is $N(\mathbf{O},\ \mathbf{Q})$.

In this case, the parameter estimation algorithm is represented as follows instead of Eq. (13)-Eq. (15).

$$\hat{\mathbf{P}}_{k+1/k+1}=\hat{\mathbf{P}}_{k+1/k}+\mathbf{K}^p_{k+1}(Z_{k+1}-\hat{\mathbf{H}}_{k+1/k+1}\hat{\mathbf{P}}_{k+1/k}) \tag{25}$$

$$\mathbf{K}^p_{k+1}=\mathbf{S}^p_{k+1/k}\mathbf{H}_{k+1/k}(\mathbf{H}_{k+1,k}\mathbf{S}_{k+1/k}\mathbf{H}_{k+1/k}+R)^{-1} \tag{26}$$

$$\mathbf{S}^p_{k+1/k+1}=(\mathbf{I}-\mathbf{K}^p_{k+1}\mathbf{H}_{k+1/k+1})\mathbf{S}^p_{k+1/k} \tag{27}$$

$$\mathbf{P}_{k+1/k}=\mathbf{P}_{k/k} \tag{28}$$

$$\mathbf{S}^p_{k+1/k}=\mathbf{S}^p_{k/k}+\mathbf{Q} \tag{29}$$

The selection of the value of $\mathbf{Q}$ is rather difficult, but its effective value can be determined by trial and error.

Numerical Example

The 3rd order system is considered as a numerical example.

The system is described as follows.

$$\mathbf{X}_{k+1}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -0.5 & -0.8 & -0.3\end{bmatrix}\mathbf{X}_k+\begin{bmatrix}1\\ 0.2\\ 0.7\end{bmatrix}\mathbf{U}_k$$

$$Z_k=[1\quad 0\quad 0]\mathbf{X}_k+V_k$$

We employ the following data.

Initial true state: $\mathbf{X}=[1\quad 1\quad 1]^T$

A priori state: $\hat{\mathbf{X}}=[0.8\quad 1.2\quad 0.8]^T$

Error covariance of initial state: *diag.* [0.05 0.05 0.05]

True parameter: $\mathbf{P}=[-0.5\quad -0.8\quad -0.3\quad 1.0\quad 0.2\quad 0.7]^T$

A priori parameter: $\hat{\mathbf{P}}=[-0.4 \quad -0.6 \quad -0.2 \quad 0.8 \quad 0.3 \quad 0.6]^T$
Error covariance of parameter: *diag.* [0.01 0.05 0.01 0.05 0.01 0.01]
Measurement noise: $N(0, 10^{-6})$

Results and Discussion

The performance index for the comparison of results of estimation is defined as normalized average error concerning all unknown parameters.

$$\text{Performance index}=\frac{1}{6}\sum_{i=1}^{6} |\tilde{P}_k^i|/|\tilde{P}_0^i|$$

P_k^i is the error of the i-th element of P_k at time k and P_0^i is the one of the initial estimate P_0.

The comparison of the performance index is shown according to different fictitious noise inputs in Table 1.

The diagonal elements of the covariance $\mathbf{Q}$ in Table 1 have been selected from 0.0 to 10^{-5} with step 10^{-1}.

We may select 10^{-4} as the proper value in this example from Table 1. Of course, this value should not be the optimal value for fictitious noise inputs, but it is near to the optimal one in this simulation.

In Fig. 2-Fig. 7, the normalized errors of estimates of parameters which are defined as,

$$\varepsilon_i=|\tilde{P}^i|/|\tilde{P}_0^i|$$

are shown.

The estimates of a_1, a_2, a_3 and b_1 are near to their true values respectively, but the ones of b_1 and b_3 do not approach their true values. The effects of fictitious noise inputs are denoted by the dotted lines, but they are not very distinguished.

However, after all, if the value of $\mathbf{Q}$ is selected properly, the estimation is rather better. Therefore the fictitious noise inputs are available for the correction of the estimates of parameters. It is rather difficult to select the proper value of the covariance of fictitious noise. The convenient method to select the covariance is to take the square of 1 per cent value of unknown parameter.

Table 1. Performance index versus covariance of fictitious noise Q.

Stage \ Q	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	0.0
4	0.558E+0	0.558E+0	0.558E+0	0.558E+0	0.558E+0	0.558E+0
6	0.122E+1	0.374E+0	0.374E+0	0.374E+0	0.374E+0	0.374E+0
8	0.354E+0	0.411E+0	0.288E+0	0.285E+0	0.285E+0	0.285E+0
10	0.326E+0	0.294E+0	0.246E+0	0.222E+0	0.225E+0	0.226E+0
12	0.270E+0	0.143E+2	0.256E+0	0.201E+0	0.199E+0	0.199E+0
14	0.285E+0	0.264E+0	0.254E+0	0.200E+0	0.203E+0	0.204E+0
16	0.283E+0	0.219E+0	0.255E+0	0.198E+0	0.202E+0	0.203E+0
18	0.243E+0	0.223E+0	0.255E+0	0.196E+0	0.200E+0	0.201E+0
20	0.424E+0	0.231E+0	0.255E+0	0.196E+0	0.200E+0	0.201E+0
25	0.387E+0	0.237E+0	0.255E+0	0.196E+0	0.200E+0	0.201E+0
30	0.111E+1	0.238E+0	0.255E+0	0.196E+0	0.200E+0	0.201E+0
50	0.561E+0	0.238E+0	0.255E+0	0.196E+0	0.200E+0	0.201E+0
100	0.514E+0	0.238E+0	0.255E+0	0.196E+0	0.200E+0	0.201E+0
500	0.513E+0	0.238E+0	0.255E+0	0.196E+0	0.200E+0	0.201E+0
1000	0.513E+0	0.238E+0	0.255E+0	0.196E+0	0.200E+0	0.201E+0

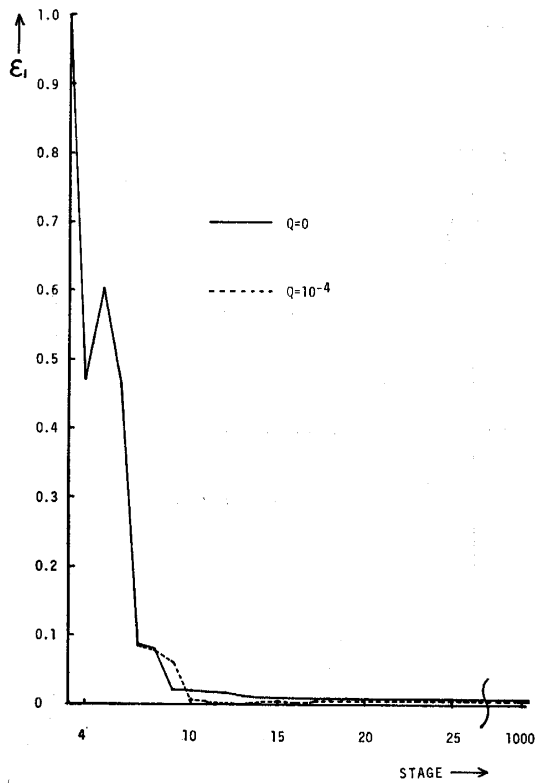

Fig. 2 Normalized error of a_1.

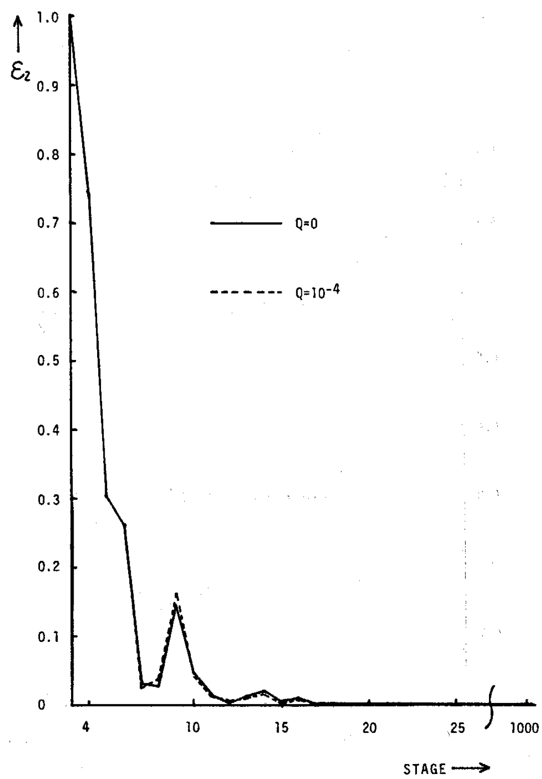

Fig. 3 Normalized error of a_2.

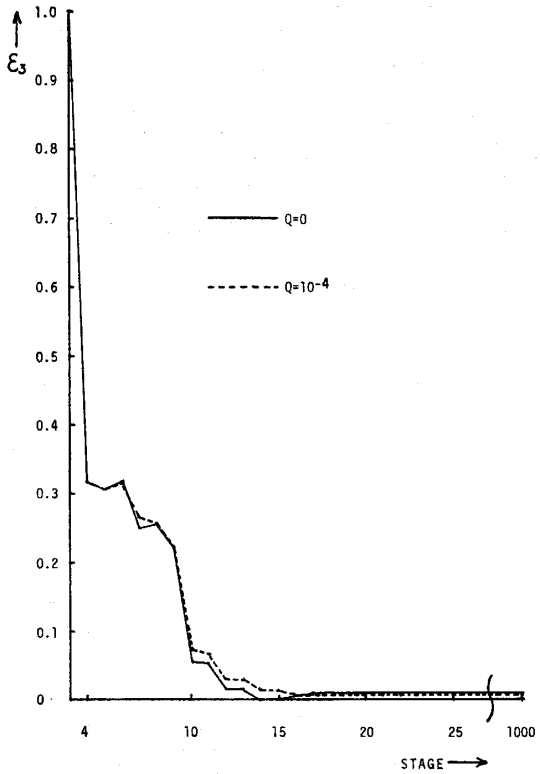

Fig. 4 Normalized error of a_3.

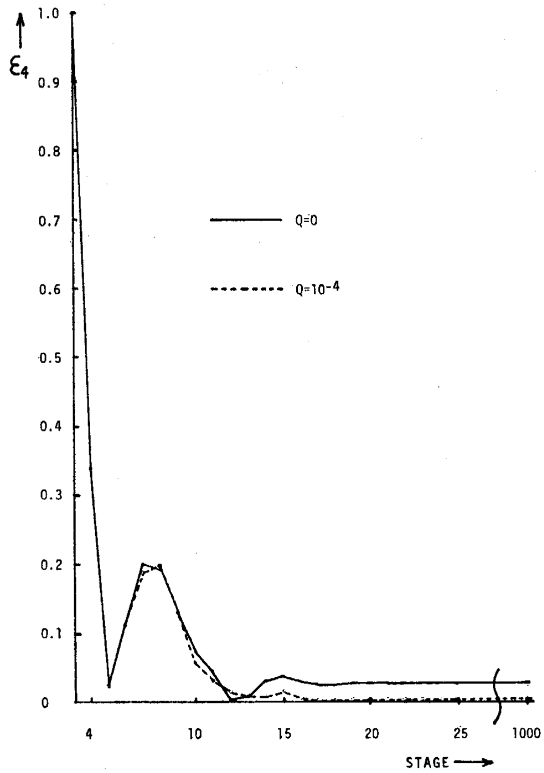

Fig. 5 Normalized error of b_1.

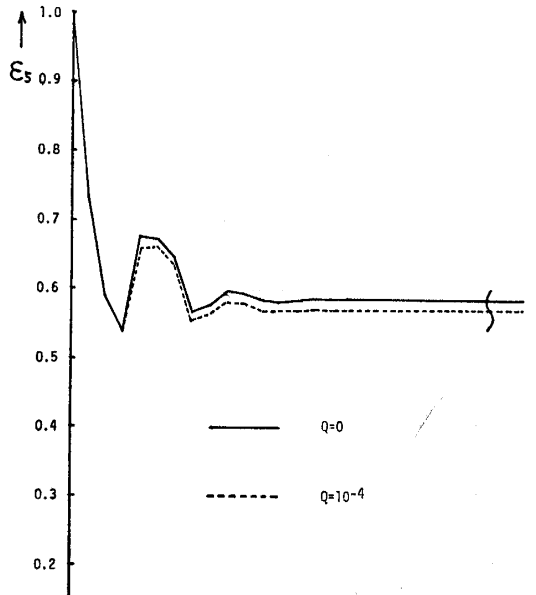

Fig. 6 Normalized errorf b_2.

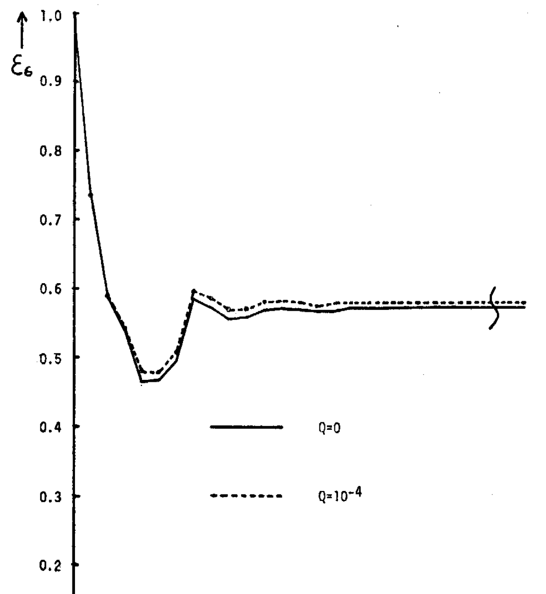

Fig. 7 Normalized error of b_3.

Conclusion

A method of identification and state estimation in the linear discrete-time system which has single input has been represented in the canonical form. The procedure requires the Kalman filter which is applied to state estimation and parameter estimation at each stage and the smoother for state.

The smoother is utilized to decide the observation matrix in the Kalman filter for parameter identification.

If an a priori information is given, the estimates of the state and the parameter is obtained alternatively. However, the assumption requires the knowledge of the order of the system and the noise statistics.

We have shown two approaches with parameter additive noise and without it. The concept of fictitious noise inputs has been originally employed in the area of state estimation of the system which has model errors and has been able to be effective, in especial, in preventing divergence. It is shown that it is useful for the identification problem considered in this paper. The effective choice of fictitious noise inputs has led rather better estimation in the numerical example.

References

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