The Determination of Flow Velocity Porous Media, Using the Allen Method[1)]

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Abstract

One of the most important causes of the destraction of dikes by waves is that the sand particles inside the dikes are drawn out by the waves. To analyze this kind of phenomenon theoretically, it becomes necessary to know the the flow velocity through the porous media. In the past, several studies have seen made on this problem. Here, an investigation is made on the possibility of measuring the flow velocity through porous media by the Allen method applying the variation of electrical conductivity by using salt water solutions.

As the results, the accuracy of this method was found to be theoretically 98.4% and experimentally 97%, and it was found that the Allen method can be applied to measure the flow velocity through porous media.

Introduction

There are many permeable structures along shore-lines. These structures are badly dammaged by Suction (which is called Suidashi in Japanese). The purpose of this study is to analyse the Suction phenomenon in permeable structures. Permeable structures on shore-lines have conciderable porocity and a high transmittance of flow. There are a number of studies on Darcy's law on flow through these kinds of rough porous media. In order to elocidate the mechanism of Suction on these structures, it is necessary to evaluate the effect of the friction of the flow through them.

In order to evaluate the effect of this friction, it is necessary to measure the flow velocity. The author adopted the Allen method to measure this velocity. In this paper, the use of the Allen method, its accuracy of measurement and its application, together with an example is dresented.

Application of the Allen method

We shall briefly describe our application of the Allen method in this section. Fig. 1 shows the equipment used.

When fresh water flows through the pipe, the electrical resistance between C and D is high so that the ammeter indicates low values. But, as soon as a salt water solution is introduced into the pipe, the electrical resistance disappears and the ammeter indicates high values. This variation of electric current is recorded on a pen-recorder.

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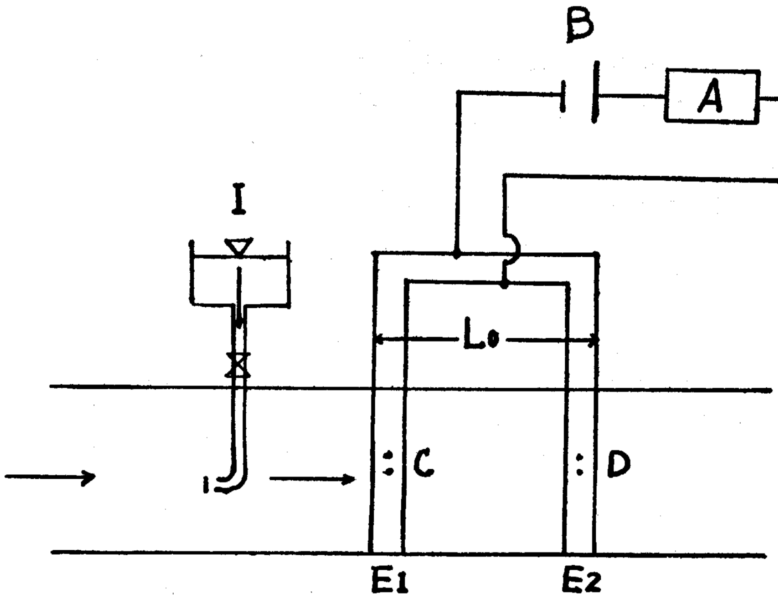


Fig. 1. Diagram of equipment

A: An ammeter
B: The electric power source
E1: The first electrode
E2: The second electrode
I: The salt water mixing equipment

Fig. 4, 5 and 6 shows an example of measurements taken using the Allen method. The distance between the center of gravity of first peak and the second peak on the pen-recorder indicates the flow time (t_m). The vlocity is determined by the next equation.

$$V_m = \frac{L_1 - L_0}{t_m}$$

V_m : Velocity
$L_1 - L_0$ = Distance between the first electrode and the second electrode

The salt water solution used in this experiment has a consistency of 13.2% and conductivity of $16.8 \times 10^4 \mu\mho/\mathrm{cm}$.

The method and the example

The experimental equipment

Fig. 2 and 3 show general views of the electrode and the circuit for taking measurements.

The example[2)]

Figs. 4, 5 and 6 show examples of measurements taken in the pipe.

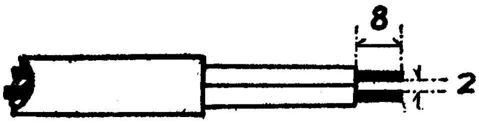


Fig. 2. Electrode

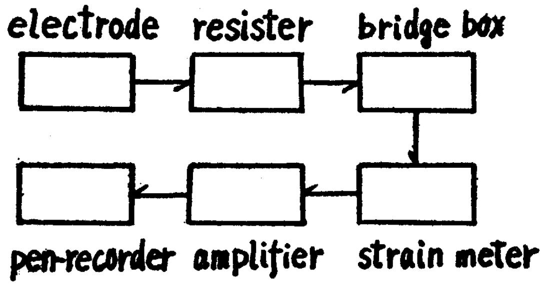
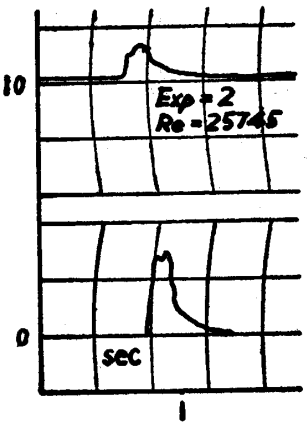
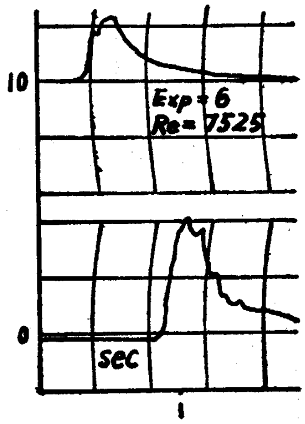


Fig. 3. Equipment for measurement





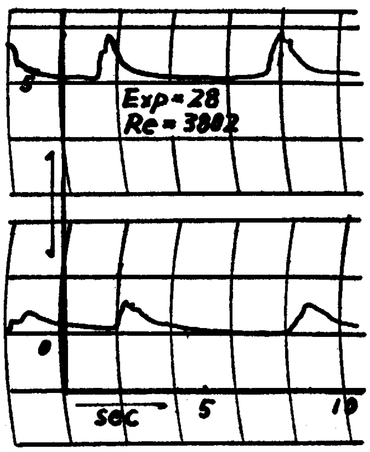


Fig. 4~6. Measurements taken as on example of Allen's method

Theoretical and experimental investigation of the accuracy of Allen's method[1)]

Theoretical investigation of the accuracy of Allen's method

The following four assumption are basic to the devlepment of a theory on the accuracy of Allen's method.

1. The pipe has a circular cross section.
2. The voltex length is negligible, compared with the diameter of the cross section.

3. The salt water solution is poured into the pipe rapidly.
4. The velocity distribution in the pipe is defined by the equation:

$$V=V_{max}(1-x^2)^{\frac{1}{n}} \qquad x=\frac{r}{R} \tag{1}$$

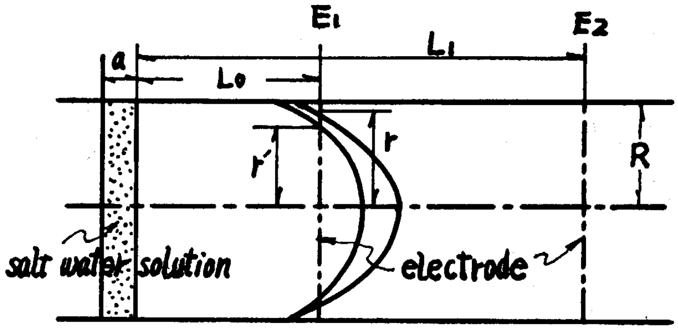


Fig. 7.

The time required for a particle of salt water solution to flow from point E on electrode E1 to point F on electrode E2 is given in the next equation.

Thus,

$$t=\frac{L_0}{V} \tag{2}$$

From equations (1) and (2),

$$x^2=1-\left(\frac{1}{\tau}\right)^n \qquad \tau=\frac{t}{t_0} \qquad t=\frac{L_0}{V_{max}} \tag{3}$$

where τ is a non-dimensional term.
The same was applied to point D.

Thus,

$$x^2=1-\left(\frac{1+\alpha}{\tau}\right)^n \qquad x'=\frac{r'}{R} \qquad \alpha=\frac{a}{L_0} \tag{4}$$

Further on, in this section, we will make the assumption that the recorded amperage is in direct proportion to the electrodes immersed in the salt water solution.

On these assumptions, the next equation was obtained,

$$i=x-x'=\sqrt{1-\left(\frac{1}{\tau}\right)^n}-\sqrt{1-\left(\frac{1+\alpha}{\tau}\right)^n} \tag{5}$$

Equation (5) gives a compound curve of the electric current.
Fig. 8 shows this compound curve. ($\alpha=0.2$, $\lambda=4.0$)

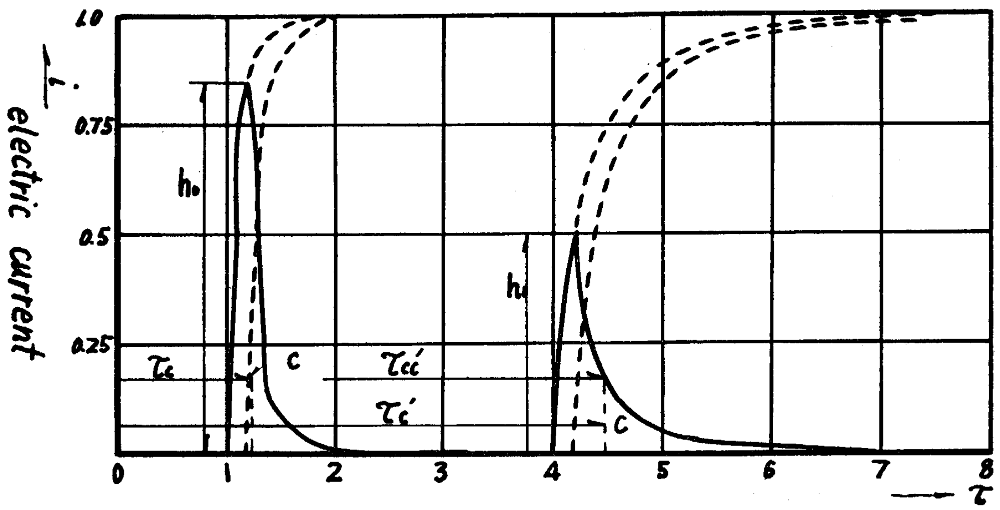


Fig. 8. A theoretical compound curve of the electric current

It is evident from Fig. 8 that this compound curve resembles closely the curves in Fig. 4, 5 and 6. This resemblance indicates that the assumptions used in hte thoery are correct.

The accuracy of the Allen method will be taken up next. First, we will consider electrode E1. The area A which is defined by a curve of electric current and abscissa is obtained by the next equation.

$$A=\int_1^\infty\sqrt{1-\left(\frac{1}{\tau}\right)^n}d\tau-\int_{1+\alpha}^\infty\sqrt{1-\left(\frac{1+\alpha}{\tau}\right)^n}\ d\tau=\left(1+Z_n\right)\alpha \qquad (6)$$

$$Z_n=\int_0^1\frac{1-\sqrt{1-Z^n}}{Z^2}\ dZ=0.1008 \qquad (7)$$

The value of Z_n was calculated by the Simpson method ($n=7$). The goemetrical moment of this curve is given in the next equation.

$$G=\int_1^\infty\sqrt{1-\left(\frac{1}{\tau}\right)^n}\ \tau d\tau-\int_{1+\alpha}^\infty\sqrt{1-\left(\frac{1+\alpha}{\tau}\right)}\tau d\tau=\left(\frac{1}{2}+Z_n'\right)(2+\alpha)\alpha \quad (8)$$

$$Z_n'=\int_0^1\frac{1-\sqrt{1-Z^n}}{Z^3}dZ=0.1185 \qquad (9)$$

Hence, the center of gravity is found from:

$$\tau_0=\frac{G}{A}=\left(\frac{1+2Z_n'}{1+Z_n}\right)\left(1+\frac{\alpha}{2}\right) \qquad (10)$$

In the same way, the center of gravity for electrode E2 was obtained.

$$\tau_c'=\frac{1+2Z_n'}{1+Z_n}\left(\lambda+\frac{\alpha}{2}\right) \qquad (11)$$

From equation (10) and (11), the distance between the centers of gravity was obtained.

$$\tau'_{cc}=\tau'_c-\tau_0=\frac{1+2Z'_n}{1+Z_n}(\lambda-1) \tag{12}$$

We assumed the velocity distribution in the pipe as in equation (1). The mean velocity is given in the next equation.

$$V_m=\frac{1}{r}\int_0^r V_{\max}(1-x^2)^{\frac{1}{n}}dr=\frac{n}{n+1}V_{\max} \tag{13}$$

The required time, t when a particle of salt water solution flows from electrode E1 to electrode E2 is given in the next equation:

$$t=\frac{L_1-L_0}{V_m}$$

and a dimentionless quantity is seen next.

$$\tau_m=\frac{t}{t_0}=\frac{n+1}{n}(\lambda-1) \tag{14}$$

Hence, we can define the accuracy of the Allen method by equation (15).

$$\xi=\frac{\tau'_{cc}}{\tau_m}=\frac{n(1+2Z'_n)}{(n+1)(1+Z_n)} \tag{15}$$

If we assume that $n=7$ then we obtain

$$\xi=0.984 \tag{16}$$

That is, we can say that the experimental results by the Allen method have a 98.4% accuracy.

The experimental investigation of the accuracy of measurements[2)]

We assume velocity distribution to be log-distribution so that

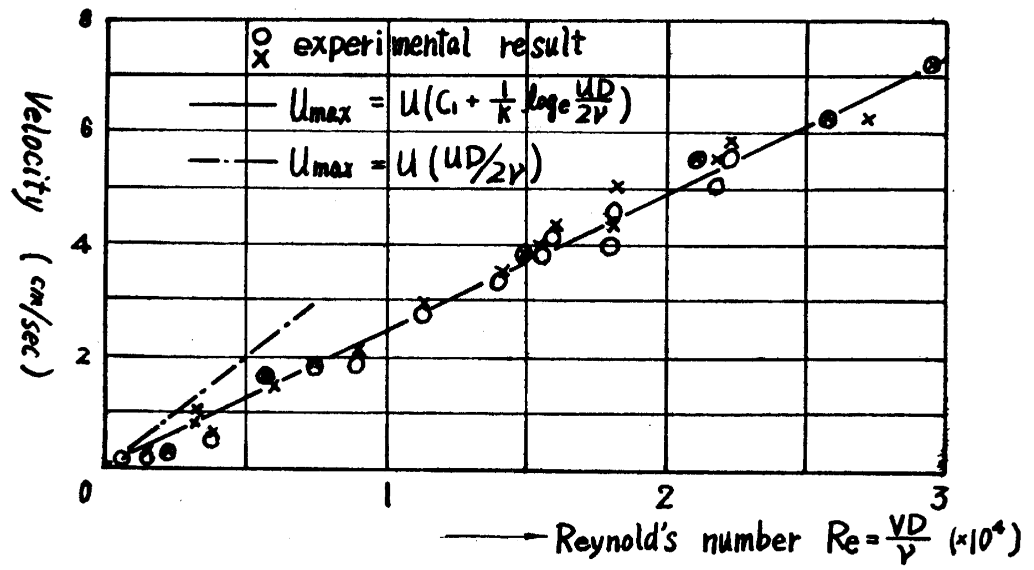


Fig. 9.

$$\frac{V_{max}}{u}=C_1+\frac{1}{K}\log_e\frac{VD}{2r} \tag{17}$$

This investigation covers a wide range of Reynold's numbers (700<Re<30,000). The experimental results which were obtained in the way described above are shown in Fig. 9. What is evident from Fig. 9 is that the Allen method met our expectations, especially with high velocity flow. It's accuracy is about 97% within the limits of this experiment.

An application of the Allen method

A profile of flow in porous media

Fig. 10 shows the experimental equipment. Figs. 11, 12 and 13 show experimental electric current curves in porous media. The porous media were made up of glass balls whose diameter were 2.45 cm, 1.69 cm and 1.2 cm. Experiments were conducted in areas measuring 50 cm and 30 cm in length. Fig. 14 gives the relationship between the experimental result U_s by the Allen method, and the mean velocity U_t. U_t is defined in the next equation.

$$U_t=\frac{Q}{\lambda A}$$

Q =The flow quantity
A =Cross section area
λ =Porosity

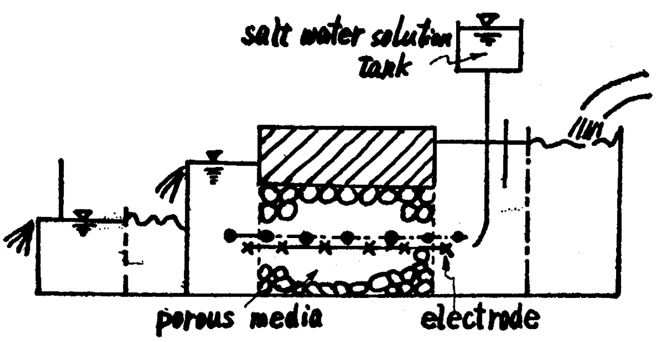


Fig. 10. Experimental equipment

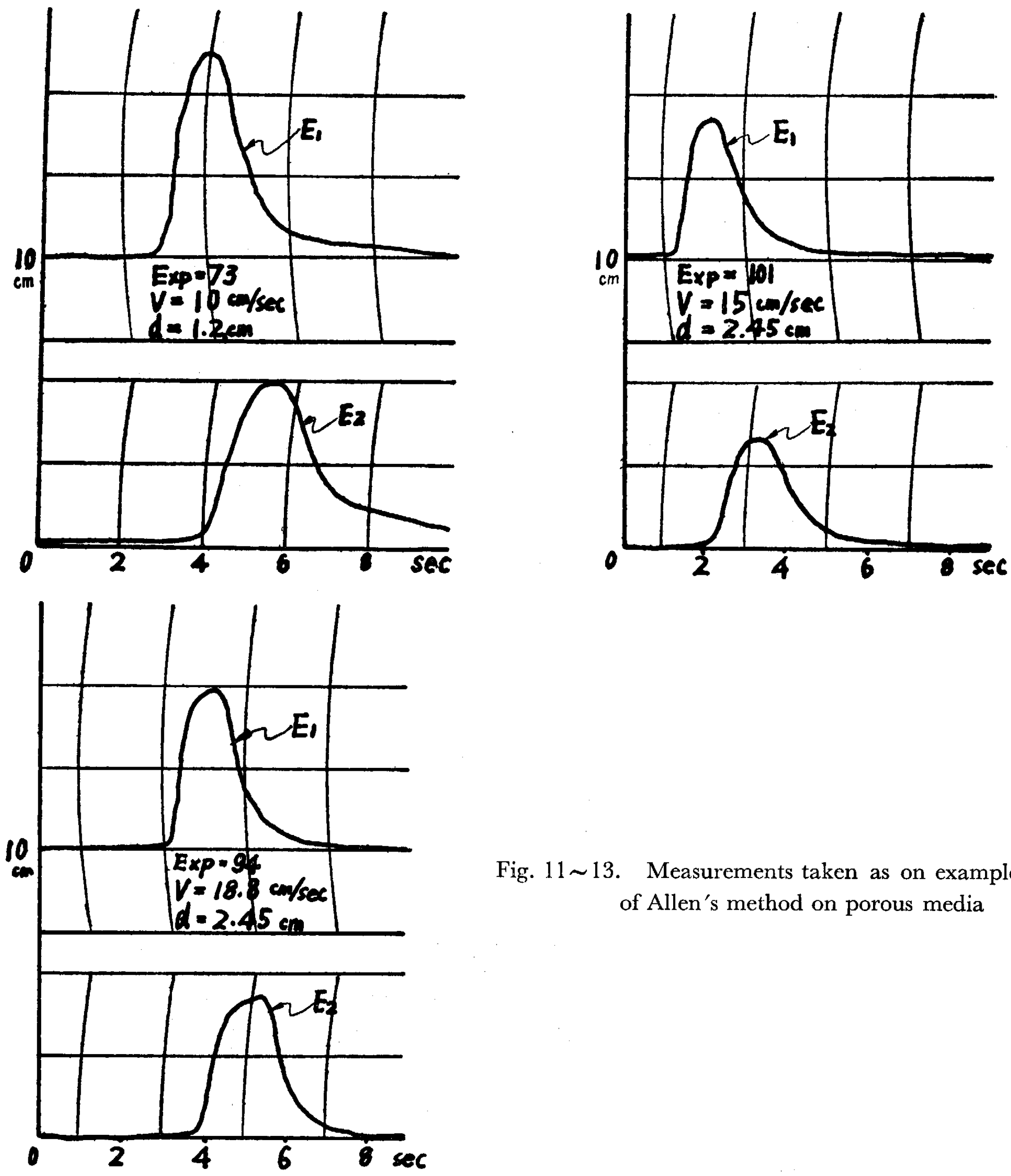


Fig. 11~13. Measurements taken as on example of Allen's method on porous media

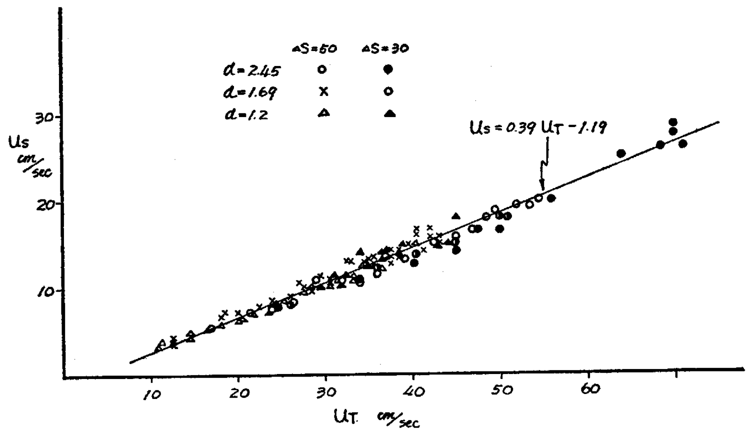


Fig. 14. The relationship between U_s and U_t

We obtained the next relationship from Fig. 14.

$$U_s = 0.39 U_t - 1.19 \tag{18}$$

From equation (18), it is evident that water particles in porous media move in a zig-zag manner, so that,

$$\frac{l}{t} = U_s \qquad \frac{l + \Delta l}{t} = U_t = \frac{Q}{\lambda A}$$

$$\frac{U_t}{U_s} = \frac{l + \Delta l}{l} = \frac{1}{0.39} \qquad L = l + \Delta l$$

$$l = 0.39L \qquad L \doteqdot 2.5l \tag{19}$$

From equation (19), it is calculated that water particles in porous media move $2.51 \times l$ length, during their passage through the porous media. These results are in agreement with the results of Dr. KIMURA.[3)]

Conclusion

As shown in Figs. 4, 5, 6 and Figs. 11, 12, 13, it is possible to obtain a smooth compound curve which almost coincides with a theoretical compound curve of electric current. The following assumption were made in working out the theory of accuracy.

1. The pipe has a circular cross section.
2. Voltex length is negligible, compared with the diameter of cross section.
3. The salt water solution is poured into the pipe in a short time.
4. Velocity distribution in the pipe is defined by the next equation.

$$V = V_0(1 - x^2)^{\frac{1}{n}} \qquad x = \frac{r}{R} \tag{1}$$

Under the above assumptions, we obtained that the Allen method's theoretical accuracy is defined by the equation:

$$\xi=\frac{\tau'_{cc}}{\tau_m}=\frac{n(1+2Z'_n)}{(n+1)(1+Z_n)} \tag{15}$$

If we assume $n=7$, then,

$$\xi=\frac{\tau'_{cc}}{\tau_m}=0.984 \tag{16}$$

From equation (16), it is clear that the Allen method has a 98.4% accuracy. This result agrees very well with the experimental. From these facts, it may be concluded that the Allen method may be used to measure the flow velocity through porous media.

References

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3) H. Kimura: Trans. Jap. Soc. IDR Eng. **40,** 30 (1971)