On the Plastic Deformations of Some Steel Beams under Equally Distributed Impulse (2nd Report)

Sachio MOGAMI*, Yōichi TAMURA* and Kazuhiro KOJIMA**

(Received July 2, 1976)

Abstract

We wrote the study on the plastic deformations of some steel beams under equally distributed impulse acting on the symmetrical positions in the first report[1] and this time treated the cases in which the equally distributed impulse acts on the unsymmetrical positions in this report.

1. Introduction

We have been making the theoretical and experimental studies on the plastic deformations of some steel beams[2),3)], i.e., canti-lever beams, simple beams and fixed beams on both ends, under the equally distributed impulse.

In this report we treated the cases in which the equally distributed impulse acts on the unsymmetrical positions of several beams. Some assumptions in our theoretical treatments are as follows:

(1) The plastic deformations of steel beams are considerably large, so we can neglect the elastic deformations, i.e., the rigid-plastic theory is introduced.

(2) After the collision the impulsive body moves together with the beam.

(3) At the impulsive time the dynamic total plastic moment in the section of the steel beam is considerably larger than the static one, and the relation is shown in the next equation.

$$M_a = M_0\{1+(v/2Dl)^{1/P}\}$$

where,

M_d : Dynamic total plastic moment

M_o : Static total plastic moment

v : Vertical velocity in the impulsive point

l : Bending length of the span

D, P: Constants of materials ($D=40.4\ \mathrm{sec}^{-1}$, $P=5$ for steel)

(4) The vertical moving velocity at the impulsive point is generally expressed as follows:

$$v = v_0\{1-(t/t_f)^n\}$$

where,

* Department of Civil Engineering

** Sonnou Ryokuchi Co., Ltd.

v : Above mentioned
v_0 : Initial vertical velocity at the impulsive moment
t : Arbitrary time $(0 \leqq t \leqq t_f)$
t_f : Time from the impulsive moment to the completion of the plastic deformation
n : Constant decided from the experiment

2. The General Outlines of Experiments

The general outlines of experiments are almost same as the first report, and the contents are as follows: i.e., at first for the experiments of the canti-lever beams the equally distributed impulse was acted on the point of quarter of the span from the tip end for convenience sake.

For the equally distributed impulse we used the drop weights, i.e., 3.09, 5.12, 7.07 and 10.01 Kg. The spans are 30~70 cm (changed for every 10 cm) and the drop heights are 20~60 cm (changed for every 10 cm). For steel beams we used square sections, 0.95 × 0.95 cm.

Next for the experiments of simple beams and fixed ones on both ends the equally distributed impulse was acted on the point of quarter of the span from the one side end, and the drop weights were 7.07, 10.01, 12.22 and 15.03 Kg (every distributed length was 15 cm); the drop heights of the weights were 30~70 cm (changed for every 10 cm).

We repeated the experiments five times in the same conditions and tried once more again for the abnormal results. We represented the experimental results considering the average values of five times to be the required experimental values.

3. Theoretical Considerations

In our analyses held hither-to the vertical moving velocities of the plastic deformations were assumed approximately to be linearly changed and the calculated results were quite in good agreement with the experimental values. But for the cases of acting on the arbitrary points of this time we could not obtain the good agreement with the experimental results, so the next equation was assumed for the velocity change.

$$v = v_0\{1-(t/t_f)^n\} \tag{1}$$

Signs are above mentioned.

Thus we can obtain the appropriate velocity changes by deciding n values which are in agreement with the experimental values.

But from the standpoint of the experimental apparatus the real conditions of the vertical velocity changes can't be measured, so that the conformity of the velocity changes must be checked in future.

This time we developed the theoretical equations under the assumptions above. Details for the cases of several beams are as follows:

(1) The case of the canti-lever beam (Fig. 1)

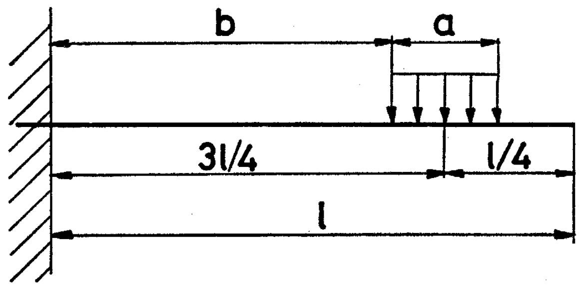

Fig. 1 Canti-lever beam under equally distributed impulse on the arbitrary point.

Considering the equilibrium equations of the momentum in the case of the canti-lever beam under equally distributed impulse and applying the assumptions above, we obtain next expressions.

$$\delta/l = \frac{n}{n+1}\alpha\gamma(LN)^{-1} \tag{2}$$

$$\left.\begin{aligned} &\alpha = I^2/(M_0 ma), \quad \gamma = l'/l, \quad l' = a/2 + b, \\ &L = 1 + \overline{m}'/(ma), \qquad \overline{m}' = m'l/3\cdot(l/l')^2, \\ &N = 1 + (v_0/2Dl')^{1/P}S, \quad S = \int_0^1 (1-x^n)^{1/P}dx, \\ &v_0 = I/(ma + \overline{m}'), \qquad I = ma\sqrt{2gh} \end{aligned}\right\} \tag{3}$$

Where,

δ : The vertical plastic deformation of the impulsive point
l : The length of the span of the canti-lever beam
n : The same as eq. (1)
I : The impulsive momentum
δ : The vertical plastic deformation of the impulsive point
l : The length of the span of the canti-lever beam
n : The same as eq. (1)
I : The impulsive momentum
a, b, l': Refer to Fig. 1
v_0 : The first velocity in the moment of the impulse
D, P : Constants of materials ($D=40.4\ \text{sec}^{-1}$, $P=5$ for the steel)
m : Mass of the weight per unit length
M_0 : The total plastic moment of the steel beam's section
g : Gravity acceleration
h : Drop height of the weight

And the integral values S are indicated in Table 1.

(2) The case of the simple beam (Fig. 2)

In the case of the simple beam refering to Fig. 2, we divide impulsive momentum

Table 1 The integral values of $S=\int_0^1(1-x^n)^{0.2}dx$

n	S	n	S	n	S
0.10	0.5725	1.45	0.8679	2.80	0.9183
0.15	0.6172	1.50	0.8709	2.85	0.9194
0.20	0.6500	1.55	0.8738	2.90	0.9205
0.25	0.6759	1.60	0.8765	2.95	0.9215
0.30	0.6973	1.65	0.8791	3.00	0.9226
0.35	0.7154	1.70	0.8816	3.05	0.9236
0.40	0.7310	1.75	0.8840	3.10	0.9246
0.45	0.7447	1.80	0.8863	3.15	0.9225
0.50	0.7560	1.85	0.8885	3.20	0.9264
0.55	0.7678	1.90	0.8906	3.25	0.9274
0.60	0.7777	1.95	0.8927	3.30	0.9282
0.65	0.7866	2.00	0.8946	3.35	0.9291
0.70	0.7948	2.05	0.8965	3.40	0.9299
0.75	0.8024	2.10	0.8983	3.45	0.9308
0.80	0.8093	2.15	0.9001	3.50	0.9316
0.85	0.8157	2.20	0.9018	3.55	0.9323
0.90	0.8217	2.25	0.9034	3.60	0.9331
0.95	0.8273	2.30	0.9050	3.65	0.9338
1.00	0.8325	2.35	0.9065	3.70	0.9346
1.05	0.8374	2.40	0.9080	3.75	0.9353
1.10	0.8420	2.45	0.9094	3.80	0.9360
1.15	0.8464	2.50	0.9109	3.85	0.9367
1.20	0.8505	2.55	0.9121	3.90	0.9373
1.25	0.8543	2.60	0.9134	3.95	0.9380
1.30	0.8580	2.65	0.9147	4.00	0.9386
1.35	0.8615	2.70	0.9157		
1.40	0.8648	2.75	0.9171		

I in x section and $(a-x)$ section and express them I_x and I_{a-x} respectively.

Using these expressions and constituting the equilibrium equations of the impulsive momentum to the sections b and c respectively, we obtain next expressions.

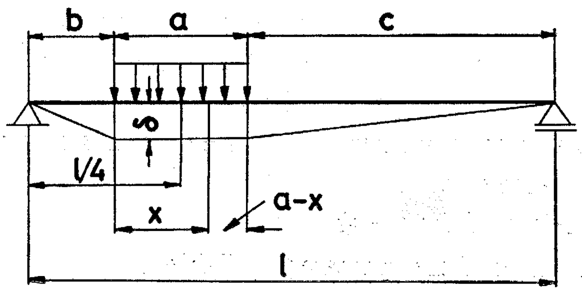

Fig. 2 Simple beam under equally distributed impulse on the arbitary point.

For the section b,

$$\delta/l=\frac{n}{n+1}\alpha_1\frac{2bx}{al}(L_1N_1)^{-1} \tag{4}$$

$$\left.\begin{aligned}&\alpha_1=I^2/(2M_0ma), \qquad L_1=1+(a/\beta l)(1+b/2x),\\&N_1=1+(v_0'/2Db)^{1/p}S, \quad \beta=ma/(m'l),\\&v_0'=\sqrt{2gh}/L_1\end{aligned}\right\} \tag{5}$$

For the section c,

$$\delta/l=\frac{n}{n+1}\alpha_1\frac{2c(a-x)}{al}(L_2N_2)^{-1} \tag{6}$$

$$\left.\begin{aligned}&L_2=1+(a/\beta l)\{1+c/2(a-x)\},\\&N_2=1+(v_0''/2Dc)^{1/p}S,\\&v_0''=\sqrt{2gh}/L_2\end{aligned}\right\} \tag{7}$$

where, signs are almost same as eq. (3) and others are indicated in Fig. 2. N must be decided that eq. (4) may coincide with eq. (6) by the trial and error method.

(3) The case of the fixed beam on both ends (Fig. 3)

In the case of the fixed beam on both ends we can treat in the same method as the simple beam. Theoretical equations are expressed in the same ones as eq. (4) ~eq. (7) by substituting $\alpha_2=\alpha_1/2$ for α_1.

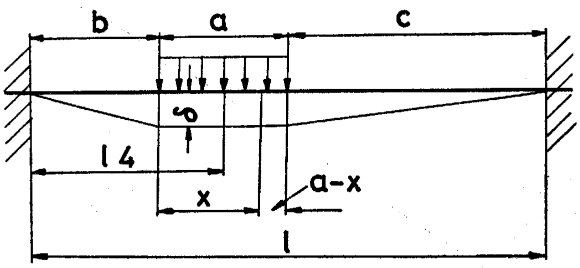

Fig. 3 Fixed beam on both ends under equally distributed impulse on the arbitrary point.

4. Analyses and Comparison wlth Experimental Results

Comparing the analytical results above described with the experimental values, some representative samples are indicated in Fig. 4~Fig. 9.

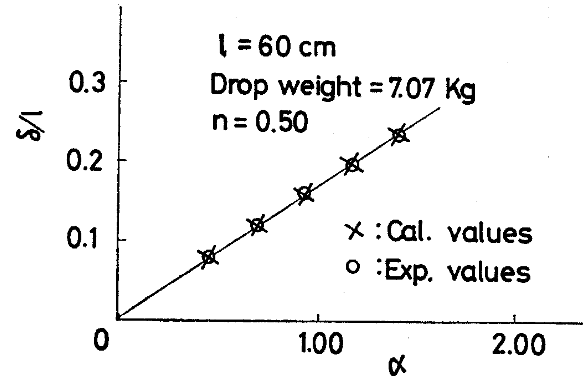

Fig. 4 Comparison of the calculated values with the experimental results (canti-lever beam).

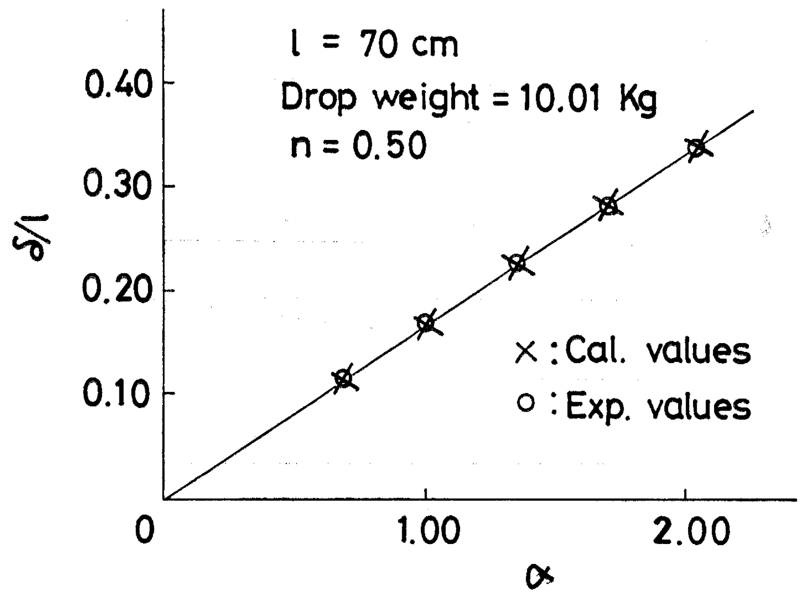

Fig. 5 Comparison of thecalculated values with the experimental results (canti-lever beam).

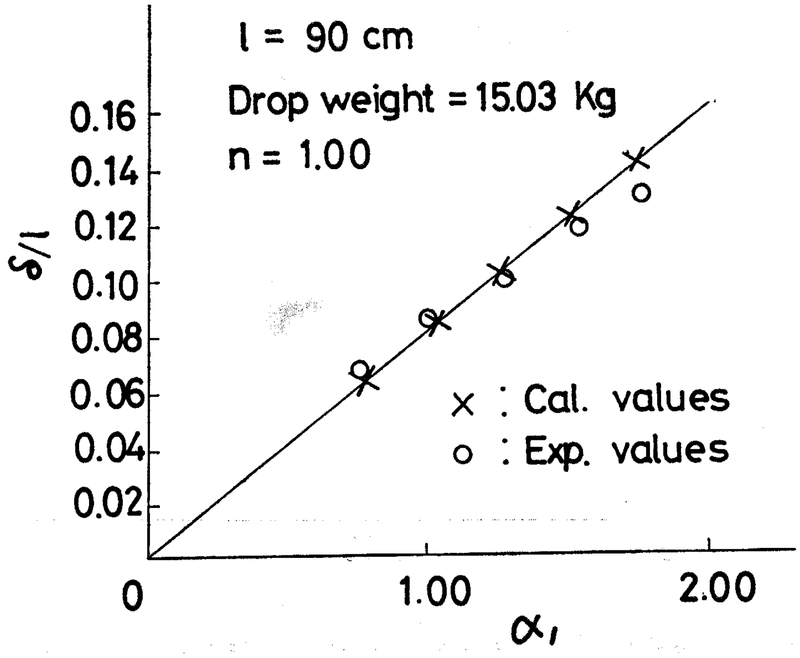

Fig. 6 Comparison of the calculated values with the experimental results (simple beam).

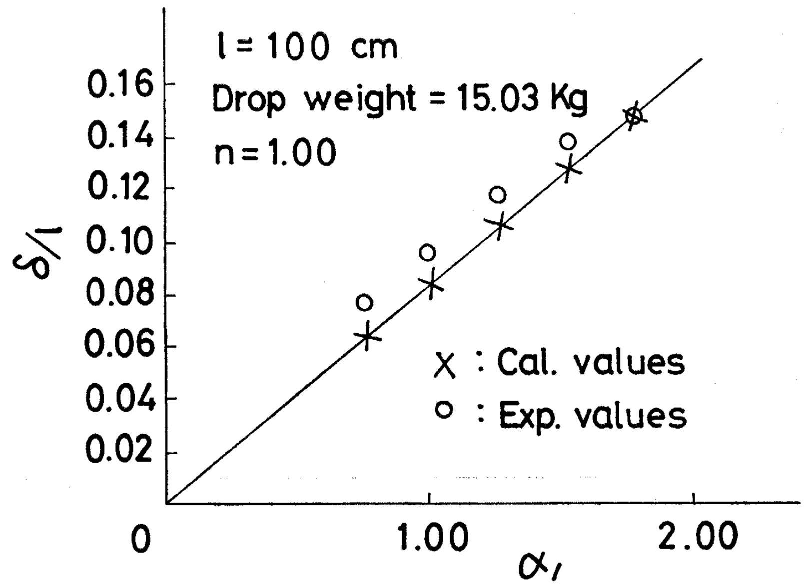

Fig. 7 Comparison of the calculated values with the experimental results (simple beam).

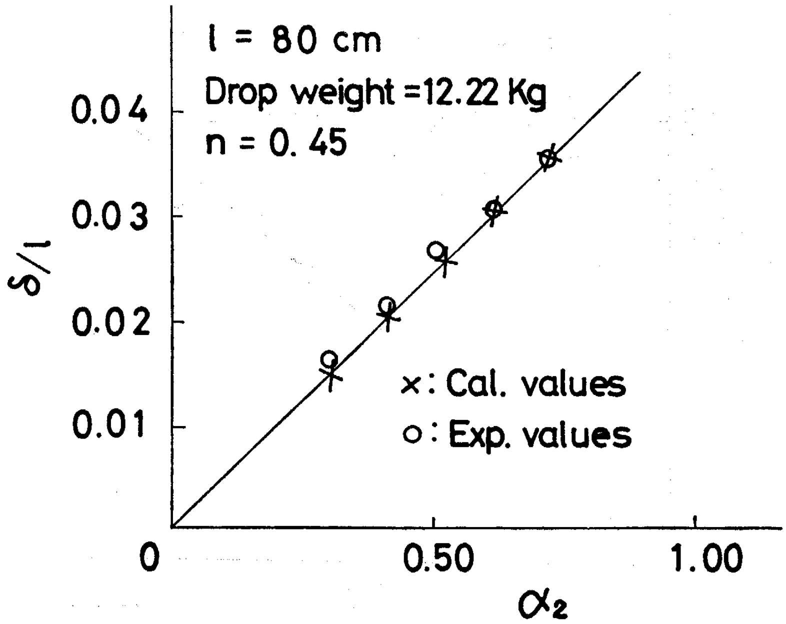

Fig. 8 Comparison of the calculated values with the experimental results (fixed beam on both ends).

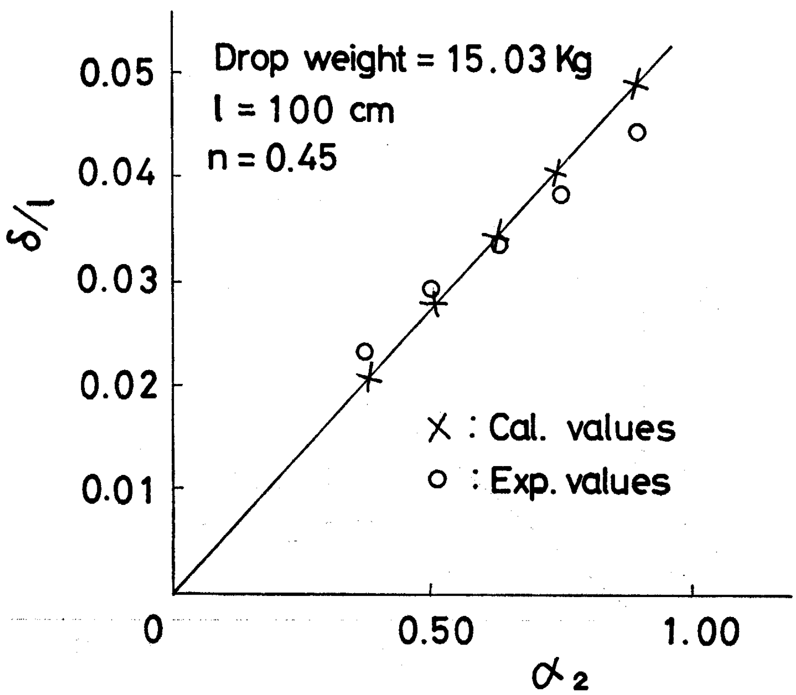

Fig. 9 Comparison of the calculated values with the experimental results (fixed beam on both ends).

5. Conclusions

We studied the theoretical analyses and experiments on the plastic deformations of the steel beams, i.e., the canti-lever beam, the simple beam and the fixed one on both ends when the equally distributed impulse acts on the arbitrary point of the span and the conclusions are summarized as follows:

(1) Canti-lever beam:

For this case if we adopt $n=0.50$ in eq. (2) and (3), the calculated values are quite in good agreement with the experimental results.

(2) Simple beam:

In this case if we adopt $n=1.00$, i.e., linear relations of the vertical velocity in eq. (4) to (7), theoretical results are considerably in agreement with the experimental values.

(3) Fixed beam on both ends:

For this case if we assume $n=0.45$, we can expect considerably good results as indicated in Fig. 8 and Fig. 9. But the experimental results show curvilinear relations, and as a result it seems necessary to consider the effect of axial force in order to obtain better results.

At any rate we found that the theoretical equations proposed above were generally applicable in calculating the plastic deformations of several beams under equally distributed impulse which acted on the arbitrary point of the span. But the unknown problems must be studied in future.

References

1) S. Mogami, Y. Tamura and K. Kojima: "On the Plastic Deformations of Some Beams under Equally Distributed Impulse (1st Report)", Memo. Facul. Eng. Yamaguchi Univ., **26**, 21 (1975) (in Japanese).
2) S. Mogami and M. Sakamoto: "A Study on the Plastic Deformations of Some Beams under Impulsive Loading (1st Report)", Memo. Facul. Eng. Yamaguchi Univ., **20** No. 2, 25 (1965) (in Japanese).
3) S. Mogami and K. Kure: "A Study on the Plastic Deformations of Some Beams under Impulsive Loading (2nd Report)", Memo. Facul. Eng. Yamaguchi Univ., **21** No. 2, 1 (1970) (in Japanese).