On the Strain History of Asphalt Mixture

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Abstract

Usually, asphalt mixture is looked on as viscoelastic body. So, author gives time to learning about viscoelastic nature of asphalt mixture.

It have a nature of stress relaxation which is different from the elastic body.

In order to study this nature, this experiment was performed with a dinamic loading machine of asphalt mixture. This dynamic machine was designed to give a various kinds of stress or strain wave and to measure the resulting strain or stress wave. Sinusoidal, square, triangle and tooth waveforms were obtained from a function generator and this equipment are controlled by a closed loop feed back system.

At first, constant rate of strain and constant strain tests are practiced under constant temperature. And the viscoelastic constant of asphalt mixture could be decided from their stress-time relationship.

In studying dynamic properties, a stress which varies sinusoidally with time is imposed, and the resulting strain, which also varies sinusoidally but in general is out of phase with the stress, is measured as a function of frequency. In this experiment, the use of square wave strain-time pulse has been discussed.

1. Introduction

It is possible to treat asphalt mixture as visco-elastic body, but the definition of rheology takes in a very broad range of phenomena, which might include the elastic deformation of ideal solids, where the strains observed are proportional to the stress applied, (Hooke's law of elasticity), and viscous flow of ideal liquids, where the rate of deformation or flow is likewise proportional to the applied force or stress (Newton's law of viscosity). However, these two types of behavior are treated in detail in the classic fields of elasticity theory and hydrodynamics, including the more complicated behavior where the simple laws of proportionarity no longer hold and where such phenomena as turbulent flow are encountered.

Rheology is therefore ordinary regarded as appling especially to the more complicated types of deformation and flow behavior that do not follow the simple laws of elasticity theory and hydrodynamics. This still includes a wide variety of systems, since ideal solids and liquids are only simple approximate idealizations of certain classes of real materials.

The most important rheological systems are those that show a combination of viscous and elastic effects. The British physicist, James Maxwell, recognized that asphalt behaves in this way; in 1867 he suggested that a mechanical analog model

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consisting of an elastic spring connected in series with a viscous unit (a dashpot) could provide a useful representation of such a visco-elastic system.

The German physicist, Woldemar Voigt, and the British physicist Lord Kelvin, suggested the alternate possibility of connecting the elastic and viscous element in parallel.

This type of representation has since been elaborated into complicated networks that must be described by mathematical distribution functions. Nonlinear spring and dashpots, which do not obey Hooke's and Newton's laws, have also been employed on occasion.

A wide variety of time-dependent behavior can be represented in this way; the corresponding mathematical theory is identical with that used to describe electric networks made up of capacitors and resistors in similar series and parallel combinations.

Materials such as steel and glass, often regarded as elastic at ordinary temperatures, on close observation are found also to exhibit small amounts of viscous behavior. This can be observed as creep (increase of deformation at constant stress), or stress relaxation (decay of elastic stress at constant deformation), and these effects may be important under certain conditions of use.

So, in this experiment, constant rate of strain and constant strain test are practiced under constant temperature, and stress relaxation was observed.

In order to decide their viscoelastic constants, author used relaxation modulus which is differential ratio of stress to strain applied and nearly equal to the opposite number of creep compliance.

Viscoelastic solution obtains from the following, that is to say, at first, we have a desire to get for elastic solution in agreement with a number of experimental conditions, (for example, four point bending test such as our case). And, then, put young's modulas into relaxation modulus.

2. Experimental method and Apparatus

The nature of asphalt and the proportion of asphalt mixture are illustrated in Table 1.

It's proportion of asphalt mixture gives the maximum marshall stability, maximum density and it's particule size accumuration curve is equal to Talbot's equation, namely.

$$P=100 \quad (d/D)^n \qquad \cdots\cdots\cdots(2\cdot1)$$

In upper equation, $D=10$ mm, $n=0.25$ and binder content is 8%. Size of specimen is $4.0\times4.0\times40$ cm, and the four point bending in a span length of 30 cm was employed. In order to make this size of specimen, we at first made slab type asphalt concrete which size is $20\times40\times6$ cm by compressing at 66 Kg/cm^2 statically, and get it cut into that size by cutting machine.

Manufacturing precision of test piece is represented in Table 2.

Loading test is separating static experiment from dynamic one, that is to say, responses of bituminous mixtures were investigated under two types of loading con-

Table 1. Nature of asphalt and composition of mixture.

Sieve Opening (mm)	% Passing	Binder
10	100	Straight
5	84.1	Asphalt: 80/100
2.5	70.7	Pen. (25°C, 100 g, 5 sec)
0.6	49.5	: 89
0.3	41.0	Tr & b (°C): 41.6
0.15	35.0	P. I.: 0.17
0.074	29.3	

Table 2. Manufacturing precision of test piece.

Item	Void content	Apparent density
Max. value	5.87	2.298
Min. value	0.51	2.164
Arithmetical mean	3.044	2.230
Standard deviation	1.406	0.001
95% probable range	2.974–3.026	
Asphalt content: 8.0 (%), Number of test piece: 150, Theoretical density: 2.313		

ditions.

A stress-strain relationship and stress-time relationship were investigated at various temperatures by means of a beam flexure test, and the relaxation modulus by the constant rate of strain test was computed and a relaxation mastercurve was obtained. A dynamic electro-hydraulic machine was developed to investigate the dynamic response of mixtures and to determine the fatigue property.

This dynamic machine was designed to give a various kinds of stress or strain wave and to measure the resulting strain or stress wave. Sinusoidal, square, triangle and tooth waveforms were obtained from a function generator and this equipment are controlled by a closed loop feed back system. A ramp function also enables measurement of creep and stress relaxation test. A symmetrical two point loading system of the repeated loading apparatus applies unidirectional loading flexural stresses to the simply supported $4 \times 4 \times 35$ cm beam specimen. The capacity of load is plus and minus 200 Kg and the maximum deflection of 5 mm was given. A vishigraph enables continuous or intermittent recording of the input waveform, the actual load or deflection waveform in the specimen and the resulting deflection or load. These testing apparatus is shown at Fig. 1 and the line-up of dinamic machine is also illustrated in Fig. 2.

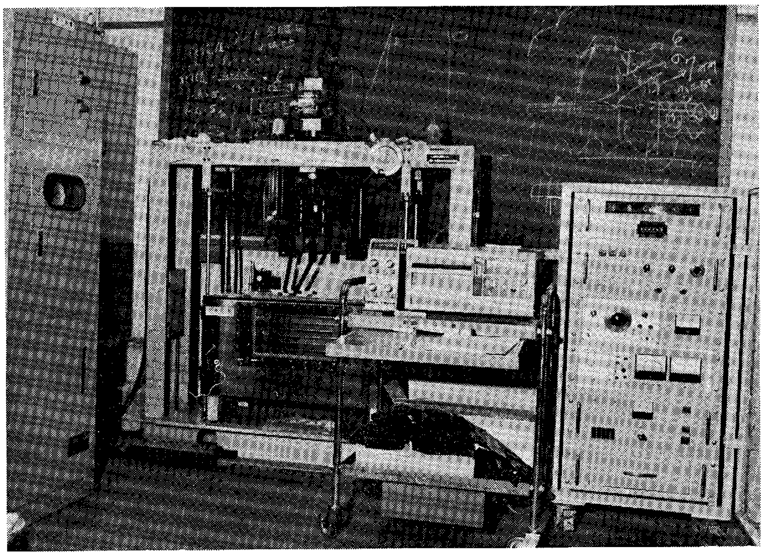

Fig. 1 Testing apparatus.

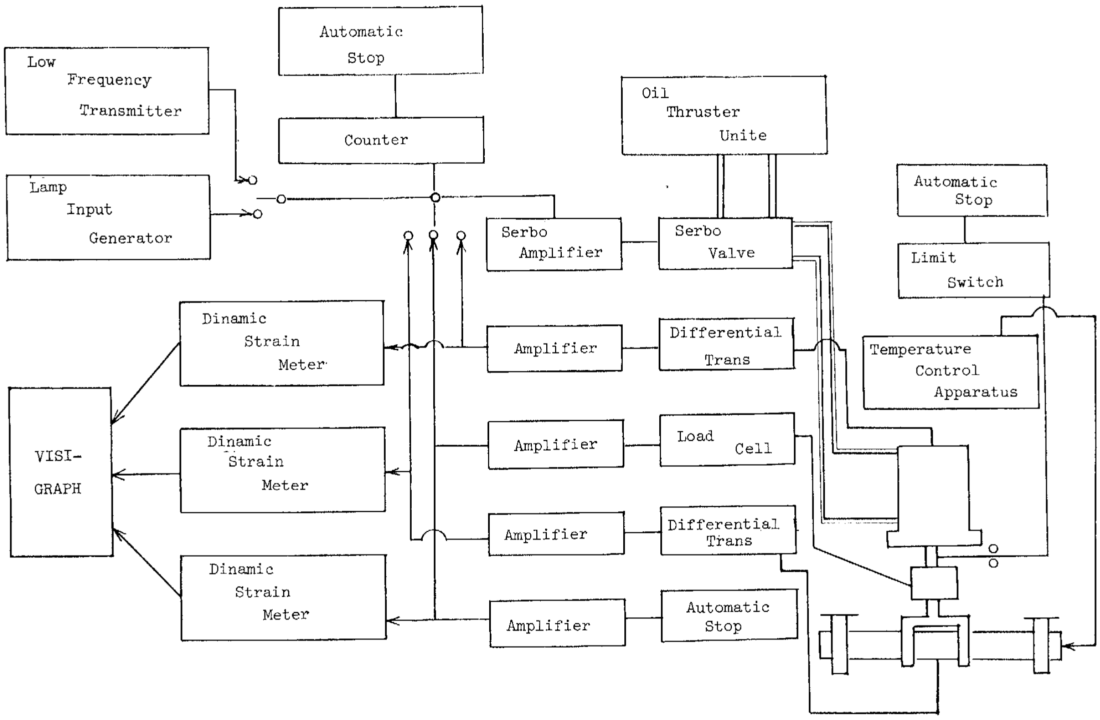

Fig. 2 Line-up of testing apparatus.

3. Theoretical Consideration

In studying stress relaxation, a strain is suddenly imposed and then held constant, while the stress required to maintain it is measured as a function of time. In studying creep, a constant stress is applied and the strain is measured as a function of time. These transient methods are obviously most appropriate for the long end of the time scale, and they also have the advantages of much simpler apparatus and greater ease of obtaining a wide range of time scale.

In studying dynamic properties, a stress which varies sinusoidally with time is imposed, and the resulting strain, which also varies sinusoidally but in general is out of phase with the stress, is measured as a function of frequency.

In some cases the stress-strain ratios are measured directly without knowlege of the individual magnitude of either. Dynamic methods provide twice as much direct experimental information as transients, because at each frequency two independent measurements can and should be made.

These are usually expressed by the ratio of the stress component in phase with strain to the strain and the corresponding out-of-phase ratio (the real and imaginary parts of a complex modulus), as explained in more detail subsequently.

Dynamic measurements are obviously most appropriate for the short end of the time scall. The use of square wave strain-time pulse has also been discussed[1]. So, also, in this paper, the use of square wave strain-time pulse has been discussed. This is equivalent to a repeatedly reversing stress relaxation experiment, in which the rate of strain does not attain a constant value before reversal occurs. The ratios of stress to strain in either of these periodic experiments would bear no simple relation to G' and G'' at a comparable frequency, where G' and G'' are equal to Real part of shear modulus and Imaginary part of shear modulus. The latter quantities are defined only for sinusoidally varying stress and strain.

An ideally elastic substance subjected to simple tension obeys Hooke's law in the region of small strains:

$$\text{Stress} = E \times \text{Strain} \qquad \cdots\cdots\cdots(3.1)$$

The quantity E is a constant known as the Young's modulus. It is very important property of the material in question, being a measure of stiffness or resistance to deformation. The Young's modulus of a substance generally varies with the temperature.

The concept of modulus can be generalized to nonideally elastic substances by recognizing that the modulus is a function of time and also to some extent a function of the method of measurement. For example, experiments may be conducted in which the substance is subjected to a fixed extension, and the stress is measured as a function of time. The stress per unit strain necessary to maintain the sample at constant extension will be defined as the stress relaxation modulus $Er(t)$.

As elastic solution of this experiment (four point bending) is the following,

$$P = (162/5)(I/l^3)Ey \qquad \cdots\cdots\cdots(3.2)$$

where, it's solution represent the resulting load at appried strain, and

P = resulting load

l = span length

I = geometrical moment of inertia

E = Young's modulus

y = appried deformation

So, in agreement with covespondence principle[2], viscoelastic solution is expressed as follows:

$$P = Ay(t) \cdot E(t) \qquad \cdots\cdots\cdots(3\cdot 3)$$

where

$$A = 162I/5l^3 \qquad \cdots\cdots\cdots(3\cdot 4)$$

$E(t)$ represent the relaxation modulus, and $y(t)$ is represented as the function of time.

Now, we calculate the stress of two types strain history, where viscoelastic model is based on the three-parameter nodel with one viscous and two elastic elements.

If it is possible to make strain clear by use of the function of time, namely strain history, stress is represented as follows[2].

$$\sigma(t) = \varepsilon(t)E(0) + \int_0^t \varepsilon(t')\{dE(t-t')/d(t-t')\}dt' \qquad \cdots\cdots\cdots(3\cdot 5)$$

In this experiment, strain history in stress relaxation test is able to represent as Fig. 3, namely $\varepsilon(t)$ is equal to $\varepsilon_1 t/t_1$ or ε_1, when $0<t<t_1$ or $t_1 \leq t$ respectively.

Three parameter model with one viscous and two elastic elements is applied as viscoelastic model. So, relaxation modulus is given by

$$E(t) = (q_1/p_1)e^{-t/p_1} + q_2(1-e^{-t/p_1}) \qquad \cdots\cdots\cdots(3\cdot 6)$$

It is obtained from putting the equation of motion such as followings

$$\sigma + p_1\sigma_t = q_0\varepsilon + q_1\varepsilon_t \qquad \cdots\cdots\cdots(3\cdot 7)$$

Where the subscript t denotes differentiation partially with respect to the time t.

We can immediately get the solution of equation (3·5) from substituting equation (3·6) for equation (3·5), and it is
when $0<t<t_1$

$$\sigma(t) = q_0\varepsilon_0 t - \varepsilon_0(q_0p_1 - q_1) + \varepsilon_0(p_1q_0 - q_1)e^{-t/p_1} \qquad \cdots\cdots\cdots(3\cdot 8)$$

and, when $t_1 \leq t$

$$\sigma(t) = \varepsilon_0 t q_0 + \varepsilon_0(q_0p_1 - q_1)(1-e^{t_1/p_1})e^{-t/p_1} \qquad \cdots\cdots\cdots(3\cdot 9)$$

where $\varepsilon_0 = \varepsilon_1/t_1$

And another strain history is square wave strain-time pulse and sinusoidal wave.

Strain history of square wave strain-time pulse is given by Fig. 4, and $\varepsilon(t)$ is equal to ε_0 or $-\varepsilon_0$, when $2nt_1 < t < (2n+1)t_1$ or $(2n+1)t_1 < t < 2(n+1)t_1$ respectively. In this case, if the viscoelastic model is the same, stress at the point of $t = (2nt_1)^-$ is easily derived from equation (3·5)

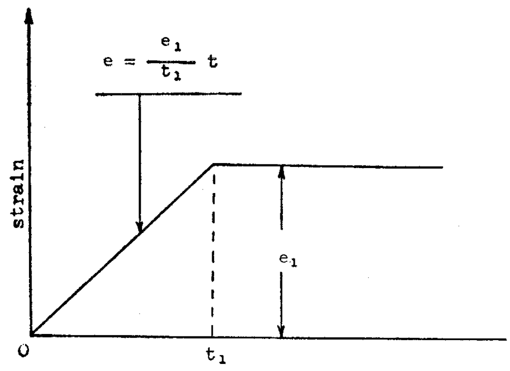

Fig. 3 Relationship between strain and time due to static test.

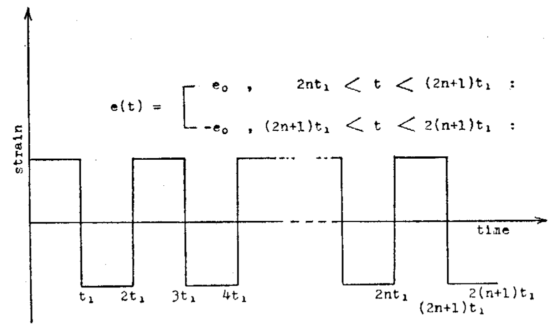

Fig. 4 Relationship between strain and time due to square wave.

$$\sigma(t) = -\varepsilon_0 q_1/p_1 - \frac{\varepsilon_0(q_0 - q_1/p_1)(e^{t_1/p_1} - 1)}{(e^{t_1/p_1} + 1)}(1 - e^{-2nt_1/p_1}) \cdots\cdots\cdots (3\cdot10)$$

Equation (3·10) represents different stress at every alternating cycles.

4. Experimental results of stress relaxation

The results of stress relaxation test at five kinds of temperature is illustrated in Fig. 5. Fig. 5 represents stress-time relationship at the rate of strain being 6×10^{-5}, 3×10^{-5}, and 2×10^{-5}, respectively.

Fig. 6 represent resultant stress curves at three kinds of strain rate due to the temperature being -2°C and 2°C, respectively.

From these figure, it can be easily found that the tangent of stress-time curves gradually decrease with increasing temperature and decreasing the rate of strain. Stiffness is the relationship between stress and strain expressed as a function of loading conditions and temperatures; this relationship between stress, strain and time is also

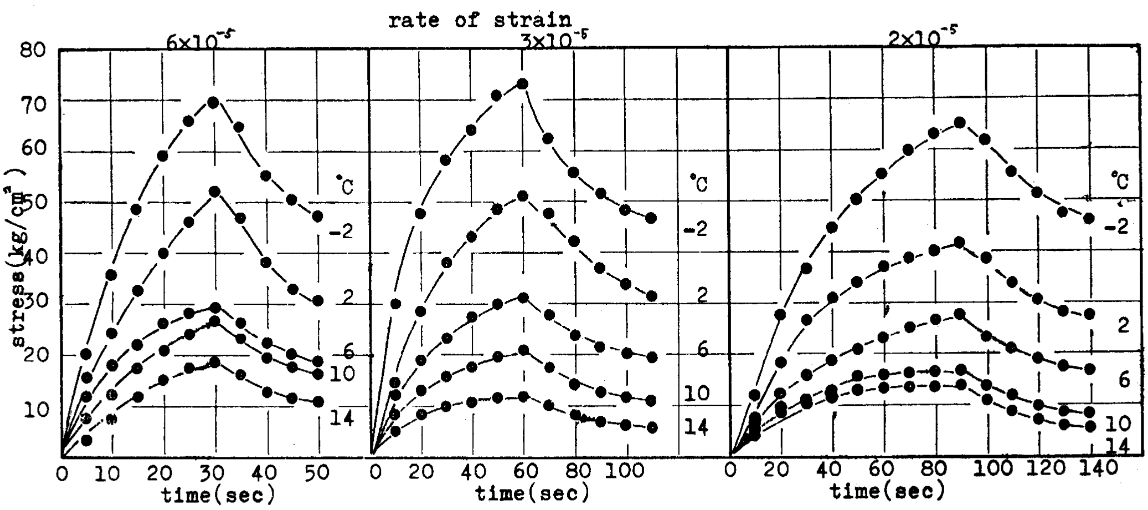

Fig. 5 Stress-Time relationship at every strain rate.

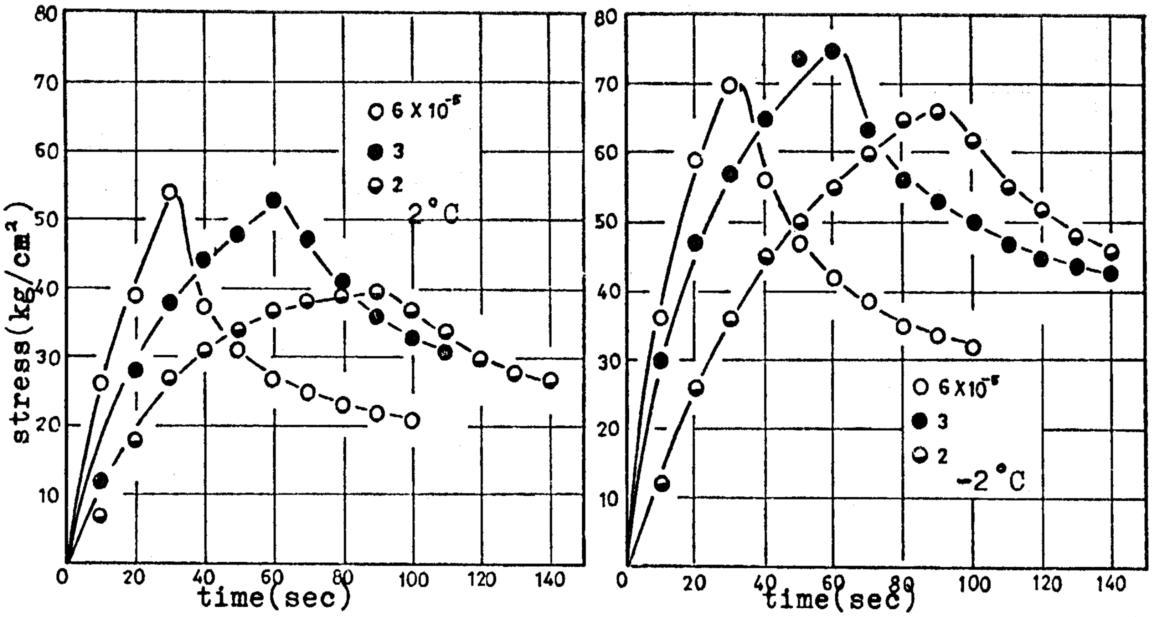

Fig. 6 Resultant stress curves due to the temperature being −2°C and 2°C.

referred to as the rheological property of paving mixtures. Stiffness characteristics must be known not only to assess the behavior of the mixture itself, but also to evaluate the performance of pavement structure. Generally, at any short loading time, namely a very faste rate of strain or at low temperature, it will be noted that stiffness is essentially time independent; in this case the stiffness approaches the elastic modulus. For an intermediate range of loading, and higher temperatures, the Stiffness decreases with increase of loading time.

Van der Poel recommended that attention should be concentrated on a single stress and it's resultant strain, because for many purposes this is reasonably adequate. He suggested a single parameter termed the Stiffness "s".

$$S(t, T)=\sigma/\varepsilon \qquad \cdots\cdots\cdots(4.1)$$

The test result which were obtained from the constant rate of strain flexure test is discussed in this paper. In the beam flexure test, a constant rate of deflection is applied and the resulting load is recorded throughout the loading time as a function of time. From the stress-strain curve obtained at various temperatures and rates of strain, the relaxation modulus, $Er(t)$ can be computed as:

$$Er(t)=d\sigma/d\varepsilon \qquad \cdots\cdots\cdots(4.2)$$

$Er(t)$: Relaxation Modulus (Kg/cm^2)

$d\sigma/d\varepsilon$: Slope of stress-strain curve at particular loading time

Stiffness of this experiment is represented in Fig. 7. Fig. 7 is the stiffness of the constant rate of strain flexure test and the constant strain after applying strain rate of

3×10^{-5} (sec^{-1}). From this figure, it can be easily found that the stiffness decreases with increase of loading time, and it's tendency is the same at the both range. Viscoelastic constants are computed by using equation of (3·8) and (3·9). From it's result, viscoelastic constants are about following values,

Spring constant of pure elastic element: $1\times10^4\sim7\times10^4$ $Kg{\cdot}cm^{-2}$

Spring constant of Voigt model : $2\times10^3\sim4\times10^4$ $Kg{\cdot}cm^{-2}$

Viscous constant of Voigt model : $3\times10^5\sim5\times10^6$ $Kg{\cdot}sec{\cdot}cm^{-2}$

Viscous constant of Voigt model decreases with increase of temperature and the rate of strain. This is illustrated in Fig. 8. And equation of this curve is calculated by using the Principle of Least Squares.

It's equation is

$$\eta=10^{m/(T-\alpha)},\quad m\fallingdotseq600,\quad \alpha\fallingdotseq90 \qquad \cdots\cdots\cdots(4{\cdot}3)$$

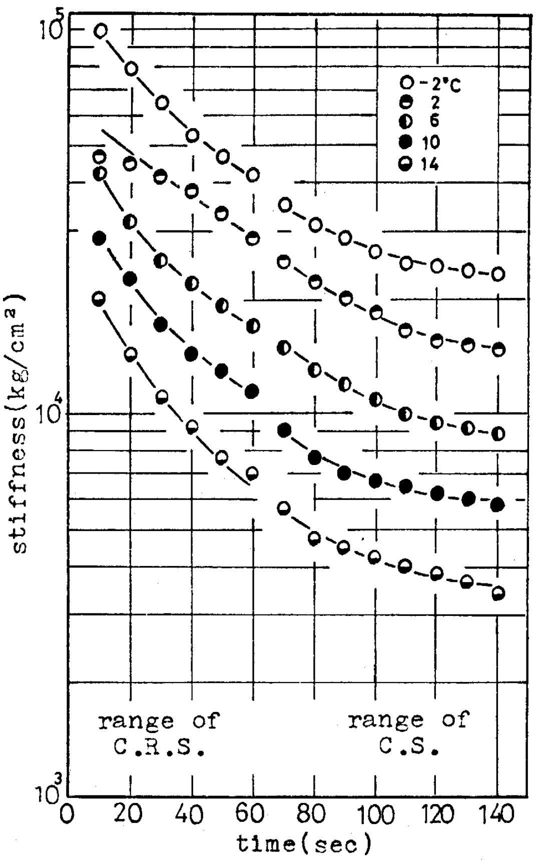

Fig. 7 Characteristic of Stiffness for loading time.

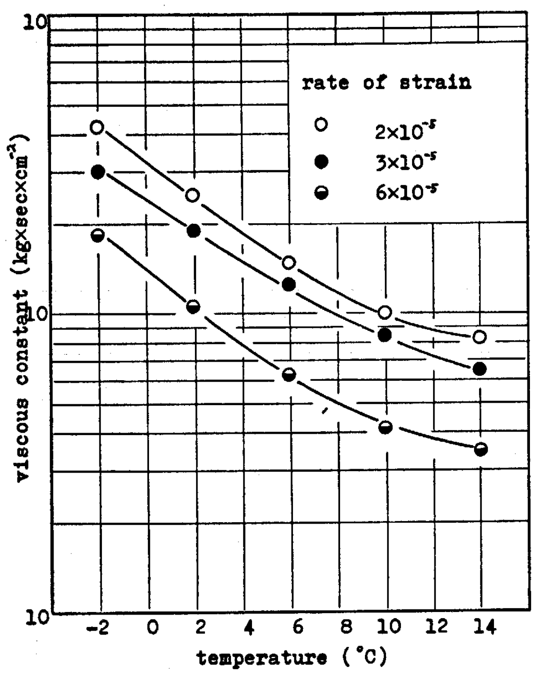

Fig. 8 Relationship between viscous constant and temperature.

5. Results of Dynamic Test

In studying dynamic properties, a stress which varies sinusoidally with time is imposed, and the resulting strain, which also varies sinusoidally but in general is out of phase with the stress, was measured as a function of frequency. The use of square wave strain-time pulse has also been discussed in this paper. Relationship between strain and the number of altermating frequency is shown in Fig. 9.

Number of cycles to failure is defined as the intersection point of both tangent, one is in the range of stress relaxation and other is in the range of stress which shows immediate decrease.

In this way, we defined the number of cycle to failure. It's results are illustrated in Fig. 10. These figures represent relationship between strain and number of cycles to failure in every temperature range and two types of wave form. And it's line form approximate to straight line, so it is possible to decide the equation of straight line by using the Principle of Least Squares. Equations of straight line at every temperature are given by

In case of temperature being 14°C

Sinusoidal wave: $N_f=(10)^{-17.149}(1/\varepsilon)^{6.570}$ ·········(5–1)

Square wave : $N_f=10^{-12.183}(1/\varepsilon)^{4.780}$ ·········(5–2)

In case of temperature being 6°C

Sinusoidal wave: $N_f=10^{-21.346}(1/\varepsilon)^{7.794}$ ·········(5–3)

Square wave : $N_f=10^{-13.181}(1/\varepsilon)^{5.032}$ ·········(5–4)

In case of temperature being −2°C

Sinusoidal wave: $N_f=10^{-43.083}(1/\varepsilon)^{14.686}$ ·········(5–5)

Square wave : $N_f=10^{-23.055}(1/\varepsilon)^{8.114}$ ·········(5–6)

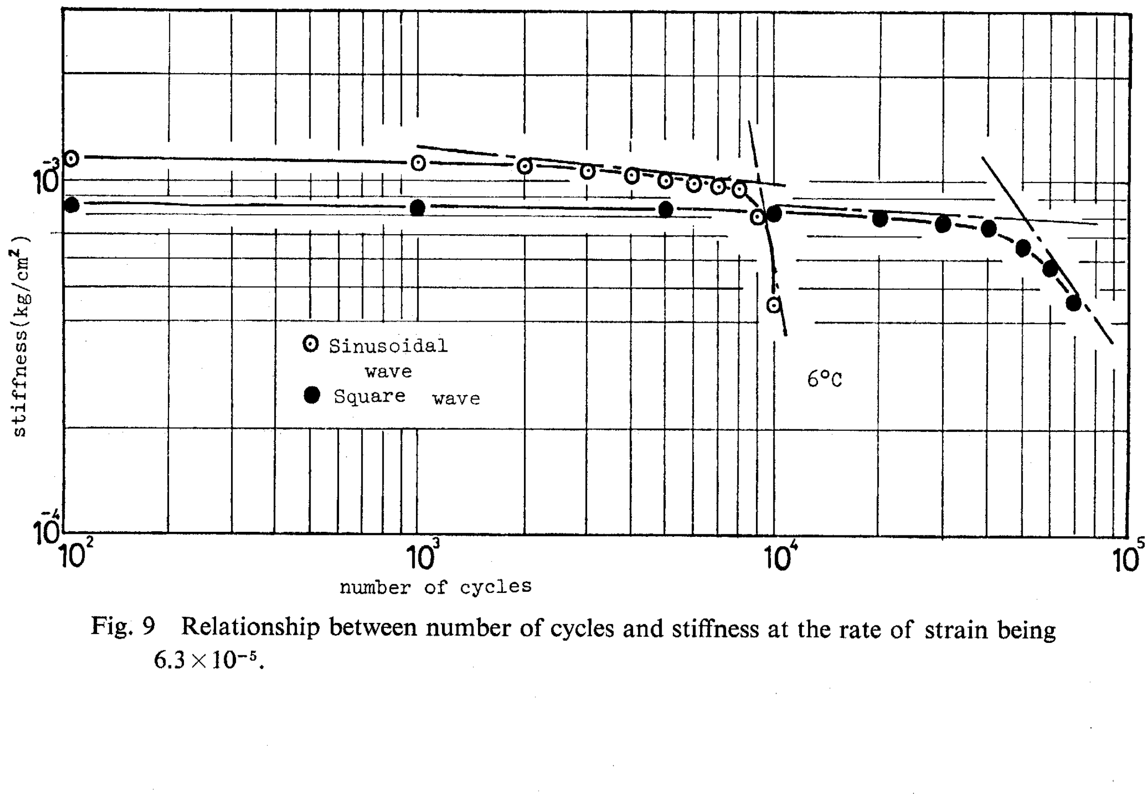

Fig. 9 Relationship between number of cycles and stiffness at the rate of strain being 6.3×10^{-5}.

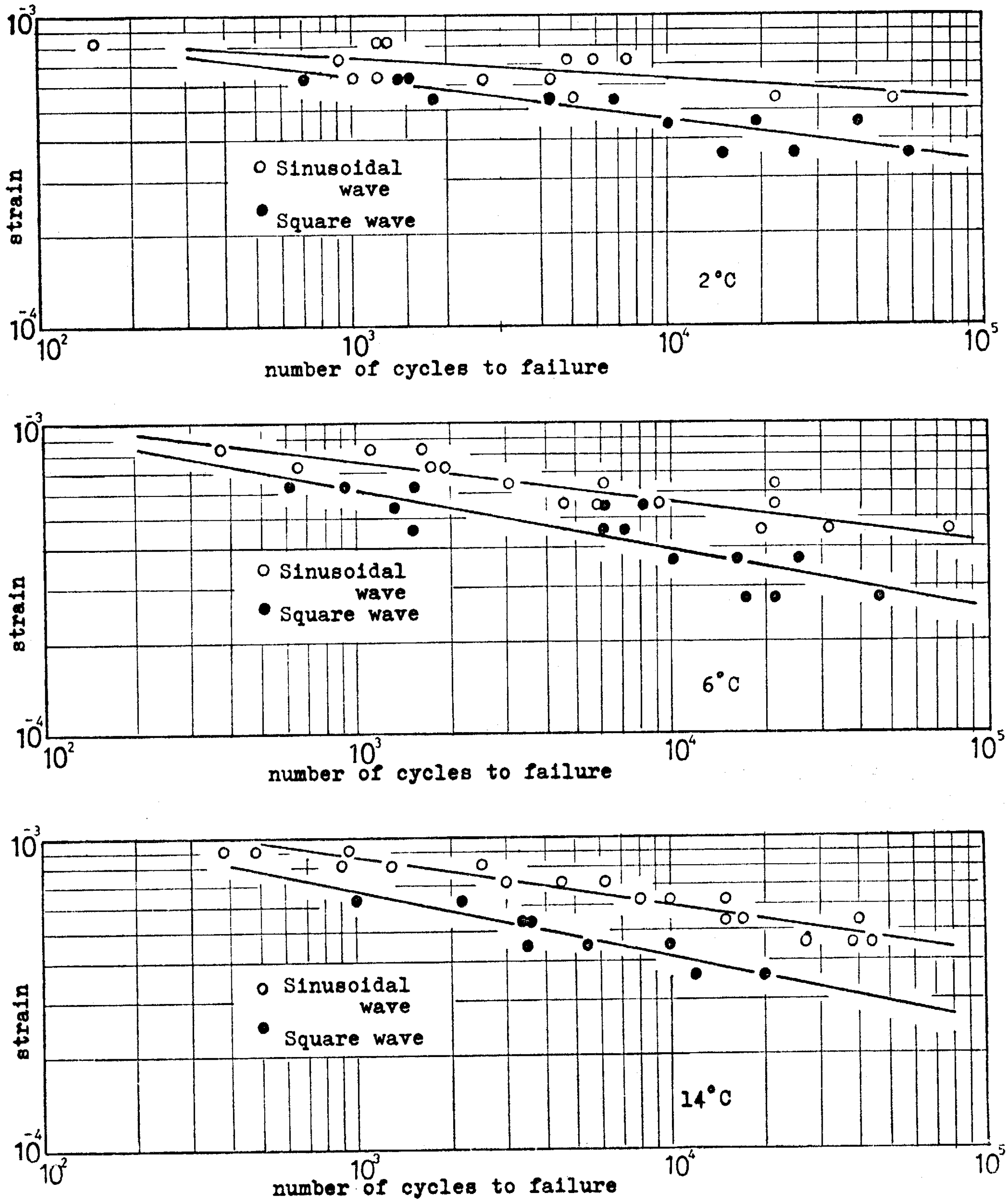

Fig. 10 Relationship between number of cycles to failure and strain at every temperatures.

And these correlation coefficient is about the rang of $-0.88 \sim -0.99$. From these figures, number of cycles to failure of sinusoidal wave is larger than that of square wave, so asphalt mixture has study nature about smooth load. Namely, when strain level is 6×10^{-4}, number of cycles to failure of sinusoidal wave is about seven times as much number as that of square wave. And, it can be obviously observed that there exist an phenomenon of stress relaxation in resulting wave of dinamic test.

6. Conclusion

The stress-strain relationship of bituminous mixture was measured by means of flexure at various rates of strain. From the analysis of stress-strain curve, the master-curves of relaxation modulus were obtained. The tangent of stress-time curves gradually decrease with increasing temperature and decreasing the rate of strain, and the stiffness decreases with increase of loading time and it's tendency is the same at both range. Viscous constant of Voigt model decreases with increase of temperature and the rate of strain. The equation of this curve is able to calculate by using the Principle of Least Squares, and this equation is represented by $\eta = 10^{m/T-\alpha}$.

From dynamic test, number of cycles to failure was defined as the intersection point of both tangent, one is in the range of stress relaxation and other is in the range of stress which shows immediate decrease. And $\varepsilon - N_f$ curve form approximate to straight line, then the equation of straight line is able to represent by $N_f = 10^{-\alpha}(1/\varepsilon)^{\beta}$, where α, β equal to some constant. Number of cycles to failure of sinusoidal wave is larger than that of square wave, namely, when strain level is 6×10^{-4}, number of cycles to failure of sinusoidal wave is about seven times as much number as that of square wave. So, asphalt mixture has sturdy nature about smooth load.

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