Control of Outflow Quantity from a Hopper
— Application of Adaptive Control Method —

Kenzo WADA*, Nobuo HAYANO* and Hiroshi MORIOKA**

(Received July 15, 1989)

Abstract

In this paper, we discuss the control of the outflow quantity of a known amount of granules from a hopper. First, for the control algorithm, a model reference adaptive control algorithm extended to a nonminimum phase system by compensating zeros of the system is used. Next, the adaptive algorithm for a process with unknown deterministic disturbance is investigated using an AR model with time delay. Results show the proposed algorithm to be very useful for processes with deterministic disturbances and time variant parameters.

1. Introduction

In our previous report [1], we discussed the characteristics of the flowmeter used as a detecting element in the control system of an outflow quantity. We showed that a holdup on the detecting plate of the flowmeter can be approximated by a first-order lag element, and that the dynamic behavior of granules on the flowmeter can be approximated with white noise. In this experiment, sand was used as the granules because of its good flowability, and the LQ control method was applied to the system as a control scheme. Good response waves were obtained for a reference input and a disturbance input by changing the values of a weighting matrix, in spite of the presence of a modeling error and the fluctuation of parameters.

The good flowability of granules means that the outflow rate from the hopper is almost constant and the fluctuation of the outflow rate is very small. Generally, however, in a process handling granules and powders, the particle size of granules in the hopper is not uniform ; i.e., granules usually have a wide size distribution. It may be difficult to ascertain the exact dynamic characteristics and obtain a good performance using conventional control schemes for such a process because the parameters of the process are always changing with variation in outflow quantity and the size distribution of granules. In this paper, we shall discuss the application of a model reference adaptive control which is very useful for processes with the time variant unknown parameters. Presently, there are two control methods for the adaptive control schemes. One is a model reference adaptive control (MRAC) and the other is a self-tuning regulator(STR).

The MRAC can be roughly classified into two groups from the point of view of stability theorems : one based on Lyapunov's stability theorem and the other on

---

*Department of Industrial Mechanical Engineering
**Mitsubishi Electric Co.

PoPov's hyperstability theorem. Since our process turns to a nonminimum phase system when the sampling period is small, the model reference adaptive control based on Lyapunov's direct method is extended to permit application to the nonminimum phase system by compensating zeros of the system [(2)]. Furthermore, since description of the process by the ARMA model leads the process to the nonminimum phase system, next, a process model based on the AR model is used. We shall investigate the design method of a system in which no steady state offset occurs for the disturbance input. The response waves of the process will also be examined.

2. Controlled Object

Figure 1 shows a schematic diagram of the experimental apparatus. This is similar to that used in the previous work. First, we examine the shift of the zeros of the process in accordance with changes in the sampling period. The process is described by the following ARMA model in which the parameters are estimated using a least square estimation method :

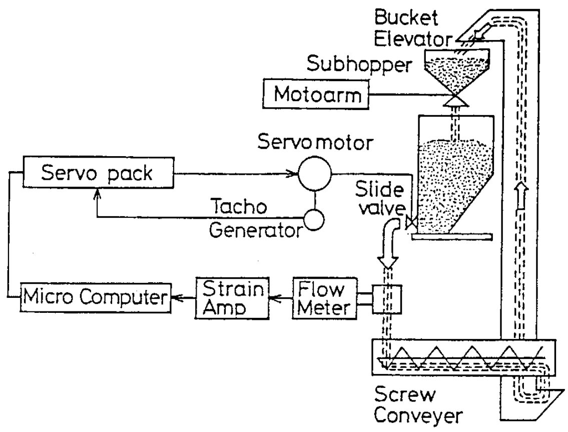


Fig. 1 Schematic diagram of the experimental apparatus

Table 1 value of zeros

	Value of Zeros
$\gamma=0.5$sec	-1.095 $0.433547\pm0.13418j$
$\gamma=1$sec	-0.61750 $0.16324\pm0.11701j$
$\gamma=2$sec	-0.43691 $-0.01112\pm0.01291j$
$\gamma=3$sec	0.08630 -0.23396 -0.29245

$$y(k)=\frac{\sum_{i=1}^{4} b_i z^{-i}}{1-\sum_{i=1}^{4} a_i z^{-i}} u(k) .$$

The values of the zeros of the estimated model are shown in Table 1. Table 1 shows that the zeros of the process lie outside the unit circle in the z-plane, and that the process turns to a nonminimum phase system when the sampling period τ becomes small. As τ grows large, these zeros approach the center of the unit circle. On the basis of facts, we use the adaptive control algorithm [2] extended to the nonminimum phase system by compensating zero-points. (we shall call this algorithm "MRAC-1".)

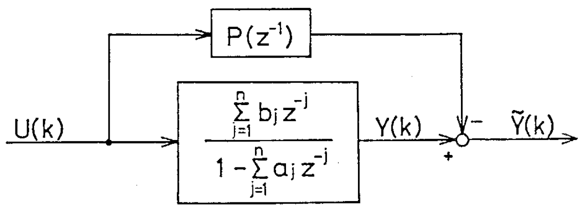


Fig. 2 Block diagram of the augmented process with compensator

3. MRAC System for the Nonminimum Phase System

A compensating element $P(z^{-1})$ is introduced in parellel to the process so that all zeros of the process lie inside the unit circle in a z-plane, as shown in Fig. 2. From Fig. 2, the output $\tilde{y}(k)$ of the process compensated for is expressed by

$$\tilde{y}(k)=y(k)-P(z^{-1})u(k) . \tag{2}$$

Here, $P(z^{-1})$ is a polynomial of z^{-1} described by the following equation :

$$P(z^{-1})=\sum_{j=1}^{m} p_j z^{-j} \tag{3}$$

and p_j $(j=1\cdots\cdots n)$ is determined so that roots of the following equation lie insede the unit circle :

$$\sum_{i=1}^{n} b_i z^{-i}-P(z^{-1})(1+\sum_{i=1}^{n} a_i z^{-i})=0 . \tag{4}$$

It must satisfy the following assumptions regarding the process :

(1) $b_1-p_1\neq 0$,

(2) All zeros of the process compensated lie inside the unit circle in the z-plane,

(3) n is known and $m\leqq n$.

The following system is considered to be the reference model for the process expressed by Eq. (1) :

$$y_M(k)=\sum_{i=1}^{m}(a_i'(k)z^{-i}\, y_M(k)+b_i'(k)z^{-i}\, r(k)) . \tag{5}$$

Here, $r(k)$ is a reference input for the reference model. We determine the control input $u(k)$ as follows :

$$u(k)=\{y_M(k+1)-\sum_{i=1}^{n}\alpha_i(k)z^{-i}\ y(k+1)-\sum_{i=1}^{n}\ \beta_i(k)z^{-i}u(k+i)\}/\beta_1(k), \tag{6}$$

where, $\alpha_i(k)$ and $\beta_i(k)$ are adaptive gains. And we define the output error $e(k)$ and the augmented error $\xi(k)$, respectively, as follows :

$$e(k)=y_M(k)-y(k), \tag{7}$$

$$\xi(k)=y_M(k)-\tilde{y}(k). \tag{8}$$

The output error $e(k)\to 0$ as $k\to\infty$ is guaranteed by implementing the following adaptive algorithm :

$$\Phi(k+1)=\Phi(k)-\lambda\frac{\xi(k+1)}{\omega(k)^T\omega\ (k)}\ \omega(k), \tag{9}$$

$$(0<\lambda<2)$$

where

$\Phi(k)^T=\{(\alpha_1(k)-a_1),\cdots,(\alpha_n(k)-a_n),(\beta_1(k)-g_1),\cdots,(\beta_n(k)-g_n)\}$

$\omega(k)^T=\{(y(k),y(k-1),\cdots,y(k+1-n),u(k),\cdots,u(k+1-n)\}$

$g_i(k)=\beta_i(k)-(b_i-p_i)\quad (i\leqq m)$

$g_i(k)=\beta_i(k)-b_i\qquad (i\leqq m)$

T denoting the transportation of the vector.

3.1 Experimental Results

The step input is given to the following second-order system, and the output $y_M(k)$ is used as the reference response.

$$y_M(s)=\frac{\omega_n^{\ 2}}{s^2+2\zeta\ \omega_n s+\omega_n^{\ 2}}R(s) \tag{10}$$

Here, $\zeta=0.5$, $\omega=0.2$. It has already been confirmed through simulation experiments[2] that this algorithm operates excellently. Experimental response waves of the process in the case where the sampling period τ is 0.5 sec and $P(z^{-1})=-200z^{-1}$ are shown in Fig.3. The upper part of the figure indicates the output of the process, and the lower part shows the change in the amount of valve opening with time.

When τ is 0.5 sec, the process becomes a nonminimum phase system. But we see

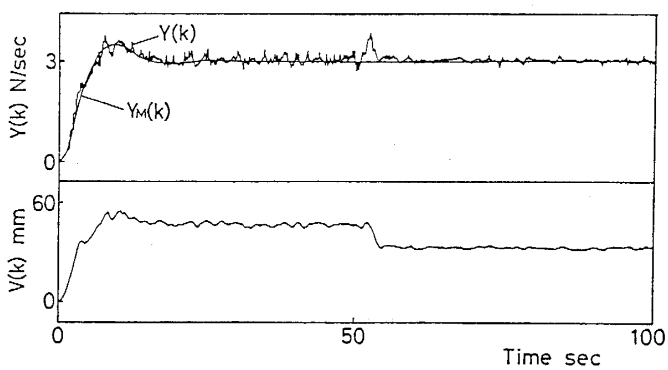


Fig. 3 Response of the process for a reference response and the disturbance input (Experiment)

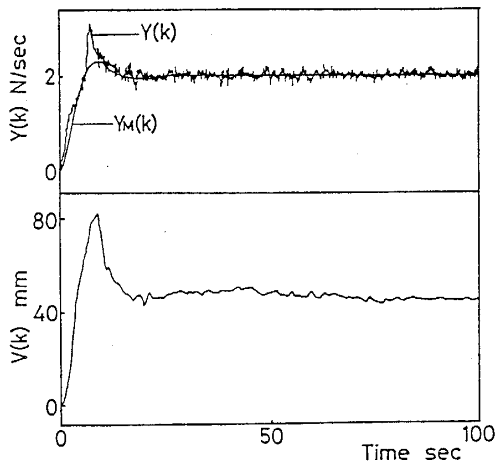


Fig. 4 Response of the process with time variant parameters (Experiment)

that a good response wave can be obtained for the reference input or the disturbance input by compensating the zeros of the process. Next, as an example of a case in which the process has time variant parameters, the following experiment was implemented.

Nylon chips and sand, which differ in size and density, were used as granules. They were put one on top of the other in alternating layers, each 20cm deep inside the hopper. An example of the response wave when $\tau=0.5$ sec and $P \doteq -200z^{-1}$ is given in Fig.4.

Some overshoot is seen in the early stages of the response, but after that, the response wave follows the output of the reference model.

4. Construction of the MRACs for a Process with an Unknown Deterministic Disturbance

In the previous section, the MRACs for a nonminimum phase system were constructed by compensating the zeros of the process. This time, the ARMA model was used as the process model. As one of the factors by which the process becomes the nonminimum phase system, the process can be considered to be described by the ARMA model. Therefore, we expressed the process by the AR model with delay time and discussed the design method of a stable MRACs based on the Fujii's method(3).

We were able to a obtain a good response in the simulation experiment, but could not get a good response when the control method was applied to our process. It was not feasible to apply Fujii's algorithm to our process because the outflow quantity was always changing and the time constant of the process was shorter than that of heat systems. Furthermore, a lot of a priori information on the process is needed to apply its algorithm to the real process. Consequently, in order to take into account the facts mentioned above, we assume that an unknown disturbance enters into the process, and we discuss the construction of a robust control system for the process of handling granules by considering the disturbance beforehand.

4.1 Description of the Process

If an adaptive control system is constructed for a process with an unknown disturbance, it is necessary to make a model of the disturbance. We consider disturbances to be separated into two groups: disturbance expressed as the output of a stable autonomous system (i.e., disturbances with dynamics), and those expressed as polynomial functions of time [4]. In this section, the process is described by the AR model with time delay, and the adaptive control algorithm is derived based on Lyapunov's stability theorem. The algorithm posited in this section is easier to implement than Fujii's algorithm.

The process is described by the following AR model:

$$y_p(k+1)=\sum_{i=1}^{n_p} a_{pi}y_p(k+1-i)+b_pu(k). \tag{11}$$

If the disturbance $w_1(k)$ expressed as the output of the stable autonomous system and the disturbance $w_2(k)$ described by the polynomial function of time are added to the process as unknown disturbances, Eq. (11) is expressed as follows:

$$A(z^{-1})y_p(k)=b_pu(k)+w_1(k)+w_2(k), \tag{12}$$

where

$$A(z^{-1})=1-\sum_{i=1}^{n_p} a_{pi}z^{-1} \tag{13}$$

in which

$y_p(k)$: the output of the process,
$u(k)$: the input of the process,
a_i, b_1 : unknown parameters of the process,
$w_1(k)$: the disturbance with dynamics,
$w_2(k)$: the disturbance described by the polynomial of time.

It is assumed that the disturbances $w_1(k)$ and $w_2(k)$ have the following characteristics, respectively:

$$D_1(z^{-1})w_1(k)=0, \tag{14}$$

$$D_2(z^{-1})w_2(k)=0. \tag{15}$$

Here,

$$D_1(z^{-1})=1+\sum_{i=1}^{n_p} c_iz^{-i}, \tag{16}$$

$$D_2(z^{-1})=(1-z^{-1})^{nD2}, \tag{17}$$

and $D_1(z^{-1})$ is a stable polynomial; the upper limit of the order nD1 is known and c_i is unknown. From Eqs. (12) and (14), the following equation is derived:

$$D_1(z^{-1})A(z^{-1})y_p(k)=D_1(z^{-1})b_pu(k)+D_1(z^{-1})w_2(k). \tag{18}$$

The above equation can be rewritten by the following difference equation:

$$y_p(k)=\sum_{i=1}^{n'} a_i'y_p(k-i)+\sum_{i=0}^{nD_1} b_iu(k-1-i)+w_2'(k). \tag{19}$$

Here,

$$\begin{aligned} D_1(z^{-1})A(z^{-1})&=\sum_{i=1}^{n'} a_iz^{-i},\\ D_1(z^{-1})b_p&=\sum_{i=0}^{nD_1} b_iz^{-i},\\ D_1(z^{-1})w_2(k)&=w_2'(k), \end{aligned} \tag{20}$$

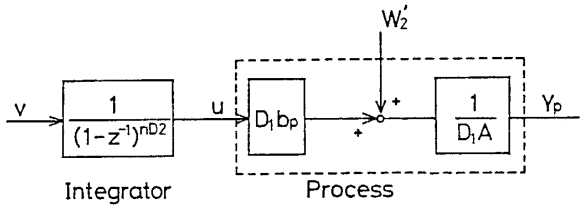


Fig. 5 Block diagram of the process including an integral element

and $n'=np+nD1$. The system expressed by Eqs. (18) and (19) is an augmented system in which the order is nD1 higher than that of the system described by Eq. (11). Next, we consider the design of the control system for the process expressed by Eq. (19). From Eq. (20),

$$D_2(z^{-1})\ w_2'(k)=D_2(z^{-1})D_1(z^{-1})w_2(k)=0. \tag{21}$$

Consequently, the system expressed by Eq. (19) can be considered to be influenced by the disturbance of the polynomial function of time. First, we consider the system shown in Fig.5 that involves the nD2 order discrete time integral characteristics for the process expressed by Eq. (19).

From Fig.5 the transfer function $G_{uv}(z^{-1})$ from signal v to the output y is as follows:

$$G_{uv}(z^{-1})=\frac{1}{(1-z^{-1})^{nD2}}\frac{\sum_{i=1}^{n_p} b_i z^{-1-i}}{1-\sum_{i=1}^{n_p} a_i z^{-i}} \tag{22}$$

assuming that all zeros of the system expressed by Eq. (22) lie inside the unit circle of z-plane. The characteristics from v to y_p are described by the following difference equations:

$$y_p(k+1)=(1-z^{-1})^{nD2}\sum_{i=1}^{n'} a_i y_p\ (k+1-i)+\ \{1-(1-z^{-1})^{nD2}\}*$$

$$y_p(k+1)+\sum_{i=0}^{nD_1} b_i v(k-i). \tag{23}$$

Here, the equation error and the parameter error are defined by $e(k)=y_m(k)-y_p(k)$ ($y_m(k)$: a reference), (24)

$$d_i=\theta-a_i'(i\cdots\cdots,n), \tag{25}$$

and the amnipulating variable v(k) is synthesized as follows:

$$v(k)=\{y_m(k+1)-(1-z^{-1})^{nD2}\sum_{i=1}^{n'}\ \{\theta_i-\phi_i(k)\}y_p\ (k+1-i)$$

$$-\{1-(1-z^{-1})^{nD2}\}\ y_p\ (k+1)-\sum_{i=1}^{n^{D1}}\ \pi_i(k)v(k-1)\}\ /\pi_0(k). \tag{26}$$

From Fig.5, Eq(17) becomes

$$D_2(z^{-1})u(k)=v(k). \tag{27}$$

Using the above equation, Eq. (26) is rewritten as

$$y_m(k+1)=(1-z^{-1})^{nD2}\sum_{i=1}^{n'}\ \theta_i-\phi_i(k)\}\ y_p(k+1-i)+$$

$$\{1-(1-z^{-1})^{nD2}\}\ y_p(k+1)-\sum_{i=0}^{nD_1}\pi_i(k)v(k-i). \tag{28}$$

Consequently, from Eqs. (23), (27) and (28), the equation error e(k) is expressed as follows :

$$e(k+1)=y_m(k+1)-y(k+1)$$
$$=D_2(z^{-1})\{\sum_{i=1}^{n'}\{d_i-\phi_i(k)\}\ y_p(k+1-i)+\sum_{i=0}^{nD_1}\pi_i(k)-b_i\}\ u(k-i)\}$$
$$=D_2(z^{-1})r^T\delta\ (k). \qquad (29)$$

Here,

$$r(k)=\{d_1-\phi_1(k),\cdots,d_n-\phi_n(k),\pi_0(k)-b_0,\cdots,\pi_{nD1}(k)-b)$$
$$\delta^T(k)=\ \{y_p(k),\cdots,y_p(k+1-n),u(k),\cdots,u(k-n)\}.$$

We shall select the Lyapunov function for the system expressed by Eq. (29) as follows :

$$v(k)=\gamma^T(k-1)\gamma\ (k-1). \qquad (30)$$

Then, the adaptive algorithm is given by

$$\gamma(k)=\gamma(k-1)-\varepsilon\ (k)e(k)\delta(k-1) \qquad (31)$$
$$\varepsilon(k)=\rho(k)/(\delta^T(k-1)\delta(k-1)+\eta(k)$$
$$0<\rho(k)<2,\ 0\leqq\eta$$
$$\Delta v(k)=v(k+1)-v(k)$$
$$=\frac{e^2(k)}{\delta^T(k-1)\delta\ (k-1)}\ \rho^2(k)-2\rho\ (k)\frac{\{1-D_2(z^{-1})\}}{D_2(z^{-1})}.$$

From $0<\rho(k)<2$, $\rho^2(k)-2\rho(k)<0$ and from $\rho(k)>0$, the following inequality is derived :

$$-2\rho(k)\{1-D_2(z^{-1})\}/D_2(z^{-1})>0.$$

Therefore, if δ (k) is unbounded and $V(k)<0$, $e(k)\rightarrow 0$ as $k\rightarrow\infty$ is guaranteed from Lyapunov's stability theorem.

4.2 Simulation and Experimental Results

To confirm the stability of the system for the disturbance using the control method mentioned above, four kinds of disturbances were applied to the process. The responses of the process were then examined through experiments and simulations. In this experiment, the order of the system was np=5, nD1=2 and nD1=1, respectively, and this means that a first-order integral element was involved in the system. The reference value was 2.94N/sec (300g/sec), and the sampling period τ was 1 sec. The initial value of each parameter was set at zero, except for π (0)=30.

Types of disturbances

① A step disturbance $w_2(k)$ with a magnitude of 0.98N/sec (100g/sec)
(This disturbance was added to the flowmeter after 100 steps)

② A rampwise disturbance $w_2(k)$

$$w_2(k)=\begin{cases}2k & (0\leqq k<50)\\ 2(k-50) & (50\leqq k<100)\\ 2(k-100) & (100\leqq k<150)\\ 2(k-150) & (150\leqq k<200)\end{cases}$$

③ A sinusoidal wave disturbance

$$w_1(k)=30\sin(2\pi k/100)\ (0\leqq k<100)$$
$$50\sin(2\pi/100)\ (150\leqq k<200)$$

④ As a example of a process with time variant parameters, the value of gain K_V was

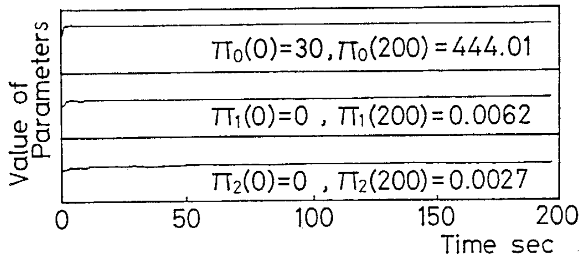


Fig. 6 Response of the process for the step disturbance (Simulation)

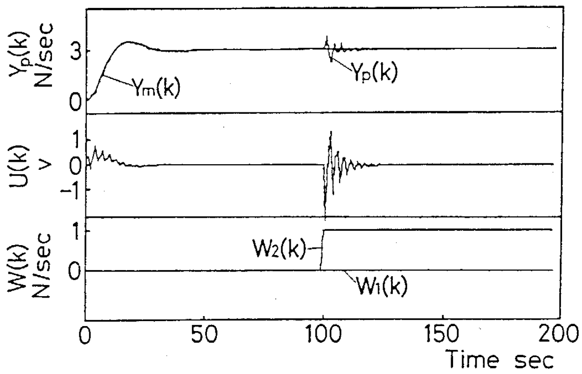


Fig. 7 Response of the process for the rampwise disturbance (Simulation)

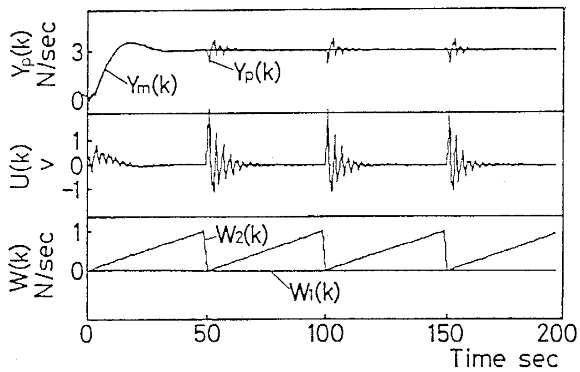


Fig. 8 Response of the process for the sinusoidal disturbance (Simulation)

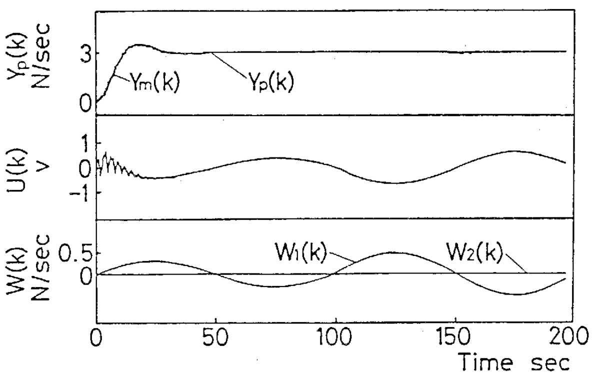


Fig. 9 Response of the process with time variant parameters (Simulation)

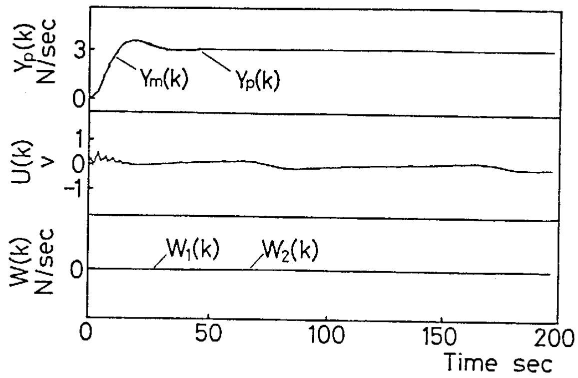


Fig.10 Response of the process with time variant parameters (Algorithm : MRAC-1)

changed by the following equation :

$K_V = K_V + (K_V/2)\sin(2\pi kT/N)$ (N ; period).

The response waves for the disturbances described above are illustrated in Figs. 6 ~9. From these simulation results, we see that the output $y_P(k)$ of the process follows closely the output $y_m(k)$ of the reference model, and that no steady-state offset occurred. The manipulating variable became slightly oscillating at the time when the disturbance $w_2(k)$ described by the polynomial function of time was added to the process (in case ①) and at the time when the disturbance changed to zero in case ②. But after about 20 sec, the output $y_m(k)$ of the process agreed with $y_P(k)$. In case ③, in which the disturbance $w_1(k)$ was added to the process, the manipulating variable changed sinusoidally so as to cancel it. However, the output of the process agreed well with the output of the reference model. In case ④, the manipulating variable changed slightly, but the output $y_P(k)$ also agreed well with the output $y_m(k)$ of the reference model. The simulation results obtained by the control method MRAC-1 used in section 3 confirm the advantage of the algorithm mentioned above and are shown in Fig. 10. Comparing both simulation results, we found that the algorithm proposed in this section was more applicable to the disturbance than the MRAC-1 algorithm.

In the algorithm, the disturbance was considered to be a combination of a disturbance with dynamics in which the order is less than nD1 and a disturbance described by the polynomial function of time in which the order is less than nD2. In the experiment, since nD1 and nD2 are given as nD1=2, nD1=1, respectively, it was possible to cancel the disturbance with dynamic of the second order or under. The system includes a first-order integral element (see Fig.5), and our process itself also has a first-order integral element, therefore the system includes a second-order integral element as the whole system. Namely, the system can theoretically cancel a second-order disturbance $w_2(k)$ described by the polynomial function of time, such as a rampwise disturbance. In practice, it is possible to cancel the disturbance so long as the order of the sum of $w_1(k)$ and $w_2(k)$ is less than nD1+nD2+1.

When only a disturbance $w_2(k)$ that can be canceled by the integral element is added to the process, the disturbance can be considered to be canceled by the composite operation of both the adaptive action and integral action. Since the augmented process is finally expressed by an ARMA model such as Eq.(19) in the construction of the adaptive control system, it can be said that the process model described by the ARMA model is more stable for the disturbance input than that described by the AR model, but the problem of the nominimum phase system for the augmented process requires future study.

Next, experiments corresponding to simulations ① and ④ were implemented. In case ①, a known amount of the outflow quantity from an another hopper was applied to the flowmeter as the step disturbance. The experimental response wave is shown in Fig. 11. And as an example of a process with time variant parameteres (case ④), nylon chips and sand were alternated in 20 cm layers inside the hopper in the same manner as in section 3. Figure 12 shows the response waves, and the behavior of the parameters over time is shown in Fig.13.

Although both response waves oscillate considerably, the output $y_P(k)$ closely follows the output $y_m(k)$ of the model. In Fig.12, the overshoot at a starting point

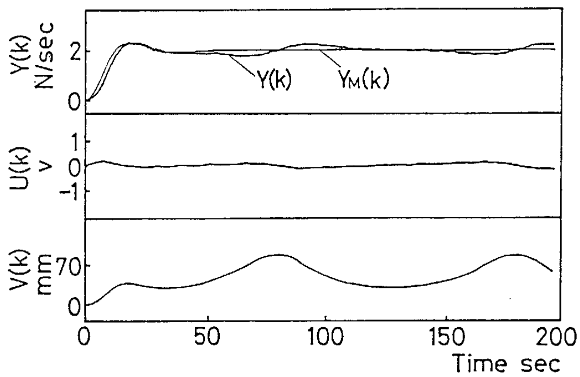


Fig.11 Response of the process for the step disturbance input (Experiment)

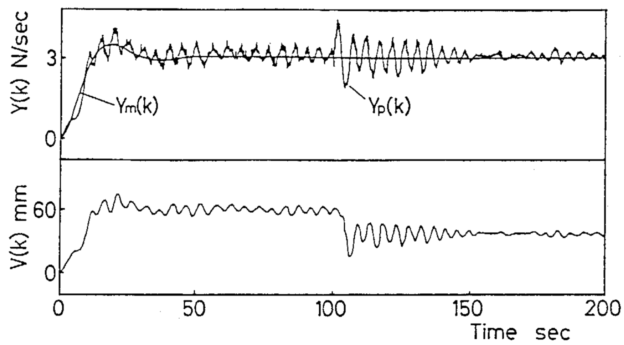
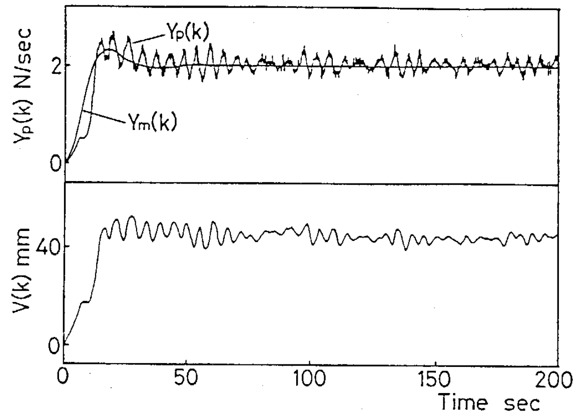
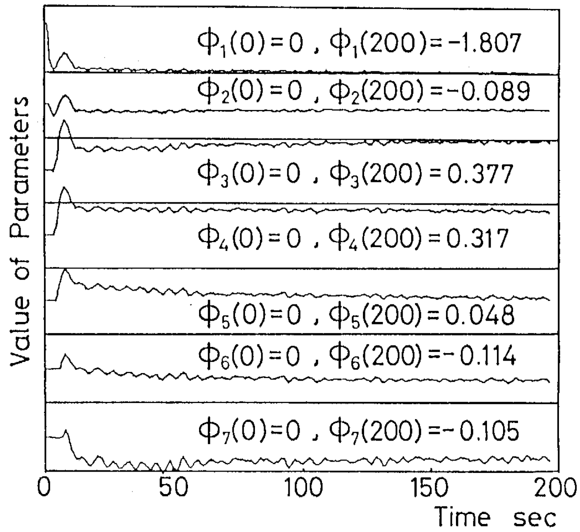


Fig.12 Response of the process with time variant parameters (Experiment)



(a) ϕ_i (k)



(b) π_i (k)

shown in Fig.4 cannot be seen, and the output $y_P(k)$ agrees with $y_m(k)$. From Fig.13, it is understood that the disturbance is canceled by the composite operation of the adaptive action and integral action. From the facts mentioned above, it is clear that the algorithm considering an unknown disturbance is very useful in the process of handling granules.

5. Conclusions

Two kinds of adaptive algorithms were applied to the process, and the advantages of these algorithms were confirmed through simulations and experiments. For the process with unknown disturbances, good response waves were obtained by using an algorithm considering this disturbance. However, there is still room for discussion of small change in the outflow rate.

References

1) Wada,K., Hayano, N and Morioka, H., Trans. Jpn. Soc. Mech. Eng., 54 (497), C, 189 (1988), (in Japanese)

2) Wada,K., Hayano, N and Sagara, S., Trans. SICE, 22(3), 329 (1986), (in Japanese)

3) Fujii, S., and Mizuno. N., Prepri. of 8th IFAC Congr., VII, 120 (1981)

4) Fujii, S., and Mizuno, N., Trans. SICE, 21(9) 914 (1985), (in Japanese)