

Drag Reduction with Heterogeneous Polymer Injection into a Turbulent Pipe Flow

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Abstract

The injection of polymer solution into turbulent pipe flow caused significant drag reduction. The injected polymer threads were visualized photographically. Combining the flow visualization results with the measurements of drag reduction caused by the centerline injection, the possible interaction between polymer threads and turbulent eddies was discussed. It was proposed that, for the centerline injection system, there existed the maximum drag reduction asymptote which was different from the maximum drag reduction asymptote for premixed polymer solutions. This was confirmed by evaluating the pipe diameter effect and the concentration effect in the centerline injection system. The annular injection in the near wall region caused more significant drag reduction than those obtained for the centerline injection. The maximum drag reduction by annular injection coincided with the maximum drag reduction asymptote for the premixed polymer solutions.

INTRODUCTION

Drag reduction with a polymer additive is important as a power saving technology in fluid transportation. In 1979, Trans-Alaska Pipe-Line started the use of polymer injection technology to reduce the pumping power consumption.¹ The other applications, such as flow improvement in sewer system² and slurry transportation³, are also reported recently. Dosing of polymer additives into the tube flow is usually accomplished by injecting a relatively concentrated polymer solution by means of a suitable dosing system. Injected polymer solution does not mix soon with the main flow. Heterogeneous flow condition is maintained over a considerably long downstream distance from the polymer dosing station. In particular, if the injected polymer solution is immiscible with the main fluid, heterogeneous flow condition will be held over the whole length of pipe-line.

Drag reduction in premixed, i. e. homogeneous, dilute polymer solutions has been well documented by a lot of investigations published during the last two decades. However, reports on heterogeneous drag reduction with polymer injection are rather scarce. Vlegaar and Tels⁴ injected a concentrated polymer solution into the core region of a turbulent pipe flow. Their results indicated that considerable drag reduction occurred before the injected polymer was convected radially into the wall region. It was suggested that the interaction between injected polymer thread and turbulent eddy in the turbulent core region might cause reduction. Vlegaar's observation was recently confirmed by Berman⁵ and Bewersdorff.⁶ They showed by LDV measure-

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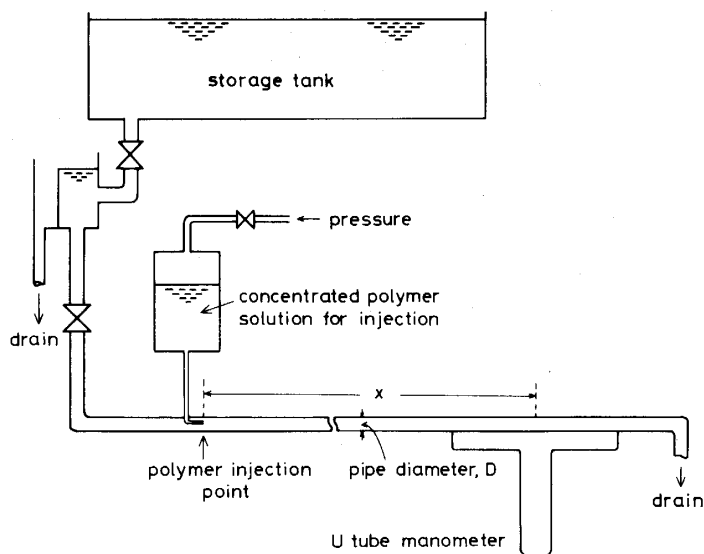


Fig. 1 Experimental apparatus.

ments that the velocity profile in turbulent core for heterogeneous drag reduction was changed significantly from those obtained for premixed drag reducing system. On the other hand, McComb and Rabie⁷ have carried out the centerline injection experiments. From the measurements of concentration profile of injected polymer, they concluded that the existence of polymer additives in the wall region was required to cause drag reduction in heterogeneous system. The other experimental works^{8,9} for heterogeneous drag-reducing system cannot give a decisive conclusion for the above-mentioned contradiction because of their different mode of polymer injection.

Main purpose of this work was to clarify the heterogeneous drag reduction behaviour caused by injection of polymer solutions. It was intended to examine experimentally the effects of polymer concentration, pipe diameter and difference of injection mode on heterogeneous drag reduction. From the results both of pressure drop measurements and flow visualization, possible interaction between injected polymer thread and turbulent eddy was discussed. Also the difference between heterogeneous and premixed drag-reducing systems was discussed.

EXPERIMENTAL APPARATUS AND PROCEDURE

The flow diagram of experimental apparatus is shown in Fig. 1. In the case of premixed drag reduction experiments, dilute polymer solutions were prepared in the storage tank (1m³), and an once-through flow system was used to minimize the effect of mechanical degradation of polymer additives. In the case of polymer injection experiments, water was forced to flow through head tank, and a relatively concentrated polymer solution contained in a pressurized vessel was injected through injection nozzle. Four kinds of acrylic glass tubes, of which inner diameters, D , are 12.3, 18.8, 25.2 and 51.3 mm, were used as test tubes. Pressure drop along the test tube was measured by means of CCl₄ U-tube manometers at $x/D=30\sim 300$, where x was the distance from injection point to measuring point.

In addition to pressure drop measurements along the test tube, flow visualization experiments were carried out. The flow behaviour of polymer thread coloured by

indian ink was observed photographically.

Polyethylene oxide (grade Alcox, E-160 supplied by Meisei Chemical Corp.) was used as polymer additive. The intrinsic viscosity of Alcox, E-160 in water was determined experimentally as 1200 cm³/g. We used two kinds of polymer concentration in this work, i. e. c_p and c_{av} . c_p was the concentration injected polymer solution. c_{av} was the averaged polymer concentration over the cross-section of the pipe. The polymer concentration in the premixed solution was also indicated by c_{av} . In the case of premixed drag reduction experiments, dilute polymer solution ($c_{av}=30\sim300$ ppm), were prepared in the storage tank. A stabilizer, Sandex-C ($c_{av}=30$ ppm), was used to prevent the chemical degradation of polymer additive. In the case of polymer injection experiments, concentrated polymer solutions containing $c_p=2000, 4000$ and 8000 ppm were prepared in the pressurized vessel. The injection flow rate was measured by observing the level meter attached to the pressurized vessel. Bulk flow rate in a test tube was measured gravimetrically.

Table 1 Comparison of injection velocity, U_p with centerline velocity, U_c at $Re=10^4$

D[mm]	d[mm]	U_p [cm/s]	U_c [cm/s]
12.3	1.0	92.9	111.4
18.8	2.0	35.7	72.9
25.2	2.0	47.7	54.4
51.3	4.0	24.2	26.7

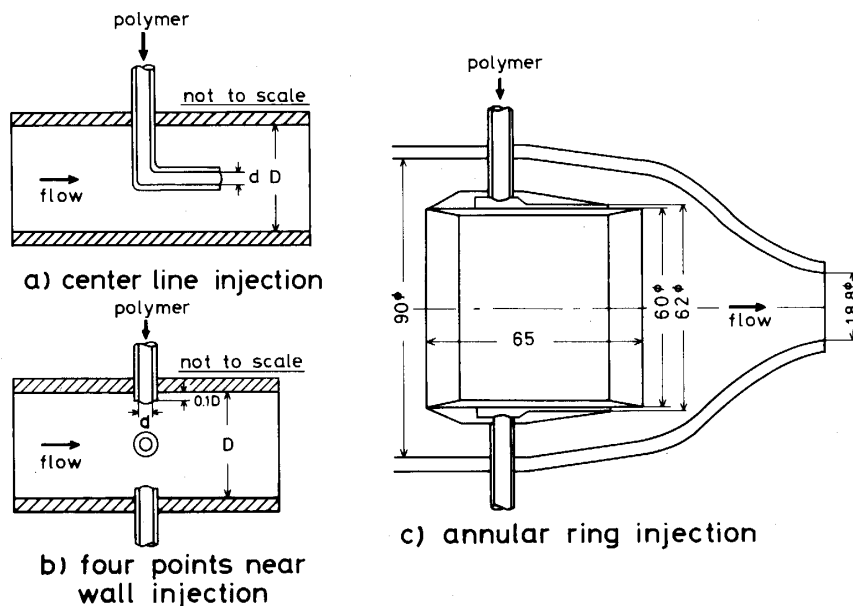


Fig. 2 Details of the polymer injectors.

The designs of polymer injector are shown in Fig. 2. The center line injection mode is accomplished by a single nozzle (Fig. 2-a). Inner diameter, d , of injection nozzle was

designed so that the outlet velocity from the injection nozzle coincide approximately with the center line velocity of tube flow. The values of inner diameter are shown in Table 1. Figure 2-b indicates the four points near wall injection mode. Nozzle diameter is the same as indicated in Table 1. The distance between nozzle outlet and tube wall was equal to 10% of test tube diameter. An annular injector (Fig. 2-c) was designed to inject an annular ring of polymer solution in the near wall region. This was specially designed for the 18.8 mm test tube. An annular ring of polymer solution, having 1 mm thickness, was injected before the main flow was contracted to 18.8 mm test tube.

EXPERIMENTAL RESULTS AND DISCUSSION

Rheology of the injected polymer solutions

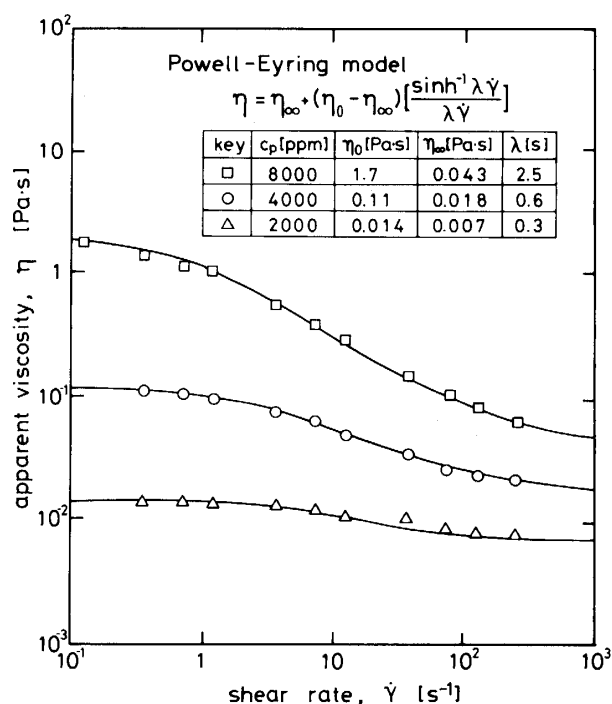


Fig. 3 Apparent viscosity of polymer solutions to be used for polymer injection.

The rheological behaviour of the injected polymer solution was investigated by a coaxial rotating cylinder rheometer (IR-200, Iwamoto seisakusho Co. Ltd.). The dependence of viscosity on shear rate is shown in Fig. 3. The flow characteristic curves are simulated by Powell-Eyring model¹⁰;

$$\eta = \eta_{\infty} + (\eta_0 - \eta_{\infty}) \left\{ \frac{\sinh^{-1} \lambda \dot{\gamma}}{\lambda \dot{\gamma}} \right\} \quad (1)$$

, where η , η_0 , η_{∞} , λ and $\dot{\gamma}$ are viscosity, viscosity at zero shear rate, viscosity at infinitely large shear rate, relaxation time and shear rate, respectively. The simulated curves are indicated by solid lines in this diagram, and the values of the best fit parameters are also shown in this diagram. These rheological parameters will be discussed in the following sections combining with the drag reduction data.

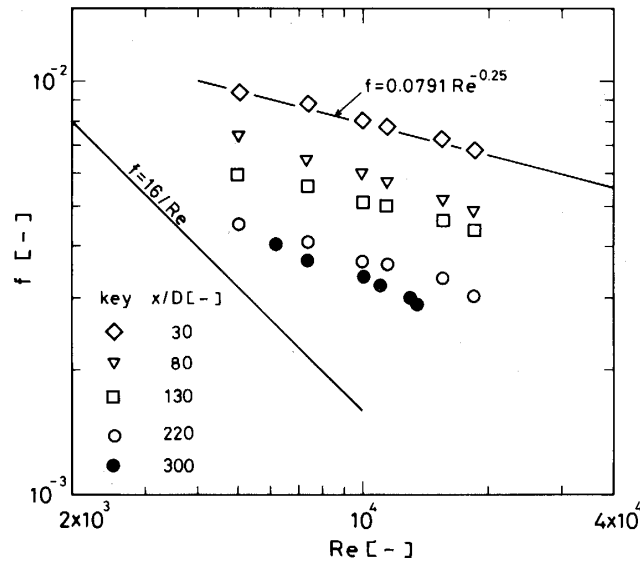


Fig. 4 Friction behaviour for heterogeneous drag reduction caused by centerline polymer injection ($D=12.3\text{mm}$, $c_p=4000\text{ppm}$ and $c_{av}=30\text{ppm}$).

Local drag reduction

Typical drag reduction data for centerline injection with $c_p=4000\text{ppm}$, $c_{av}=30\text{ppm}$ and $D=12.3\text{mm}$ are shown in Fig. 4. To evaluate the Reynolds number, the viscosity of water, not the polymer solution viscosity, was used. Drag reduction is the function of downstream distance from the injection point. Also, it should be emphasized that friction factor is reduced almost in parallel with the Newtonian Blasius law. No onset point of drag reduction is observed. Percentage drag reduction, DR, was defined as ;

$$DR = \frac{f_N - f_p}{f_N} \times 100 \quad (\%) \quad (2)$$

, where f_N and f_p are friction factors for Newtonian fluid flow and for drag-reducing flow with polymer additives, respectively.

The local percentage drag reduction data at $Re=10^4$ are shown in Fig. 5, comparing with the experimental results obtained by McComb-Rabie⁷ and Bewersdorff⁶. Bewersdorff used 50mm i. d. test tube. So, his results should be compared with the open circle symbol data of this work. McComb-Rabie⁷ used Polyox WSR-301 at $Re=4.5 \times 10^4$, $c_p=3000\text{ppm}$ and $c_{av}=10.8\text{ppm}$, while Bewersdorff⁶ used Separan AP30 at $Re=8 \times 10^4$, $c_p=3000\text{ppm}$ and $c_{av}=20\text{ppm}$. They showed that drag reduction caused by polymer injection reached an equilibrium state at $x/D > 100 \sim 150$. The difference of drag reduction at large x/D value should be attributed to the difference of injected polymer species and the difference of experimental conditions. Experimental results of this work with 51.3mm test tube indicated by open circle symbol correspond to the data of Bewersdorff. Total length of 51.3mm test tube was 8m in this experiments. So the drag reduction data at $x/D > 136$ was not obtained. However, referring to the results of the other investigations, we expect that the drag reduction at $x/D=136$ with $D=51.3\text{mm}$ has reached an equilibrium state. The drag reduction data with $D=25.2\text{mm}$ show that an equilibrium state can be obtained at $x/D > 180$. In the case of $D=12.3\text{mm}$ test tube, drag reduction becomes almost an equilibrium state at $x/D=220 \sim 300$. Although the drag

reduction data shown in Fig. 5 indicate that there exists an equilibrium state of drag reduction over a considerably long down-stream distance of a pipe flow, there is no experimental evidence to answer the question; how long does the equilibrium state persist? The experiment for long distance fluid transportation with polymer injection is expected to establish the engineering applicability of heterogeneous drag reduction technique.

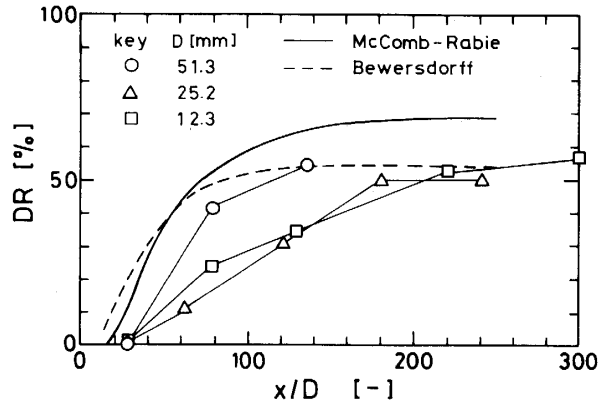


Fig. 5 Local drag reduction versus distance from the injection point at $Re=10^4$, $c_p=4000\text{ppm}$ and $c_{av}=30\text{ppm}$. (All the results were obtained for centerline injection.)

As the different development of local drag reduction is observed for the different tube diameter, it may be anticipated that the interaction between injected polymer thread and turbulent eddy is somewhat different if the pipe diameter is changed. So the behaviour of polymer thread along the test tube was observed photographically. Examples of results are shown in Fig. 6.

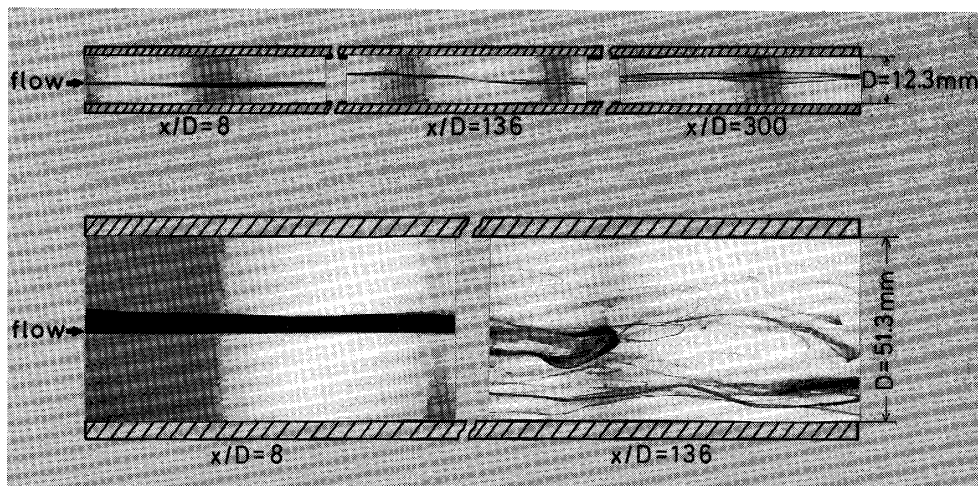


Fig. 6 Centerline injection of dyed polymer solution at $Re=10^4$, $c_p=4000\text{ppm}$ and $c_{av}=30\text{ppm}$.

This figure compares the results of the largest tube ($D=51.3\text{mm}$) and the smallest tube ($D=12.3\text{mm}$). The single thread injected at the center of the tube is observed at $x/D=8$ both for the cases of $D=51.3\text{mm}$ and $D=12.3\text{mm}$. At the downstream of larger tube (at $x/D=136$ and $D=51.3\text{mm}$), fine threads of polymer solution are observed. They are distorted and spread out radially because of the interaction between turbulent eddies.

On the other hand, the polymer thread injected into smaller tube is not distorted significantly. Although the injected polymer solution is subdivided into several threads at $x/D=300$, all polymer threads seem to flow in the turbulent core region of tube flow. It should be emphasized that, as shown in Fig. 5, the drag reductions obtained for different tube diameters ($D=51.3\text{mm}$ and 12.3mm) are almost the same, however, observed interaction between polymer threads and turbulent eddies is quite different if the tube diameter is different.

Pipe diameter effect

Pipe diameter effect in premixed drag-reducing system has been well recognized in the previous investigations^{11,12}. It has been pointed out that the dimension of polymer molecule was constant, while the turbulence scale was enlarged when the pipe diameter was scaled up. So it has been reported that the drag-reducing effect was diminished when the pipe diameter become larger under the premixed condition with constant polymer concentration¹². This is a very severe defect in drag reduction application. In the previous investigations of heterogeneous drag reduction with polymer injection, no one has reported the results of pipe diameter effect. Thus, in this study, we carried out the drag reduction measurements with different pipe diameter ($D=12.3\sim 51.3\text{mm}$). The experimental results are shown in Fig. 7. The premixed drag reduction data indicated by open symbols are well correlated by the predicted line by Mizushima-Usui¹². No drag reduction is expected in premixed system if we use larger pipe ($D>51.3\text{mm}$) and dilute polymer solution ($c_{av}<30\text{ppm}$) at $Re<4\times 10^4$. On the other hand, the drag reduction data with centerline polymer injection indicated by solid symbols do not show any pipe diameter effect. They show almost the same drag reduction level. This means that the heterogeneous polymer injection technology is quite effective in scale-up problem of drag-reducing system.

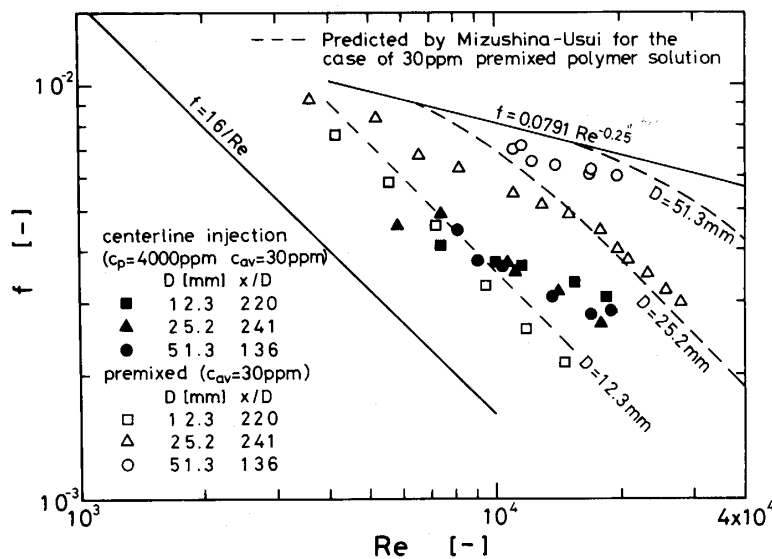


Fig. 7 Pipe diameter effect on drag reduction (Centerline injection.)

As indicated in Table 1, the scales of centerline polymer injection nozzle were

fabricated as almost proportional to the pipe diameter. So the dimension of polymer thread flowing in test tube is increased in accordance to the scale-up of pipe diameter. The diameter of polymer thread was determined photographically as several percent of tube diameter. On the other hand, the scale of largest eddy in turbulent tube flow is almost the same as tube radius. So the main interaction with polymer thread may be caused by one order smaller eddy than the largest eddy. This eddy may be thought to be energy containing eddy of turbulent pipe flow. It is evident that the possibility of interaction with micro-scale turbulent eddy is very scarce because the polymer threads are sparsely distributed in a tube flow. The relative velocity between polymer thread and turbulent eddy may be roughly equal to turbulent fluctuating velocity. The representative turbulent fluctuating velocity is the turbulent intensity, and its value is given elsewhere¹³. Weissenberg number, We_e , for the dynamic motion of polymer thread may be defined as ;

$$We = \lambda u_r' / d_p \quad (3)$$

, where λ , u_r' and d_p are relaxation time of injected polymer solution, radial turbulent intensity and diameter of polymer thread, respectively. We assume that the relaxation time is given by the rheological measurements shown in Fig. 1. Also we assume that $u_r' = 0.03 \sim 0.04 U_{max}$ according to Laufer's data¹³, where U_{max} is the centerline velocity of the tube flow. The diameter of polymer thread, d_p , just after the injection point is roughly equal to 0.1D as shown in Table 1. The polymer thread diameter at the downstream becomes several percent of the pipe diameter as shown in Fig. 6. Assuming that d_p is roughly equal to 0.03D, Weissenberg number, We_e , is estimated as $We_e = 1.1 \sim 11$ when Reynolds number changed from 5×10^3 to 5×10^4 for the case of $c_p = 4000$ ppm and $D = 51.3$ mm. In the case of $c_p = 4000$ ppm and $D = 12.3$ mm, Weissenberg number varies from 20 to 200 for the same Reynolds number change. Thus we can conclude that the strong interaction between polymer thread and turbulent eddy is expected.

It has been pointed out in Fig.6 that the degree of deformation of polymer thread was different between the largest tube ($D = 51.3$ mm) and the smallest tube ($D = 12.3$ mm). Large deformation may be caused by the interaction with the largest scale eddy which is one order larger than d_p . So the values of Weissenberg number become one order smaller than those estimated above. In the case of $D = 51.3$ mm, Weissenberg number based on large turbulent eddy becomes smaller than unity. Thus, the polymer thread can be significantly deformed. On the other hand, if the pipe diameter is small (i. e. $D = 12.3$ mm), Weissenberg number based on large turbulent eddy may be possibly larger than unity. The polymer thread is not significantly deformed, while the thread can cause a strong damping effect on the turbulent flow field. The interpretation mentioned above seems to explain well the flow visualization results shown in Fig.6.

Concentration effect

The dependence of drag reduction on the variation of polymer concentrations are shown in Fig. 8. Both concentrations c_p and c_{av} were changed. Difference in drag reduction was not observed when c_{av} is changed. Slight dependence of drag reduction on injected polymer concentration is noticed. The highly concentrated polymer solution

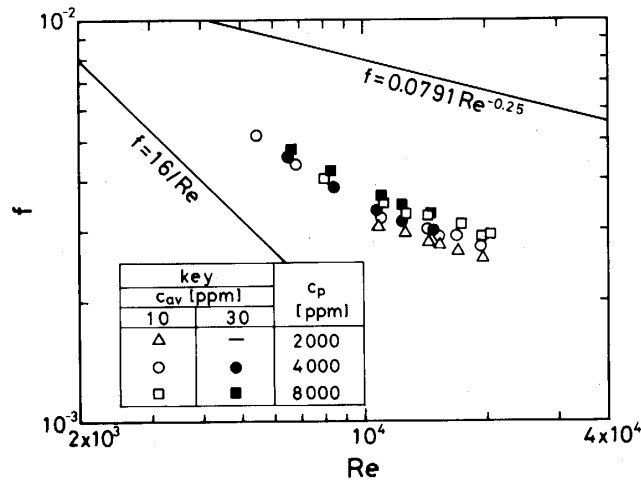


Fig. 8 Effect of centerline injected polymer concentration on drag reduction (D=18.8mm.)

is so rigid that the interaction with turbulent eddy may not be caused effectively. The same observation was reported by Berman.⁵ He also showed that lower concentration gave less drag reduction again. If the concentration of injected polymer solution is too low, the polymer thread is easily mixed by turbulent shear action, and the flow situation may become similar to premixed drag-reducing system. So it may be concluded that there exists an optimum concentration to obtain the maximum drag reduction. From Fig.8, the conditions, $c_p=2000\text{ppm}$ and $c_{av}=10\text{ppm}$ seems to be best to obtain effective drag reduction. But this conclusion should be restricted within the experimental range of this work.

The drag reduction data obtained thus far in this work indicate that both pipe diameter effect and concentration effect are very little. In the premixed drag-reducing system, pipe diameter effect and concentration effect disappear when maximum drag

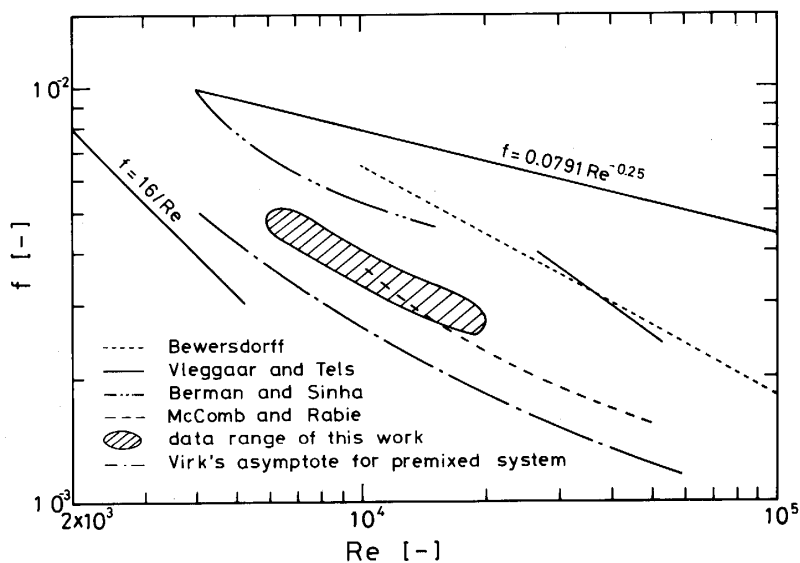


Fig. 9 Maximum drag reduction obtained for centerline polymer injection by several investigators.

reduction condition is obtained.¹¹ So it may be worth while to discuss if the present heterogeneous drag reduction data reach the maximum drag reduction asymptote. The previous drag reduction data obtained by several authors are compared in Fig. 9. All of them were obtained for centerline injection experiments. Lines except Virk's asymptote indicate the minimum friction factor observed by each investigator. Present experimental data and results of McComb-Rabie⁷ are the most effective drag-reducing data for centerline polymer injection. They do not coincide with Virk's asymptote¹¹ for premixed drag reduction. It is not necessary that the maximum drag reduction asymptote for heterogeneous drag reduction for centerline injection must coincide with Virk's asymptote. As discussed thus far, the interaction between polymer thread and turbulent eddy occurs mainly in turbulent core region, and this interaction is thought to be quite different from the polymer molecule-turbulent eddy interaction in premixed system which is normally assumed that the addition of polymers only changes the structure of turbulence in the near-wall region. Thus, we propose that the minimum friction factor line obtained by this work and McComb-Rabie is the maximum drag reduction asymptote for heterogeneous drag reduction with centerline polymer injection. This line is approximated by ;

$$1/\sqrt{f} = 16.2 \log Re\sqrt{f} - 27.6 \quad (4)$$

Comparison of drag reduction data obtained by different injection modes.

All the drag reduction results shown and discussed thus far in this paper were obtained for centerline injection. We have pointed out that the maximum drag reduction asymptote for centerline injection might be different from Virk's asymptote. Flow visualization results suggested that the polymer thread-turbulent eddy interaction occurred significantly in turbulent core region. However, if the polymer solution is injected in the near wall region, different type of interaction could occur. An annular injector and four points near wall injector, shown in Fig. 2, were used to examine if there existed different drag reduction behaviour from centerline injection mode or not. The experimental results are shown in Fig. 10. The experimental conditions of this

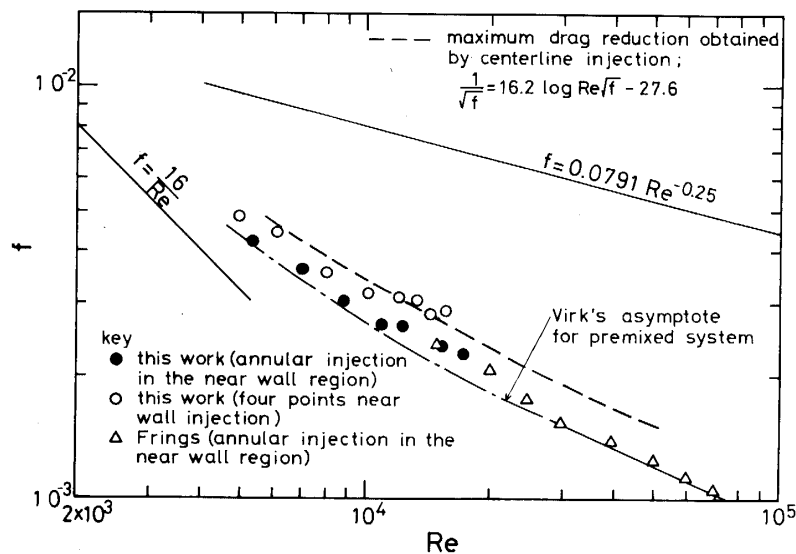


Fig. 10 Drag reduction by annular injection in the near wall region.

work are $D=18.8\text{mm}$, $c_p=4000\text{ppm}$ and $c_{av}=30\text{ppm}$. The results of friction factor obtained at $x/D=210$ are plotted. The experimental results by Frings¹⁴ for annular injection in the near wall region are compared in this diagram. His experimental conditions were $D=50\text{mm}$, $c_p=2000\text{ppm}$, $c_{av}=50\text{ppm}$, $x/D=70\sim 210$, and Separan AP45 solution was injected through 0.4mm annular slit. Both annular injection results of this work and Frings show the same drag reduction level, and they coincide with Virk's asymptote for premixed drag-reducing system. Four points near wall injection data of this work show an intermediate behaviour between annular injection and centerline injection at lower Reynolds number range (at $Re < 10^4$), but they coincide with the centerline injection results at $Re > 10^4$. These observations are convincing because the four points near wall injection mode is the intermediate mode between centerline injection and annular injection as shown in Fig. 2.

Drag reduction data for annular injection shown in Fig.10 suggest that the annular ring of polymer solution injected in the near wall region may cause a similar interaction with wall turbulence as observed in the case of premixed system. The coloured polymer solution injected through annular slit was photographed at $x/D=210$, and the result is shown in Fig. 10. Complete annular polymer layer was not obtained even at the exit of contraction nozzle just after the annular injector, but fine polymer threads were aligned in the near wall region. These polymer threads convected into down stream, while some of them was diffused into turbulent core. The photograph taken at $x/D=210$ (Fig. 11) shows that the alignment of polymer threads is considerably distorted

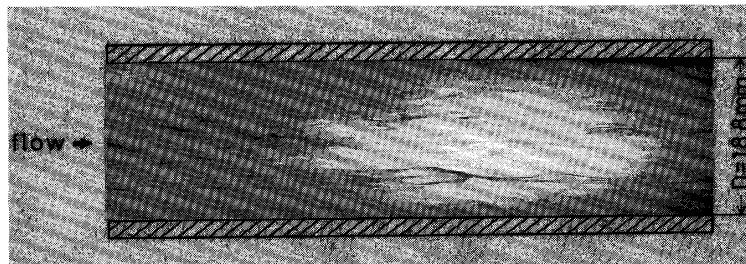


Fig. 11 Annular injection of dyed polymer solution at $Re=10^4$, $c_p=4000\text{ppm}$, $c_{av}=30\text{ppm}$ and $D=18.8\text{mm}$.

by turbulent eddy. But large parts of fine polymer threads are still interacting with wall turbulence in the near wall region. It is obvious that the effective concentration of polymer in the near wall region is diminished at the large distance downstream from injection point because of the convecting effect by turbulent eddy. So it is doubtful that the high level of drag reduction caused by annular injection is maintained for ever. Figure 12 shows the local drag reduction obtained by annular injection comparing with the experimental results both for wall slot injection and centerline injection. Experimental conditions of McComb-Rabie were $c_p=3000\text{ppm}$, $c_{av}=7.5\text{ppm}$, $D=50\text{mm}$, Polyox WSR-301. Both wall injection and near wall annular injection show larger drag reduction at shortly after the injection point if it is compared with centerline injection data. This means that the interaction between polymer-turbulent eddy can occur very fast in the near wall region. All the data obtained for wall or near wall injection (this work, Frings¹⁴ and McComb-Rabie⁷) show the maximum value of DR at $x/D=100\sim 200$,

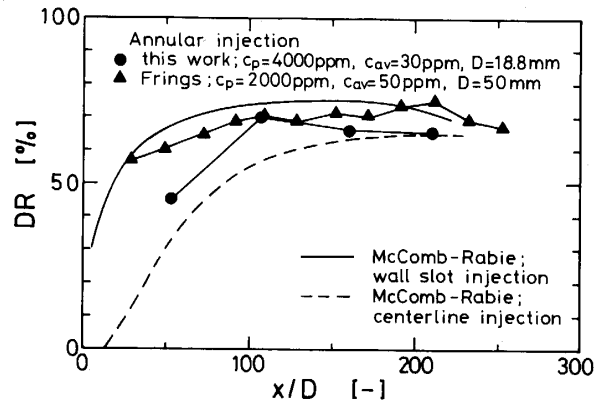


Fig. 12 Local drag reduction versus distance from the injection point obtained for annular injection.

and they show the tendency to decrease the DR value at $x/D > 150 \sim 200$. We have no evidence that the drag reduction level caused by annular injection become lower than obtained for centerline injection. Also we are not sure how long the constant drag reduction level caused by heterogeneous polymer injection will be maintained in the downstream distance. However, we observed that the fine polymer thread formed by annular injection was not easily mixed to generate the homogeneous solution within the experimental range. So it may be concluded that the injected polymer solution maintain the heterogeneous state and cause a very effective drag reduction for considerably long distance from injection point.

CONCLUDING REMARKS

1. When polymer solution was injected at the centerline of test tube, the local drag reduction increased with distance downstream from the injection point. At constant Reynolds number, the injected polymer solution was spreaded more easily in radial direction if the pipe diameter was large. This caused a rapid increase in drag reduction with the downstream distance. If the pipe diameter was small, the injected polymer maintained the same figure through the flow visualization section $x/D=0 \sim 300$ although it was deformed a little. In this case, the increase of local drag reduction along the downstream distance was rather slow, but the final drag reduction level was the same as those obtained for larger pipe diameter. Differences of drag reduction and polymer thread visualization observed for different pipe diameters were qualitatively explained by using the characteristic time measured for concentrated polymer solutions.

2. Both pipe diameter effect and concentration effect were not observed in the centerline polymer injection experiments. All the drag reduction data fell on a narrow experimental error bound which was almost parallel to Virk's asymptote and was a little upward shifted. Comparing with the centerline polymer injection data by previous investigators, we proposed the maximum drag reduction asymptote given by ;

$$1/\sqrt{f}=16.2 \log \text{Re}\sqrt{f}-27.6$$

for drag reduction caused by centerline injection. This maximum drag reduction asymptote is a little worse than those obtained for premixed drag reduction. But the maximum drag reduction was easily obtained for lower polymer concentration with

larger pipe diameter. So the centerline injection technique was concluded to be effective as a practical drag reduction application.

3. Annular injection in the near wall region caused more significant drag reduction than those obtained for centerline injection. Maximum drag reduction by annular injection coincided with Virk's asymptote for premixed drag-reducing system. However, the drag reduction caused by annular injection had a tendency to diminish a little at the downstream distance after the maximum drag reduction was obtained. It was not certified that if the diminished drag reduction level coincided with the centerline maximum drag reduction or not.

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