Estimation of the Earth Pressure Acting on a Shaft Lining

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Abstract

Various formulae for the purpose of estimating the earth pressure which acts on a shaft lining have so far been proposed. Each of all these formulae has been derived under some comparatively simple assumptions. Although some of these formulae seem to be fairly pertinent for a ground of a certain character, such as homogeneous Quarternary system, it seems to be unreasonable, in many cases, to estimate the earth pressure by the aid of these formulae, while the nature and the conditions of a ground are fairly complicated in general.

In this article, therefore, the author described a method for estimating the earth pressure which acts on the outer wall of a shaft lining utilizing the results of measurement of stresses which arise on the inner wall surface of the lining. The author further refered to estimation of the earth pressure by the nature and conditions of a ground surrounding a shaft.

1. Introduction

In order to design a shaft lining, it is essential to estimate the earth pressure acting on it. Proper estimation of the earth pressure acting on a shaft lining, however, will be difficult indeed, because the ground surrounding the shaft may consists of sand or clay in some cases, and rock strata in other cases, the nature and conditions of which are quite different, and the earth pressure which acts on the shaft lining will be diversely varied and complicated accordingly.

Since the magnitude and distribution of the earth pressure which acts on a lining, however, are the basis for planning a shaft lining, it is necessary to estimate them by some means or other.

In this article the author begines with the description of methods for estimating the earth pressure acting on a lining hitherto proposed, describes secondly how to determine the earth pressure utilizing the results of measurement of the stresses which arise in the lining, and then discusses the general means for assuming the earth pressure.

2. Methods for estimating the earth pressure which acts on the shaft lining hitherto proposed.

With regard to the methods for estimating the earth pressure which acts on a shaft lining, several investigations have been carried out and various formulae have been proposed. In the following, results of these investigations are reviewed and their pertinences are considered.

(1) Formulae obtained under assumptions that the earth pressure which acts on a lining distributes uniformly in any horizontal cross-section.

Protojakonov proposed the following equation derived from Rankine's earth pressure theory[1)]:

$$p=\gamma H \tan^2\left(\frac{\pi}{4}-\frac{\phi}{2}\right) \quad \cdots\cdots(1)$$

in which p means the earth pressure per unit area, γ weight of unit volume of the ground, and ϕ angle of internal friction of the ground. If the ground is composed of a number of strata,

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denoting the thickness of each stratum by h_1, h_2, h_3,……, h_n, the mean value of the angle of internal friction is represented by the following expression:

$$\phi_{mean}=\tan^{-1}\frac{h_1 \tan \phi_1+h_2 \tan \phi_2+\cdots\cdots+h_n \tan \phi_n}{h_1+h_2+\cdots\cdots h_n} \quad \cdots\cdots(2)$$

Protojakonov has classified rocks and soils which compose the ground into ten groups, in accordance with their strength. For examples, $\phi=87°08'$ for very hard rocks belonging to grade I, $\phi=75°58'$ for rocks of average hardness belonging to group V, and $\phi=16°42'$ for earth belonging to grade X. Accordingly, ratio of p obtained by Eq. (1) to γH, in which γ and H mean weight of unit volume of the ground and the depth from the surface respectively, comes to 0.0007 for a ground composed of rocks belonging to group I, 0.015 for group V and 0.55 for group X, According to the results of measurement of the stresses which arise in the lining of a circular shaft sunk in a Quarternary strata, the magnitudes of tangential stresses measured coincide well with the magnitudes of the stresses obtained by calculation using Rankine's earth pressure formula.

Consequently, Eq. (1) seems to be competent for a ground which is composed of comparatively homogeneous earth and sand, like a Quarternary system. However, it is considered to be unreasonable to determine the value of ϕ for a ground composed of rocks.

Dinnik treated the ground around a lining as an elastic body and proposed the following equation to obtain the earth pressure acting on a lining:

$$p=\frac{1}{m-1}\gamma H \quad \cdots\cdots (3)$$

in which m means Poisson's number of the ground. This equation shows that the earth pressure which acts on a lining is equal to horizontal initial stress in the ground. This means that even when the shaft is sunk in a very strong ground, the lining is exposed to such a earth pressure. For this reason above mentioned equation is undoubtedly unreasonable.

Tinvalewich proposed the following equation for a ground composed of a number of strata, each of which exerts different earth pressure p_n on the lining:

$$p_n=(\gamma_1 h_1+\gamma_2 h_2+\cdots\cdots+\gamma_n h_n)\ \tan^2\left(\frac{\pi}{4}-\frac{\phi_n}{2}\right) \quad \cdots\cdots (4)$$

in which h_1, h_2,……, h_n mean the thickness of each stratum, γ_1, γ_2,……, γ_n weight of unit volume of each stratum, and ϕ_n the angle of internal friction for number n stratum. This equation also seems to be competent for a ground which is composed of earth or sand stratum.

Heise-Herbst-Fritzsche state that under normal conditions only horizontal pressure corresponding to the depth from surface should be considered as the earth pressure which distributes uniformly on a lining, and in case that the strata consist of quick sand, 1.3 times of hydrostatic pressure which corresponds to the depth from the surface for depth range less than 300 m, and 1.7 times of the hydrostatic pressure for depth range more than 300 m, should be taken[2].

(2) Formulae obtained under the assumption that the earth pressure acting on a lining does not distribute uniformly.

When the earth pressure which acts on a lining distributes uniformly on a lining, no tensile stress exists in the lining. However, when the earth pressure does not distribute uniformly, tensile stresses are produced in the lining, providing that the ratio of mean value of the earth pressure to uniformly distributing component of the earth pressure is larger than a certain limit[3].

Mohr expressed the earth pressure p by the following equation as sum of uniform load and additional non-uniform load in radial directions, Fig. 1[4]:

$$p=p_0+\frac{p_1}{2}(1+\cos 2\theta) \quad \cdots\cdots (5)$$

in which p_0, p_1 and θ mean the intensity of the uniform earth pressure, maximum value of the additional earth pressure and the angle between directions of p and p_1. The value of p_1/p_0 is called "grade of non-uniformity". Mohr stated that in case of planning a tubbing, 1.3 times of the hydrostatic pressure corresponding to depth from the surface should be taken as magnitude of p_0 and that a value which is smaller than 0.1 and larger than 0.05 should be taken as the grade of non-uniformity even in case that the depth is very large (say 600 m).

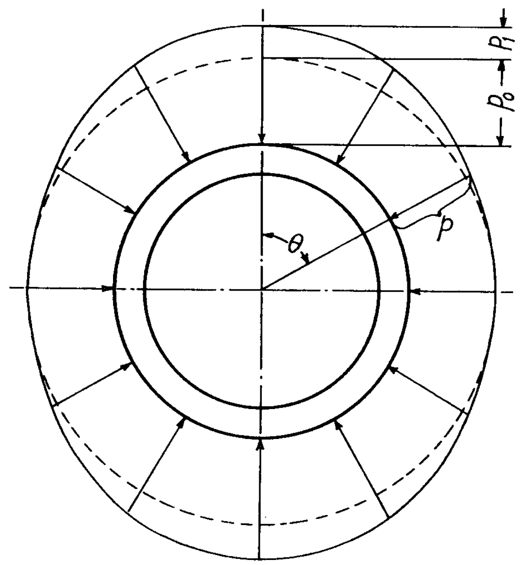

Fig. 1 Distribution of the earth pressure which is expressed as the sum of the uniform earth pressure p_0 and additional earth pressure $p_1(1+\cos 2\theta)/2$.

Heise-Herbst-Fritzsche considered the same distribution of the earth pressure as the distribution expressed by Eq. (5), and stated that as the magnitude of the grade of nonuniformity, 0.15 for shallow shafts and 0.10 for deep shafts schould be taken, being based on experiences. Wansleben as well treated the stresses in a lining considering a state of earth pressure expressed by Eq. (5)[5]. Weehuizen expressed the distribution of the earth pressure by the following equation[6]:

$$p=p_0(1+\beta+\beta\cos 2\theta) \quad \cdots\cdots (6)$$

in which β means a constant.

As stated above, the non-uniform distribution of the earth pressure on a lining has so far been expressed by a distribution which is illustrated in Fig. 1, or by some other distribution similar to this. However, it is also possible that the earth pressure concentrate on a narrow range on the outer wall of a lining, and in such cases more unfavourable stresses are produced in the lining. Accordingly the earth pressure which acts on several narrow ranges of a lining should also be taken into consideration.

3. Estimation of the earth pressure by measurement of stresses in a lining.

As a means of determining the earth pressure which acts on the lining, direct measurement of the earth pressure intensity by setting up a number of earth pressure cells between the lining and the ground is considered first of all. Krpennikov obtained the distribution of the earth pressure around a circular tubbing using above mentioned method. Although this method seems to be excellent in that the earth pressure is measured directly, whereas the area of the outer wall of the lining on which the earth pressure acts is much larger than the earth pressure cell, and its distribution is not so even in gereral, there is the risk that the results of measurement are influenced by local conditions of the ground or by the states of contacts between the pressure cells and the ground. Moreover this method will be somewhat troublesome, and will not be able to be applied in case that the shaft is sunk by well curb sinking method.

Measurement of stresses which arise on the inner wall surface of a circular shaft lining by means of photoelastic stressmeter is free from above mentioned defects, and is hardly affected by the creep of the stressmeter itself, even when the measurement lasts for a long period of time. In the following, then, a method for the purpose of estimating the earth pressure which acts on the outer surface of a lining by the results of measurement of stresses which are produced on the inner wall surface of a lining using photoelastic stressmeter is described.

In case the distribution of stresses on a whole circumference of the inner wall surface of a circular shaft lining is obtained, by measuring the horizontal tangential stresses $(\sigma_\theta)_{r=a}$ which exist in a number of stressmeters set up on the inner wall surface in a horizontal plane, the earth pressure which acts on the outer wall of the lining will be found by the following procedure.

Let us consider that s pieces of stressmeter cells P_1, $P_2, \cdots\cdots$, P_s were set up at regular intervals on the inner wall surface of a lining, and that the magnitudes of the tangential stresses $(\sigma_\theta)_1$, $(\sigma_\theta)_2$, $(\sigma_\theta)_3, \cdots\cdots$, $(\sigma_\theta)_s$, which arise in these cells were obtained by measurement, Fig. 2. Let us then take a polar coordinate (r, θ) with primitive OP_1, and express the stress distribution by the following Fourier series:

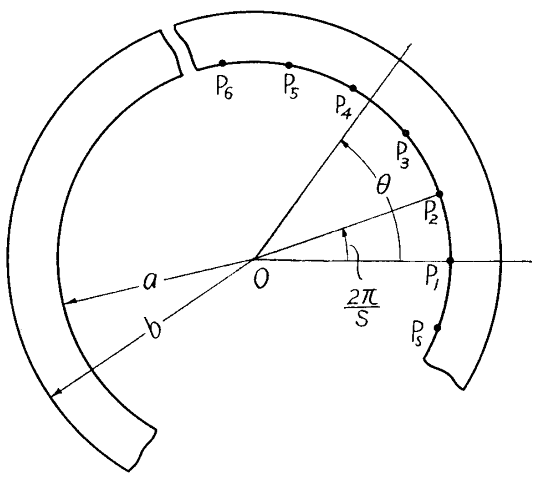

Fig. 2 The position of stressmeters set up on the inner wall surface of a lining.

$$(\sigma_\theta)_{r=a} = a_0 + a_1 \cos\theta + b_1 \sin\theta + \sum_{n=2}^{m} (a_n \cos n\theta + b_n \sin n\theta) \quad \cdots\cdots(7)$$

in which a_0, a_1, b_1, a_n, b_n mean constants, and m, n mean positive integers. In case s is odd

$$m = \frac{1}{2}(s-1)$$

and in case s is even,

$$m = \frac{s}{2}$$

Then, the constants contained in Eq. (7) are determined by substituting the magnitudes of the stresses obtained by measurement into the following equations:

$$\left.\begin{aligned}
a_0 &= \frac{1}{s}\{(\sigma_\theta)_1+(\sigma_\theta)_2+(\sigma_\theta)_3+\cdots\cdots+(\sigma_\theta)_s\},\\
a_n &= \frac{2}{s}\Big\{(\sigma_\theta)_1+(\sigma_\theta)_2\cos\frac{2n}{s}\pi+(\sigma_\theta)_3\cos\frac{4n}{s}\pi+\cdots\cdots\\
&\quad +(\sigma_\theta)_s\cos\frac{2n(s-1)}{s}\pi\Big\},\\
b_n &= \frac{2}{s}\Big\{(\sigma_\theta)_2\sin\frac{2n}{s}\pi+(\sigma_\theta)_3\sin\frac{4n}{s}\pi+\cdots\cdots\\
&\quad +(\sigma_\theta)_s\sin\frac{2n(s-1)}{s}\pi\Big\}.
\end{aligned}\right\} \qquad (8)$$

Each stress components relating to the co-ordinate at any point in a lining are expressed by the following equations[7]:

$$\begin{aligned}
\sigma_r &= 2A_0+B_0r^{-2}\\
&\quad +2\{A_1r-B_1r^{-3}+(2C_1+D_1)r^{-1}\}\cos\theta\\
&\quad +2\{A_1'r-B_1'r^{-3}+(2C_1'+D_1')r^{-1}\}\sin\theta\\
&\quad -\sum_{n=2}^{\infty}\big[\{(n-2)(n+1)A_nr^n+n(n+1)B_nr^{-n-2}\\
&\quad +n(n-1)C_nr^{n-2}+(n+2)(n-1)D_nr^{-n}\}\cos n\theta\\
&\quad +\{(n-2)(n+1)A_n'r^n+n(n+1)B_n'r^{-n-2}\\
&\quad +n(n-1)C_n'r^{n-2}+(n+2)(n-1)D_n'r^{-n}\}\sin n\theta\big], \qquad (9a)
\end{aligned}$$

$$\begin{aligned}
\sigma_\theta &= 2A_0-B_0r^{-2}\\
&\quad +2(3A_1r+B_1r^{-3}+D_1r^{-1})\cos\theta\\
&\quad +2(3A_1'r+B_1'r^{-3}+D_1'r^{-1})\sin\theta\\
&\quad +\sum_{n=2}^{\infty}\big[\{(n+2)(n+1)A_nr^n+n(n+1)B_nr^{-n-2}\\
&\quad +n(n-1)C_nr^{n-2}+(n-2)(n-1)D_nr^{-n}\}\cos n\theta\\
&\quad +\{(n+2)(n+1)A_n'r^n+n(n+1)B_n'r^{-n-2}\\
&\quad +n(n-1)C_n'r^{n-2}+(n-2)(n-1)D_n'r^{-n}\}\sin n\theta\big], \qquad (9b)
\end{aligned}$$

$$\begin{aligned}
\tau_{r\theta} &= H_0r^{-2}\\
&\quad +2(A_1r-B_1r^{-3}+D_1r^{-1})\sin\theta\\
&\quad -2(A_1'r-B_1'r^{-3}+D_1'r^{-1})\cos\theta\\
&\quad +\sum_{n=2}^{\infty}\big[n\{(n+1)A_nr^n-(n+1)B_nr^{-n-2}\\
&\quad +(n-1)C_nr^{n-2}-(n-1)D_nr^{-n}\}\sin n\theta\\
&\quad -n\{(n+1)A_n'r^n-(n+1)B_n'r^{-n-2}\\
&\quad +(n-1)C_n'r^{n-2}-(n-1)D_n'r^{-n}\}\cos n\theta\big]. \qquad (9c)
\end{aligned}$$

in which A_0, B_0, H_0, A_1, B_1,$\cdots\cdots$ are constants.

Let a and b the inner and the outer radius of the lining respectively, then on the inner wall surface we have

$$(\sigma_r)_{r=a}=(\tau_{r\theta})_{r=a}=0.$$

The magnitudes of $(\sigma_\theta)_{r=a}$ have been obtained by the measurement. Then, from Eqs. (7), (9a), (9b) and (9c) we obtain the following equations:

$$\begin{aligned}(\sigma_r)_{r=a}&=2A_0+B_0\,a^{-2}\\&+2\{A_1\,a-B_1\,a^{-3}+(2C_1+D_1)a^{-1}\}\cos\theta\\&+2\{A_1'\,a-B_1'\,a^{-3}+(2C_1'+D_1')a^{-1}\}\sin\theta\\&-\sum_{n=2}^{m}[\{(n-2)(n+1)A_n\,a^n+n(n+1)B_n\,a^{-n-2}\\&+n(n-1)C_n\,a^{n-2}+(n+2)(n-1)D_n a^{-n}\}\cos n\theta\\&+\{(n-2)(n+1)A_n'\,a^n+n(n+1)B_n'\,a^{-n-2}\\&+n(n-1)C_n'\,a^{n-2}+(n+2)(n-1)D_n'\,a^{-n}\}\sin n\theta]\\&=0, \qquad \cdots\cdots(10a)\end{aligned}$$

$$\begin{aligned}(\sigma_\theta)_{r=a}&=2A_0-B_0\,a^{-2}\\&+2(3A_1\,a+B_1\,a^{-3}+D_1\,a^{-1})\cos\theta\\&+2(3A_1'\,a+B_1'\,a^{-3}+D_1'\,a^{-1})\sin\theta\\&+\sum_{n=2}^{m}[\{(n+2)(n+1)A_n\,a^n+n(n+1)B_n\,a^{-n-2}\\&+n(n-1)C_n\,a^{n-2}+(n-2)(n-1)D_n\,a^{-n}\}\cos n\theta\\&+\{(n+2)(n+1)A_n'\,a^n+n(n+1)B_n'\,a^{-n-2}\\&+n(n-1)C_n'\,a^{n-2}+(n-2)(n-1)D_n'\,a^{-n}\}\sin n\theta]\\&=a_0+a_1\cos\theta+b_1\sin\theta\\&+\sum_{n=2}^{m}(a_n\cos n\theta+b_n\sin n\theta), \qquad \cdots\cdots(10b)\end{aligned}$$

$$\begin{aligned}(\tau_{r\theta})_{r=a}&=H_0\,a^{-2}\\&+2(A_1\,a-B_1\,a^{-3}+D_1\,a^{-1})\sin\theta\\&-2(A_1'\,a-B_1'\,a^{-3}+D_1'a^{-1})\cos\theta\\&+\sum_{n=2}^{m}[n\{(n+1)A_n\,a^n-(n+1)B_n\,a^{-n-2}\\&+(n-1)C_n\,a^{n-2}-(n-1)D_n\,a^{-n}\}\sin n\theta\\&-n\{(n+1)A_n'\,a^n-(n+1)B_n'\,a^{-n-2}\\&+(n-1)C_n'\,a^{n-2}-(n-1)D_n'\,a^{-n}\}\cos n\theta]\\&=0. \qquad \cdots\cdots(10c)\end{aligned}$$

Assuming that the earth pressure acts perpendicular to the outer wall of the lining, then we have

$$(\tau_{r\theta})_{r=b}=0.$$

Hence, from Eq. (9c),

$$\begin{aligned}(\tau_{r\theta})_{r=b}&=H_0\,b^{-2}\\&+2(A_1\,b-B_1\,b^{-3}+D_1\,b^{-1})\sin\theta\\&-2(A_1'\,b-B_1'\,b^{-3}+D_1'\,b^{-1})\cos\theta\end{aligned}$$

$$+\sum_{n=2}^{m}[n\{(n+1)A_n b^n-(n+1)B_n b^{-n-2}$$
$$+(n-1)C_n b^{n-2}-(n-1)D_n b^{-n}\}\sin n\theta$$
$$-n\{(n+1)A'_n b^n-(n+1)B'_n b^{-n-2}$$
$$+(n-1)C'_n b^{n-2}-(n-1)D'_n b^{-n}\}\cos n\theta]$$
$$=0. \qquad \cdots\cdots(10d)$$

From Eqs. (10a), (10b) (10c) and (10d) we obtain the following equations:

$$\left.\begin{aligned}&2A_0+B_0 a^{-2}=0,\\&2A_0-B_0 a^{-2}=a_0.\end{aligned}\right\} \qquad \cdots\cdots(11)$$

$$H_0=0 \qquad \cdots\cdots(12)$$

$$\left.\begin{aligned}&A_1 a-B_1 a^{-3}+(2C_1+D_1)a^{-1}=0,\\&2(3A_1 a+B_1 a^{-3}+D_1 a^{-1})=a_1,\\&A_1 a-B_1 a^{-3}+D_1 a^{-1}=0,\\&A_1 b-B_1 b^{-3}+D_1 b^{-1}=0.\end{aligned}\right\} \qquad \cdots\cdots(13)$$

$$\left.\begin{aligned}&A'_1 a-B'_1 a^{-3}+(2C'_1+D'_1)a^{-1}=0,\\&2(3A'_1 a+B'_1 a^{-3}+D'_1 a^{-1})=b_1,\\&A'_1 a-B'_1 a^{-3}+D'_1 a^{-1}=0,\\&A'_1 b-B'_1 b^{-3}+D'_1 b^{-1}=0.\end{aligned}\right\} \qquad \cdots\cdots(14)$$

$$\left.\begin{aligned}&(n-2)(n+1)A_n a^n+n(n+1)B_n a^{-n-2}\\&\quad+n(n-1)C_n a^{n-2}+(n+2)(n-1)D_n a^{-n}=0,\\&(n+2)(n+1)A_n a^n+n(n+1)B_n a^{-n-2}\\&\quad+n(n-1)C_n a^{n-2}+(n-2)(n-1)D_n a^{-n}=a_m,\\&(n+1)A_n a^n-(n+1)B_n a^{-n-2}+(n-1)C_n a^{n-2}\\&\quad-(n-1)D_n a^{-n}=0,\\&(n+1)A_n b^n-(n+1)B_n b^{-n-2}+(n-1)C_n b^{n-2}\\&\quad-(n-1)D_n b^{-n}=0.\end{aligned}\right\} \qquad \cdots\cdots(15)$$

$$\left.\begin{aligned}&(n-2)(n+1)A'_n a^n+n(n+1)B'_n a^{-n-2}\\&\quad+n(n-1)C'_n a^{n-2}+(n+2)(n-1)D'_n a^{-n}=0,\\&(n+2)(n+1)A'_n a^n+n(n+1)B'_n a^{-n-2}\\&\quad+n(n-1)C'_n a^{n-2}+(n-2)(n-1)D'_n a^{-n}=b_m,\\&(n+1)A'_n a^n-(n+1)B'_n a^{-n-2}+(n-1)C'_n a^{n-2}\\&\quad-(n-1)D'_n a^{-n}=0,\\&(n+1)A'_n b^n-(n+1)B'_n b^{-n-2}+(n-1)C'_n b^{n-2}\\&\quad-(n-1)D'_n b^{-n}=0.\end{aligned}\right\} \qquad \cdots\cdots(16)$$

Solving each simultaneous equations (11), (13), (14), (15) and (16), each constant is determined as follows:

$$\left.\begin{aligned}
&A_0=\frac{1}{4}a_0, && B_0=-\frac{1}{2}a^2 a_0,\\
&A_1=\frac{a_1}{4a(k^2-1)}, && B_1=\frac{k^2 a^3 a_1}{4(k^2-1)},\\
&C_1=0, && D_1=\frac{(k^2+1)aa_1}{4(k^2-1)},\\
&A_1'=-\frac{b_1}{4a(k^2-1)}, && B_1'=\frac{k^2 a^3 b_1}{4(k^2-1)},\\
&C_1'=0, && D_1'=\frac{(k^2+1)ab_1}{4(k^2-1)},\\
&A_n=\frac{a_n}{4n(n+1)a^n\alpha}(-k^{2n}-nk^2+n+1), &&\\
&B_n=\frac{a^{n+2}a_n}{4n(n+1)\alpha}\{(n+1)k^{2n+2}-nk^{2n}-k^2\}, &&\\
&C_n=\frac{a_n}{4n(n-1)a^{n-2}\alpha}\{k^{2n+2}+(n-1)k^2-n\}, &&\\
&D_n=\frac{a^n a_n}{4n(n-1)\alpha}\{-nk^{2n+2}+(n-1)k^{2n}+1\}, &&\\
&A_n'=\frac{b_n}{4n(n+1)a^n\alpha}(-k^{2n}-nk^2+n+1), &&\\
&B_n'=\frac{a^{n+2}b_n}{4n(n+1)\alpha}\{(n+1)k^{2n+2}-nk^{2n}-k^2\}, &&\\
&C_n'=\frac{b_n}{4n(n-1)a^{n-2}\alpha}\{k^{2n+2}+(n-1)k^2-n\}, &&\\
&D_n'=\frac{a^n b_n}{4n(n-1)\alpha}\{-nk^{2n+2}+(n-1)k^{2n}+1\}. &&
\end{aligned}\right\}\quad\cdots\cdots(17)$$

in which

$$\left.\begin{aligned}
&k=b/a,\\
&\alpha=k^{2n+2}-k^{2n}-k^2+1.
\end{aligned}\right\}\quad\cdots\cdots(18)$$

Simultaneous equations (13), (14), (15) and (16) were solved utilizing elimination by matrix grouping in order to perform the operation rapidly.

Since the earth pressure acting on the outer wall of the lining is equal to $(\sigma_r)_{r=b}$, we have the following equations from Eq. (9a):

$$\begin{aligned}
(\sigma_r)_{r=b}=\,&2A_0+B_0b^{-2}\\
&+2\{A_1 b-B_1 b^{-3}+(2C_1+D_1)b^{-1}\}\cos\theta\\
&+2\{A_1' b-B_1' b^{-3}+(2C_1'+D_1')b^{-1}\}\sin\theta\\
&-\sum_{n=2}^{n}[\{(n-2)(n+1)A_n b^n+n(n+1)B_n b^{-n-2}\\
&+n(n-1)C_n b^{n-2}+(n+2)(n-1)D_n b^{-n}\}\cos n\theta\\
&+\{(n-2)(n+1)A_n' b^n+n(n+1)B_n' b^{-n-2}
\end{aligned}$$

$$+n(n-1)C_n' b^{n-2}+(n+2)(n-1)D_n' b^{-n}\}\sin n\theta] \quad \cdots\cdots (19)$$

Substituting the constants determined by Eqs. (17) into Eq. (19), $(\sigma_r)_{r=b}$, i.e., the earth pressure which acts on the outer wall is obtained.

The above mentioned is the estimation of earth pressure by the results of measurement of the stresses which arise in photoelastic stressmeters set up on the inner wall of a concrete lining without reinforcement. In case that the lining is made of reinforced concrete, the earth pressure acting on the outer wall can be obtained by treating the results of measurement of the stresses produced in the steel bar covering the whole circumference, which were obtained using any instruments for measuring the stresses in the steel bar (e.g., Carlson type steel bar stressmeter) by some similar mathematical treatment as mentioned above.

4. Estimation of the earth pressure by the nature and the conditions of a ground around a shaft.

As stated in **2,** various formulae for the purpose of calculating the earth pressure which acts on a shaft lining have hitherto been proposed. Many of these formulae, however, have some faults when we investigate these from mechnical point of view. The author described in **3,** a method to determine the earth pressure by the results of measurement of the stresses which are produced on the inner wall surface of a lining. If many data were obtained by such stress measurement, those data treated statistically would be very helpful for a reasonable estimation of the earth pressure which acts on a shaft lining. Whereas few data concerning such stress measurement have hitherto been obtained, however, it seems to be difficult to find the general relationship between the nature and conditions of the ground and the earth pressure which acts on a lining.

In the previous article[8] the earth pressure to be assumed for planning a lining was devided into uniform part and non-uniform part, and as the non-uniform part concentrated earth pressure which acts on two narrow ranges on the both ends of a diameter as well as an elliptically distributing earth pressure were considered. Denoting then the intensity of the uniform part by p_0, and mean value of the intensity of the non-uniform part by $\bar{p}$, calculations for the purpose of planning a lining were performed using magnitudes of p_0 and $\bar{p}$. Hereupon, in the following the author expressed his opinion on the magnitudes of p_0 and $\bar{p}$ to be taken, when the nature and conditions of the ground surrounding a lining are known. In case the ground around a shaft is composed of comparatively homogeneous earth, sand or clay, the earth pressure acting on a lining will be relatively heavy and its grade of non-uniformity will be small. That is to say, it is supposed that the magnitude of p_0 will be relatively large and $\bar{p}/p_0$ will be small in most cases. For the magnitude of p_0, let us adopt the value obtained by Rankine's earth pressure theory. Namely, denoting the depth from the surface, specific weight of the ground and angle of internal friction by H, γ and ϕ respectively, let us determine the magnitude of p_0 by the following formula.

$$p_0=\gamma H \tan^2\left(\frac{\pi}{4}-\frac{\phi}{2}\right)$$

The author proposes to adopt this formula only for the cases in which the ground is composed of earth, sand or clay, though it has hitherto been proposed to determine the magnitude of the earth pressure using this formula even when the ground is composed of rocks.

Since the magnitude of ϕ for sand, pebble and clay generally exists within a range between 20° and 40°, magnitudes of p_0 obtained by above mentioned formula lie between $1/2$ and $1/5$ of the magnitude of γH, i.e., the maximum value of p_0 is nearly equal to the hydrostatic pressure corresponding to the depth from the surface, and the minimum value of p_0 is nearly equal to $1/2.5$ of this pressure. On the contrary, in case that a shaft is sunk in a ground which consists of rocks, which is not so much disintegrated, the magnitude of p_0 will be generally small and the value of $\bar{p}/p_0$ will in some cases be large. If the compressive strength S_C of rocks around a shaft is much larger than the maximum compressive stress $(\sigma_C)_{max}$ which is produced on the wall surface of rocks surrounding the shaft, both p_0 and $\bar{p}$ might be nearly equal to zero. As the ratio of $(\sigma_C)_{max}$ to S_C increases the magnitudes of p_0 and $\bar{p}$ will also increase. Taking the results of measurement of stresses which arise in the shaft lining[9] into consideration, the author suggests that in order to obtain a roughly safe estimation of the earth pressure acting on a lining which is sunk in the Neocene strata, the intensity of the uniform earth pressure p_0 might be taken as zero, and value of $\bar{p}$ might be determined within a range between zero and $1 \mathrm{kg/cm^2}$.

If some yielding materials are inserted between the lining and the ground, the grade of non-uniformity will be small as compared with the case in which the earth pressure acts directly on a rigid lining. That is, in case of yielding construction, the magnitude of p_0 and $\bar{p}$ tends to decrease and increase respectively.

5. Summary

In order to estimate the earth pressure which acts on a shaft lining, various formulas have hitherto been proposed. Each of all those folmulas has been derived under some comparatively simple assumptions. Although some of these formulas seem to be fairly pertinent for a ground, such as homogeneous Quarternary strata, it seems to be unreasonable, in many cases, to estimate the earth pressure by these formulas, while the nature and the conditions of a ground are fairly complicated in general.

In this article, therefore, a method for estimating the earth pressure which acts on the outer wall of a shaft lining, using the results of measurement of stresses which arise on the inner wall surface of the lining. In case that lots of data concerning the relationship between the nature and conditions of the ground surrounding the shaft and the earth pressure which acts on the lining are collected in the future, by carrying out numerous stress measurements, it will be most reasonable to estimate the earth pressure with reference to these data.

Since sufficient data of such kind are not available under present conditions, however, it will be necessary to estimate the earth pressure by giving consideration to the nature, conditions and other factors refering to the description in **3** of this article.

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