Dispersion Relations of Ion Acoustic Waves in Hydrogen Plasmas

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Abstract

The characteristics of dispersion of ion acoustic waves in hydrogen plasmas (a multi–ion plasma) are investigated theoretically. The dispersion relations, especially the phase velocity, are significantly affected by the addition of H^- ions. It would demonstrate the usefulness of measuring the dispersion relation of the ion acoustic wave for monitoring the ion species ratio.

1. Introduction

A multi–ion component plasma exhibits some phenomena which do not occur in a single ion plasma. For example, they are the changes in wave damping of ion acoustic wave in a two–ion plasma,[1,2] ion–ion hybrid resonance, [3] observation of two ion acoustic waves,[4] radio frequency control of the impurity ion transport based on the electrostatic ion cyclotron wave,[5] and so on. Recently, use of the two–ion hybrid as an impurity diagnostic has also been proposed.[6]

For ion acoustic waves, Fried et al.[7] previously studied theoretically the properties of these waves in a multi–ion plasma ($A_r^+ - H_e^+$ plasma). They found that there are two important modes with quite different phase velocities, i. e. the principal heavy ion mode and the principal light ion mode. Which of these is dominant depends on the relative densities of the two components. However, introduction of a small amount of light ions significantly affects the acoustic wave properties of a plasma. Therefore, if the dispersion relation in a multi–ion component plasma can be established, it would be possible to monitor the ion composition ratio by measuring the dispersion relation of the ion acoustic wave.

By the way, recently, hydrogen discharge plasma has been increasingly interested as the application of negative ion source for neutral beam injector, i. e. additional heating of fusion plasma. Because, in the future reactor, the required beam energy is so high (150–200 keV) that the neutralization efficiency for positive ions becomes too low (10%) to realize an economical injector. The use of negative ions is one possible mean of producing highly energetic neutral beams more efficiently. Namely, the neutralization efficiency for negative ions with the use of neutral gas cell remains over 60 % for all energies of interest. Now, optimization of H^- production and extraction of H^- current are studied extensively in the fusion research laboratories.[8]

Usually, hydrogen plasma produced by discharge is composed of multi–ion species, i. e. three positive ions (atomic hydrogen ion H^+, molecular ions H_2^+ and H_3^+) and the negative ion H^-.[9] Therefore, to study the plasma parameter dependence of H^- production, we must monitor the ion species ratios during the experiment. In this paper, we study theoretically

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the characteristics of dispersion of ion acoustic waves in hydrogen plasmas, and discuss the potential of propagation of ion acoustic waves for measuring the ion species ratios.

2. Theoretical Model and Basic Equations

2.1 Linear dispersion equation in a multi—ion plasma

The dielectric constant for a longitudinal wave in a multi species plasma is given by (in one dimensional case)[10)]

$$\varepsilon(k, \omega) = 1 - \sum_j \frac{\omega_{pj}^2}{k^2} \int \frac{k \cdot (\partial F_j^0/\partial v)}{k \cdot v - \omega} dv. \quad (1)$$

Here, plasma frequency $\omega_{pj} = (4\pi n_{oj} e^2/m_j)^{1/2}$, n_{oj} and m_j are the particle density and mass of the j—species, k the wave number and F_j^0 (v) the unperturbed velocity distribution function of the j—species particles.

We treat dispersion relation of ion acoustic waves mainly in hydrogen plasmas where there are four ion species, i. e. H^+, H_2^+, H_3^+ and H^-. Taking Maxwell distributions as the unperturbed distribution functions and assuming the plasma consists of four species ions and electrons, the distribution functions are expressed as follows :

$$F_e^0(v) = (\pi^{1/2} a_e)^{-1} \exp[-(v/a_e)^2],$$
$$F_1^0(v) = (\pi^{1/2} a_1)^{-1} \exp[-(v/a_1)^2],$$
$$F_2^0(v) = (\pi^{1/2} a_2)^{-1} \exp[-(v/a_2)^2],$$
$$F_3^0(v) = (\pi^{1/2} a_3)^{-1} \exp[-(v/a_3)^2],$$
$$F_4^0(v) = (\pi^{1/2} a_4)^{-1} \exp[-(v/a_4)^2], \quad (2)$$

Here, the suffix e denotes electrons, 1 H^+ ions, 2 H_2^+ ions, 3 H_3^+ ions and 4 H^- ions. Symbols a_j are the thermal velocity expressed by $a_e = (2T_e/m_e)^{1/2}$ and $a_j = (2T_j/M_j)^{1/2}$, j = 1,2,3 and 4. Setting ε (k, ω) = 0, the dispersion relation is obtained,

$$2\frac{k^2}{k_{De}^2} - \frac{n_1}{n_e}\frac{T_e}{T_1} Z'(\zeta_1) - \frac{n_2}{n_e}\frac{T_e}{T_2} Z'(\zeta_2) - \frac{n_3}{n_e}\frac{T_e}{T_3} Z'(\zeta_3)$$
$$- \frac{n_4}{n_e}\frac{T_e}{T_4} Z'(\zeta_4) - Z'(\zeta_e) = 0, \quad (3)$$

where

$$\zeta_1 = \omega/(ka_1),\ \zeta_2 = \zeta_1 \left(\frac{M_2}{M_1}\frac{T_1}{T_2}\right)^{1/2},\ \zeta_3 = \zeta_1 \left(\frac{M_3}{M_1}\frac{T_1}{T_3}\right)^{1/2}$$

$$\zeta_4 = \zeta_1 \left(\frac{T_1}{T_4}\right)^{1/2},\ \zeta_e = \zeta_1 \left(\frac{m_e}{M_1}\frac{T_1}{T_e}\right)^{1/2},\ k_{De} = (4\pi n_e e^2/T_e)^{1/2},$$

Z is the plasma dispersion function and Z′ is its first derivative. The charge neutrality is assumed, i. e. $n_1 + n_2 + n_3 = n_4 + n_e$.

2.2 General features of ion acoustic waves

In section 3, we discuss the numerical results obtained by solving eq. (3), i. e. the dispersion relation of the ion wave in a multi–ion species plasma. Here, we present the general feature of the dispersion relation of the ion waves. To this end, we consider the single ion plasma, i. e. H^+ ions. In this case, the dispersion equation reduces to eq. (4),

$$2\frac{k^2}{k^2_{De}} - \frac{T_e}{T_1} Z'(\zeta_1) - Z'(\zeta_e) = 0, \tag{4}$$

where $n_e = n_1$ and $n_2 = n_3 = n_4 = 0$.

For large T_e/T_1 (in a low–pressure hydrogen discharge plasma, $T_e/T_i \gg 1$), we expect the phase velocity of the ion acoustic wave ω/k to be large compared with the ion thermal velocity a_1, i. e. $\omega/(ka_1) \gg 1$. On the other hand, $\omega/(ka_e) \ll 1$. So, the asymptotic expansion is used for the ion Z′ function and the power expansion is used for the electron Z′ function. For simplicity, we assume that

$$Z'(\zeta_e) \simeq -2 \quad \text{and} \quad Z'(\zeta_1) \simeq \zeta_1^{-2}. \tag{5}$$

After some algebra, we obtain the following expression from eq. (4),

$$\omega^2 = \frac{k^2 \cdot k^2_{De}}{k^2 + k^2_{De}} C_s^2 \tag{6}$$

where C_s is the ion sound velocity expressed by $C_s = (T_e/M_1)^{1/2}$.

When $k^2 \ll k^2_{De}$, i. e. in the case of long wavelength limit,

$$\omega^2 = C_s^2 k^2 \quad \text{or} \quad \omega = (T_e/M_1)^{1/2} k. \tag{7}$$

So, in this case, wave phase velocity becomes C_s (ion acoustic wave region). On the other hand, when $k^2 \gg k^2_{De}$

$$\omega^2 \simeq \omega^2_{p1} \quad \text{or} \quad \omega \simeq (4\pi e^2 n_1/M_1)^{1/2}. \tag{8}$$

This short wavelength region is an ion plasma wave region. Thus, whereas in the former case, ω depends on T_e and k, in the latter it depends on n_1. In both cases, however, ω is proportional to $M_1^{1/2}$. We conclude that for the lightest and heaviest atomic ions ω differs by more than one order of magnitude.

2.3 Calculation procedure

To calculate the dispersion relation $\omega = \omega(k)$ from the dispersion equation eq. (3), we use the complex Newton method. The Newton method is a highly efficient and highly accurate iteration method to solve the transcendental equation such as eq. (3), provided that we can find a good approximate solution. For this purpose, the contour lines of $|\varepsilon(k, \omega)|$

= const. are drawn in the complex ζ_1 − plane with fixed wave number k, i. e. ω : complex and k : real.

Figure 1 shows a typical example of contour maps. The horizontal and vertical axes are the real and imaginary parts of ζ_1. There exists a solution of eq. (3) within each concentric circle. Therefore, we can treat these values of ζ_1 as an initial approximate solution.

Although there are many solutions, we do not need to obtain all the solutions. As the imaginary part of ζ_1 gives the damping rate, for simplicity the modes near to the real axis are important; the linear dispersion equation is obtained with the assumption that all first-order quantities vary as $\exp[i(kx-\omega t)]$. More precisely, the angle between the real axis and the line connecting the origin to the mode in question is physically important. Because, the damping per cycle is given by $\gamma/\omega_r = -\mathrm{Im}\,\zeta_1/\mathrm{Re}\,\zeta_1$.

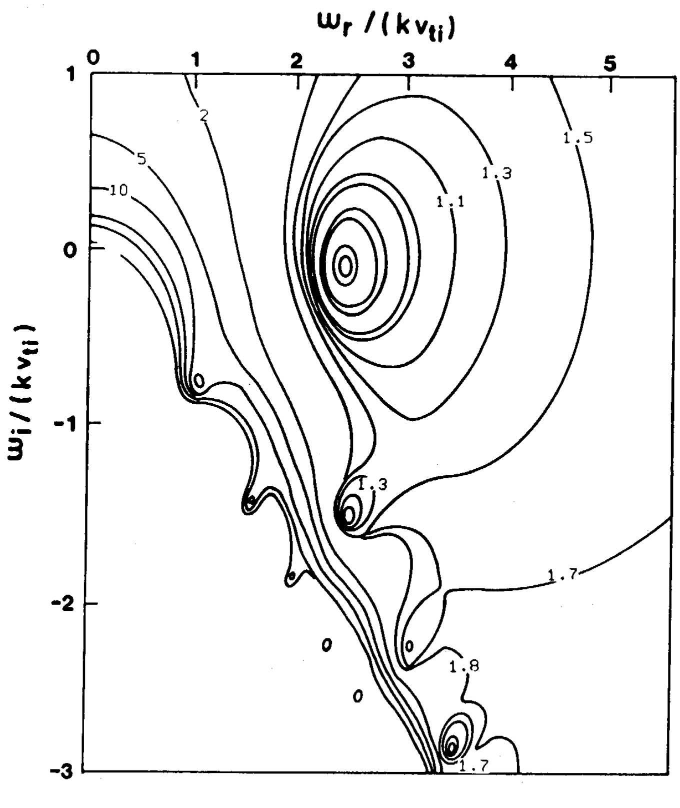

Fig. 1 The contour map of $|\varepsilon(k, \omega)|$ = const. in the complex ζ_1 plane, where wave number is fixed at k/k_{De} = 0.02. Hydrogen plasma is composed of three species of ions, i. e. H^+,H_3^+ and H^-. Plasma remains quasineutral, i. e. $n_e+n_4=n_1+n_3=n_i$. Numerical conditions are as follows : $T_1=T_3=T_4=T_i$, $T_e/T_i=10$, positive ion species ratio is $H^+ : H_2^+ : H_3^+ = 30 : 0 : 70$ and negative particles ratio is $H^- : e = 20 : 80$.

3. Numerical Results and Discussion

3.1 Dispersion relation of ion waves in a single ion plasma

First of all, we show the general features of dispersion relation of ion waves. To this end, we solve the type of eq. (4). That is the dispersion equation in a one–ion plasma.

Figure 2 shows typical examples of dispersion relation for various ion plasmas under the same temperature ratio $T_e/T_i = 10$. We assume that k is real and ω is complex, and normalize k to k_{De} and ω to ω_{pi} the ion plasma frequency for H^+ ions. As ω depends on $M^{-1/2}$ (see eqs. (7) and (8) in section 2.2), ω_r decreases with increasing ion mass.

When resonant particles whose velocities are equal or nearly equal to the phase velocity of the wave are present, wave amplitude is reduced in a collisionless plasma by Landau damping. Damping rate ω_i/ω_{pi} shown in Fig. 2 (b) is due to this damping.

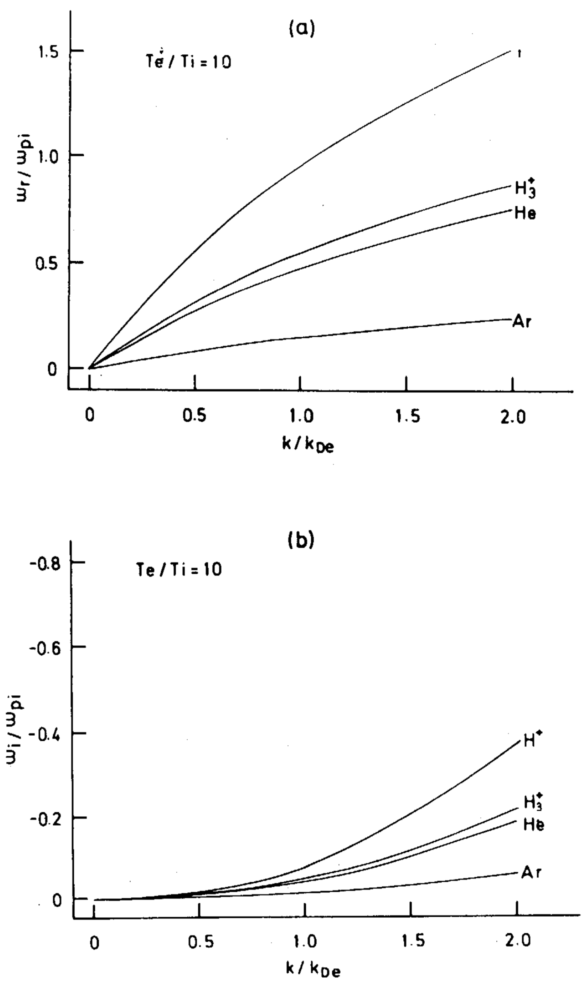

Fig. 2 Dispersion relation for various ions. They are obtained by solving the dispersion equation in one–ion plasma : (a) Real part of wave frequency Re (ω / ω_{pi}) vs wave number k/k_{De}. (b) Imaginary part of wave frequency Im (ω / ω_{pi}) vs k/k_{De}.

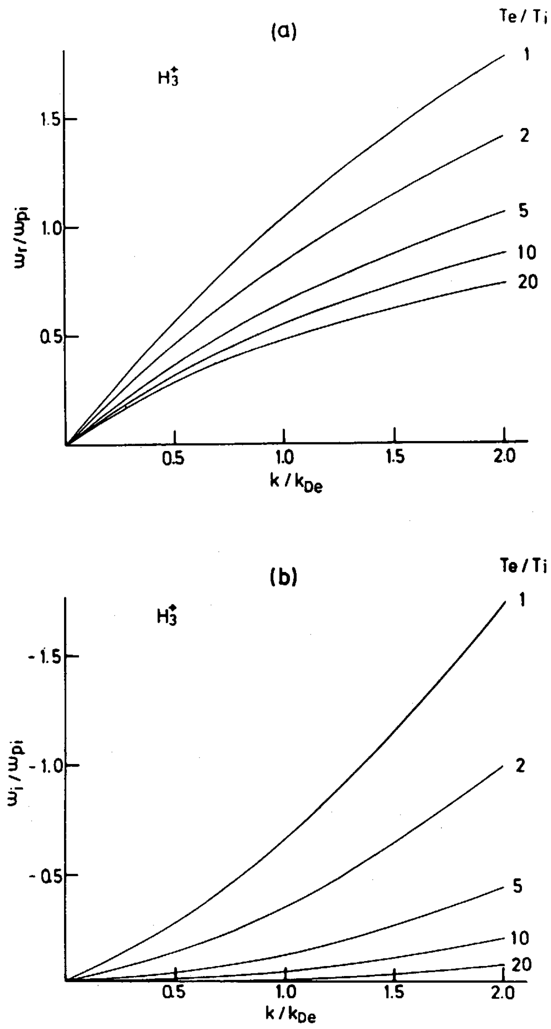

Fig. 3 Dependence of dispersion relation on electron temperature. Plasma is composed of one ion H_3^+. Parameter is a ratio of electron temperature T_e to ion temperature T_i : (a) Re (ω / ω_{pi}) vs k/k_{De}. (b) Im (ω / ω_{pi}) vs k/k_{De}.

Figure 3 shows the effect of electron temperature on the dispersion characteristics. With increasing T_e/T_i, damping rate decreases remarkably. If a wave has a slow enough phase velocity to match the thermal velocity of ions, ion Landau damping can occur. When $T_e \leqq T_i$, the phase velocity lies in the region where $F_i^0(v)$ has a negative slope. Consequently, ion waves are heavily Landau–damped. Therefore, ion acoustic waves are observable when $T_e \gg T_i$ (see Fig. 3 (b)). Namely, the phase velocity lies far in the tail of the ion velocity distribution.

When electrons are shifted relative to ions (current carrying plasma), ion acoustic waves can be excited spontaneously. Namely, the phase velocity lies in the region where $F_e^0(v)$ has a positive slope. So, ion waves are inverse Landau–damped by the electrons and Landau damped by the ions. If inverse Landau damping overcomes, ion waves are excited spontaneously. Figure 4 shows this situation. With increasing electron drift velocity v_d/v_{te}, ω_i becomes positive for a certain region of k.

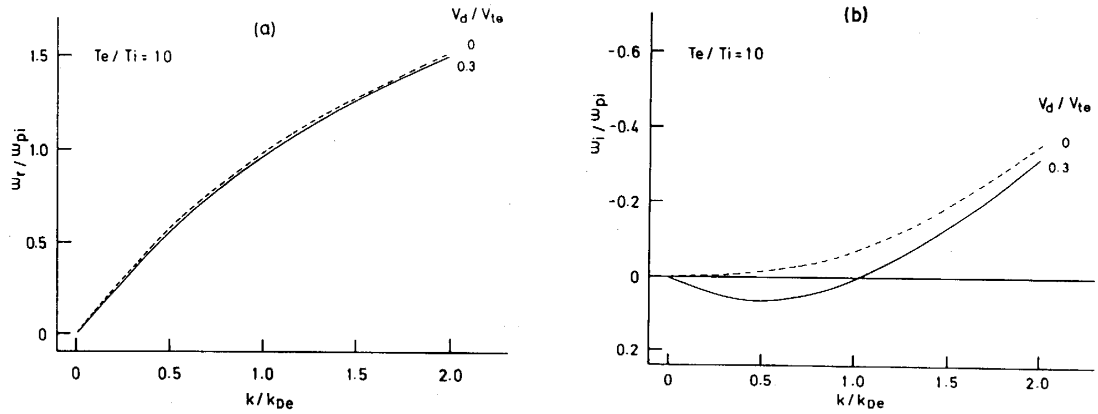

Fig. 4 Effect of electron drift velocity on excitation of ion acoustic waves. Plasma is composed of one ion H_e^+. Temperature ratio is kept constant, $T_e/T_i=10$. With increasing drift velocity (for example, $V_d/V_{te}=0.3$: solid line), ω_i becomes positive in a certain region of k/k_{de}, i. e. excitation of the ion acoustic wave. (a) Re (ω/ω_{pi}) vs k/k_{De}, and (b) Im (ω/ω_{pi}) vs k/k_{De}.

3.2 Dispersion relation for ion waves in a multi–ion plasma

The most interesting physics is associated with the case where a small fraction of light ions is added to a plasma of heavier ions.[7] Here, for example, we consider two–ion species plasma, $A_r^+ - H_e^+$. Numerical calculations are performed by using eq. (3), where $n_1 = n_{He}$, $n_2 = n_{Ar}$, $n_3 = n_4 = 0$, $M_1 = M_{He}$ and $M_2 = M_{Ar}$. We have set $T_1 = T_2 = T_i$. Of course, we assume that the plasma remains quasineutral, i. e. $n_e = n_{He} + n_{Ar} = n_i$.

Firstly, results will be discussed in the complex ζ_1 plane, the real part giving the phase velocity and the imaginary part the damping rate. Figure 5 shows the effect of the addition of H_e ions to A_r plasma. As the light ion density is increased, there is a deformation of the first heavy ion mode (the principal heavy ion mode) downward in the ζ_1 plane. Namely, the damping per cycle γ/ω_r becomes large with increasing n_{He}. On the other hand, the light ion mode manifests itself as an extension of the third heavy ion mode.

According to these results, we obtain the dispersion relation for both the principal light

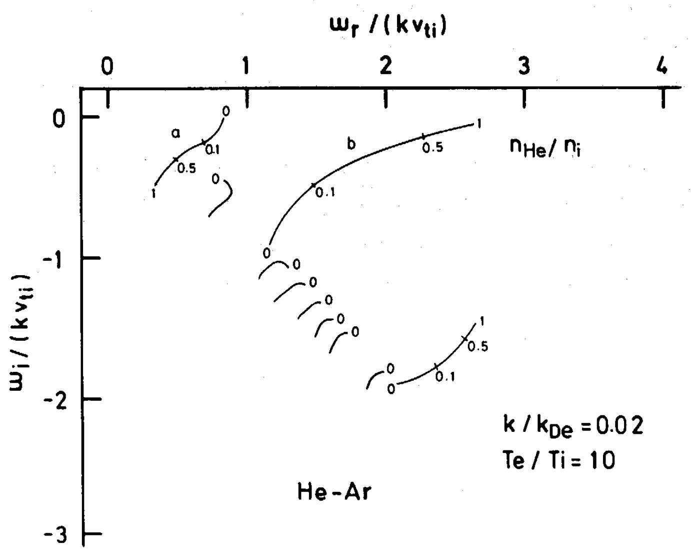

Fig. 5 Behaviors of solutions for the dispersion equation in $H_e^+ - A_r^+$ plasma, *a*: principal heavy ion mode and *b*: principal light ion mode. Parameter is the ratio of number density of ions, $n_{He}/(n_{Ar} + n_{He}) = n_{He}/n_i$. Temperature ratio and wave number are fixed.

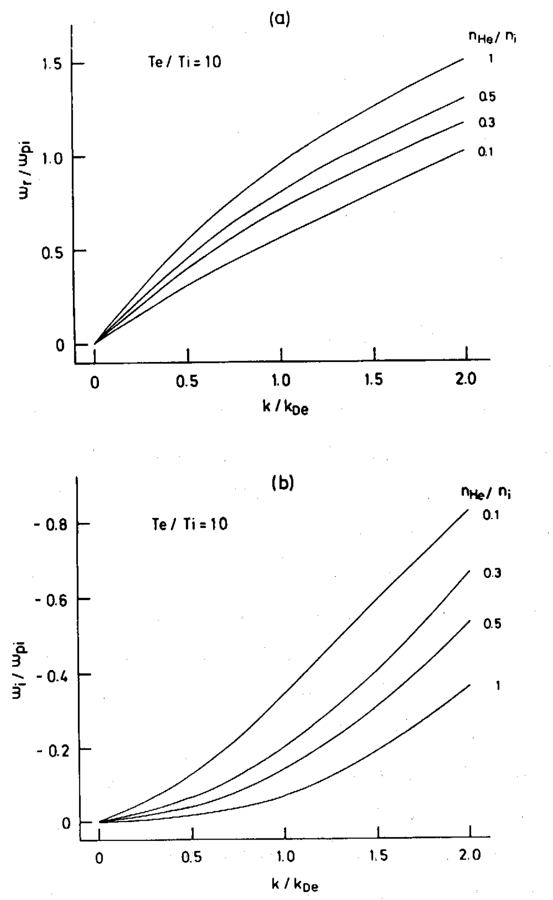

Fig. 6 Dispersion relation of principal light ion mode in $H_e^+ - A_r^+$ plasma. Numerical conditions are the same as in Fig. 5.

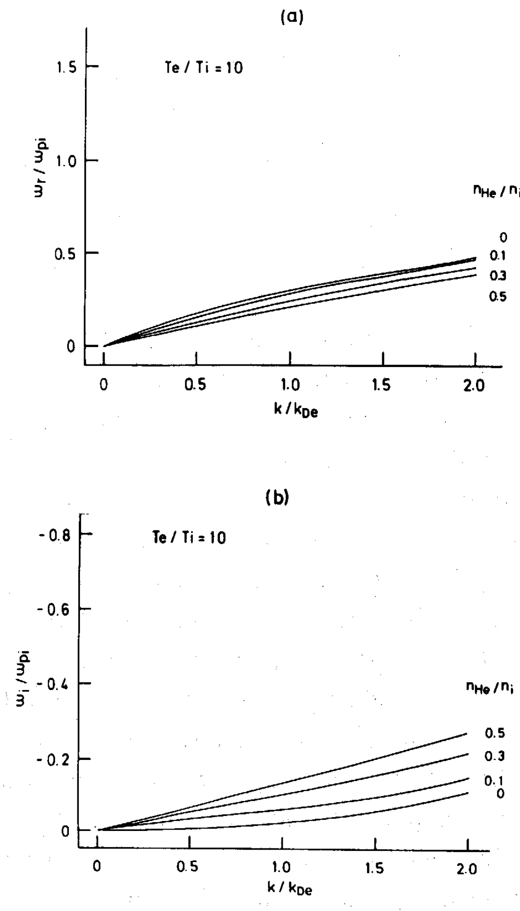

Fig. 7 Dispersion relation of principal heavy ion mode in $H_e^+ - A_r^+$ plasma. Numerical conditions are the same as in Fig. 5.

and heavy ion modes. They are shown in Figs. 6 and 7. With increasing light ions (H_e^+), wave frequency ω_r/ω_{pi} of the light ion mode increases and the damping rate decreases remarkably (see Fig. 6). On the other hand, for the heavy ion mode, damping is increased (see Fig. 7).

The damping per cycle of the two modes for two different composition ratios are shown in Fig. 8. When the light ion density is low ($n_{He}/n_i \simeq 0.1$), the damping per cycle of the two modes is comparable for small k. Under this condition, two ion acoustic waves can be observed.[4] As the light ion density is increased more ($n_{He}/n_i \gtrsim 0.5$), the principal heavy ion

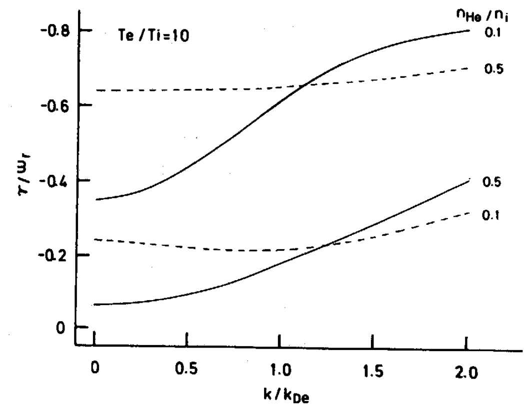

Fig. 8 Damping per cycle vs k/k_{De}. Solid and dashed lines are for principal light ion mode and principal heavy ion mode, respectively. Parameter is a density ratio, n_{He}/n_i.

mode becomes more strongly damped and the light ion mode extends up toward the real axis (see Fig. 5), becoming the most important mode in the plasma.

3.3 Propagation of the ion acoustic wave in hydrogen plasmas

(a) In the absence of H^- ions (Case I)

As is shown in the previous section, introduction of a small amount of light ions significantly affects the acoustic wave properties of a plasma. So, we discuss whether propagation of the ion aconstic wave can be used as sensitive diagnostics for the measurements of the ion species ratios in hydrogen plasmas.

First of all, we calculate the dispersion relation in a two–ion hydrogen plasma ($H^+ - H_3^+$). Figure 9 shows the behavior of the solutions of the dispersion equation (eq. (3)) in the complex ζ_1 plane, where we set $T_1 = T_3 = T_i$ and $n_2 = n_4 = 0$. With the addition of H^+ ions, light ion modes whose phase velocities increase with H^+ ions manifest itself as an extension of the first and the fifth heavy ion modes. In the $H^+ - H_3^+$ system ($M_{H3}/M_H = 3$), without reference to light ion concentration, only the principal light ion mode becomes the most important mode. This feature is quite different from that in the $H_e^+ - A_r^+$ system (see Fig. 5 where $M_{Ar}/M_{He} = 10$). In other words, the damping per cycle of the other modes are much larger than that of the principal light ion mode. Concerning this point, typical examples are shown in Fig. 10.

The change of dispersion relation for two modes (the principal light ion mode and the principal heavy ion mode) are also shown in Figs. 11 and 12. Parameter is the density ratio, i. e. the number density of H^+ions to total ion density n_i.

The heavy ion mode is significantly affected by the addition of the light ion species with

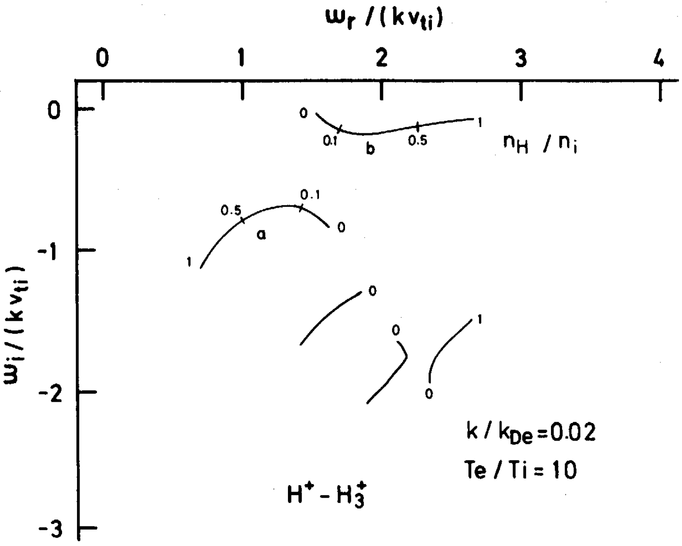

Fig. 9 Behaviors of solutions for the dispersion equation in $H^+-H_3^+$ plasma, *a*: principal heavy ion mode and *b*: principal light ion mode. Parameter is the ratio of number density of ions, $n_H/(n_H+n_{H3})=n_H/n_i$. Temperature ratio and wave number are fixed.

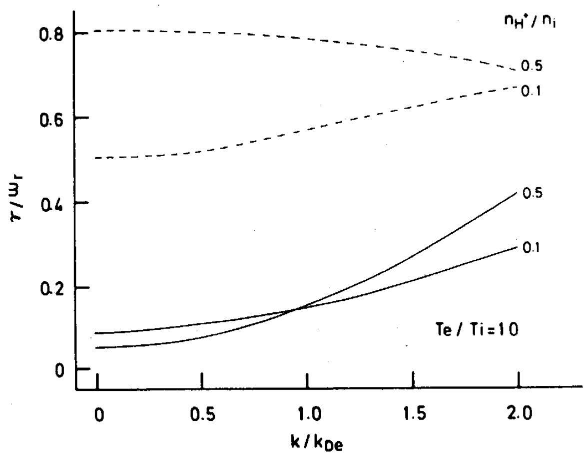

Fig. 10 Damping per cycle vs k/k_{De}. Solid and dashed lines are for principal light ion mode and principal heavy ion mode, respectively. Parameter is a density ratio, n_H/n_i.

large mass ratio. So, the addition of H_2^+ ions to $H^+-H_3^+$ plasma does not change the dispersion relation remarkably.

Figure 13 shows the dispersion relation of the light ion mode. We set $T_1=T_2=T_3=T_i$, and assume that the plasma remains quasineutral, i. e. $n_e=n_1+n_2+n_3=n_i$, where n_3/n_i is kept constant at 0.8. When $n_2/n_i=0$, the curve corresponds to the dispersion curve with $n_H/n_i=0.2$ in Fig. 11.

Figure 14 shows another example of the dispersion relation of the light ion mode. In this case, $n_e=n_1+n_2+n_3=n_i$ and n_1/n_i is kept constant at 0.8. The dispersion curve changes little although the ratio of molecular ions is varied.

These results shown in Figs. 13 and 14 reveal that we cannot determine the positive ions species ratio by the method of propagation of the ion acoustic wave.

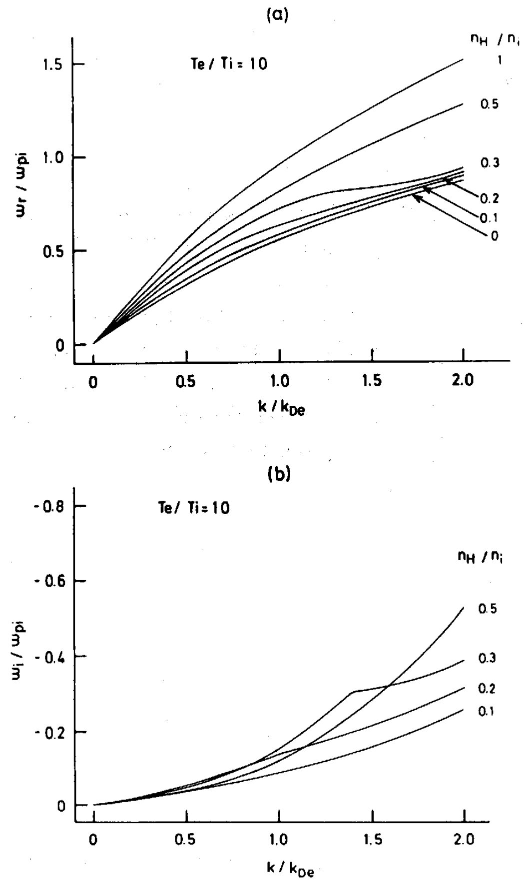

Fig. 11 Dispersion relation of principal light ion mode in $H^+-H_3^+$ plasma. Numerical conditions are the same as in Fig. 9.

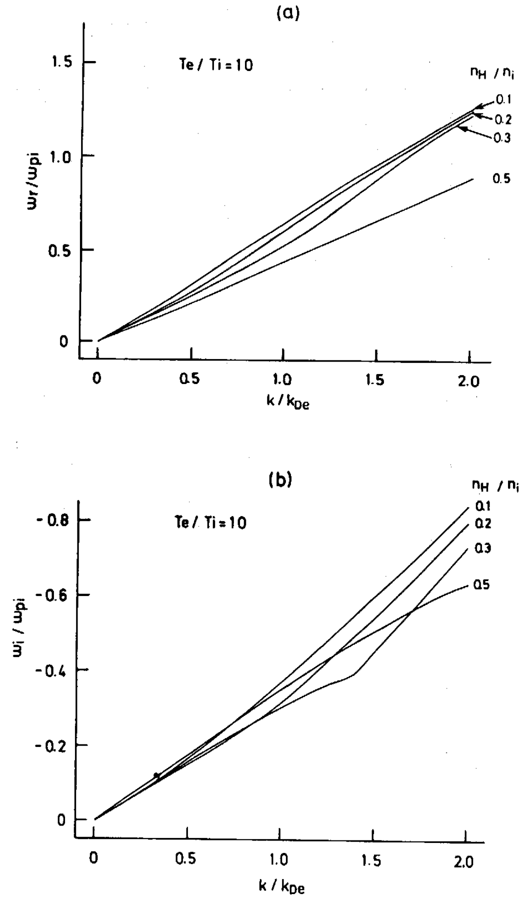

Fig. 12 Dispersion relation of principal heavy ion mode in $H^+-H_3^+$ plasma. Numerical conditions are the same as in Fig. 9.

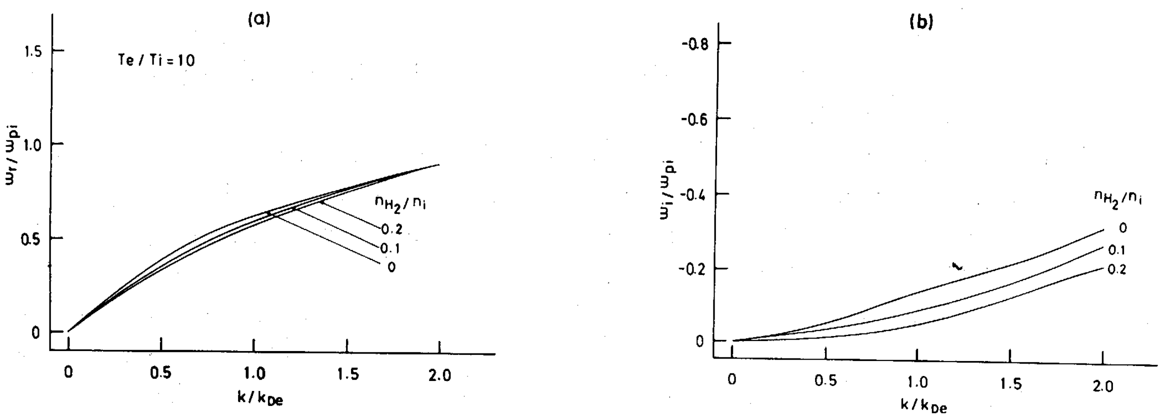

Fig. 13 Dispersion relation of light ion mode in $H^+-H_2^+-H_3^+$ plasma. Here, ratio H_3^+/n_i is kept constant at 80%. Then, ratio $(H^+ + H_2^+)/n_i$ is also kept constant at 20%. Parameter is the ratio of H_2^+, n_{H2}/n_i.

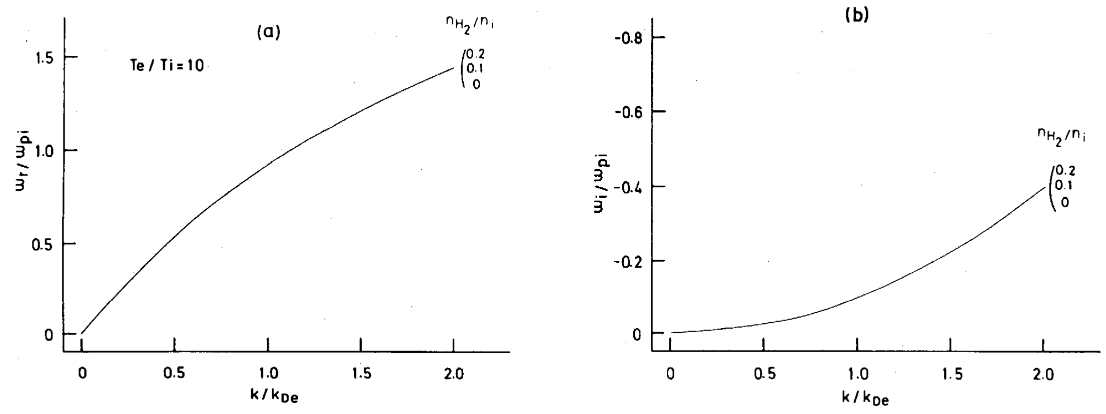

Fig. 14 Dispersion relation of light ion mode in $H^+-H_2^+-H_3^+$ plasma. Here, ratio H^+/n_i is kept constant at 80%. Then, ratio $(H_2^++H_3^+)/n_i$ is also kept constant at 20%. Parameter is the ratio of H_2^+, n_{H2}/n_i.

(b) In the presence of H^- ions (Case II)

Effect of the presence of H^- ions on propagation of the ion acoustic wave is shown in Fig. 15, where $T_1 = T_3 = T_4 = T_i$ and $n_2 = 0$. Plasma remains quasineutral, i. e. $n_e + n_4 = n_1 + n_3 = n_i$. Positive ion species ratio is kept constant, i. e. $n_1/n_i = 0.1$ and $n_3/n_i = 0.9$. With the addition of H^- ions, the behavior of the modes in complex ζ_1 plane are nearly the same as the behavior of the modes in Fig. 9, where H^+ ions are added. However, the principal light ion mode changes more remarkably than that in Fig. 9. With increasing H^- ions, the phase velocity of the mode increases significantly. Note that the phase velocity at 10% H^- ions increases up to the point which is equivalent to the phase velocity at 50% H^+ ions in Fig. 9. Namely, this principal ion mode is significantly affected by the addition of a small amount of the light ion species, i. e. H^- ions.

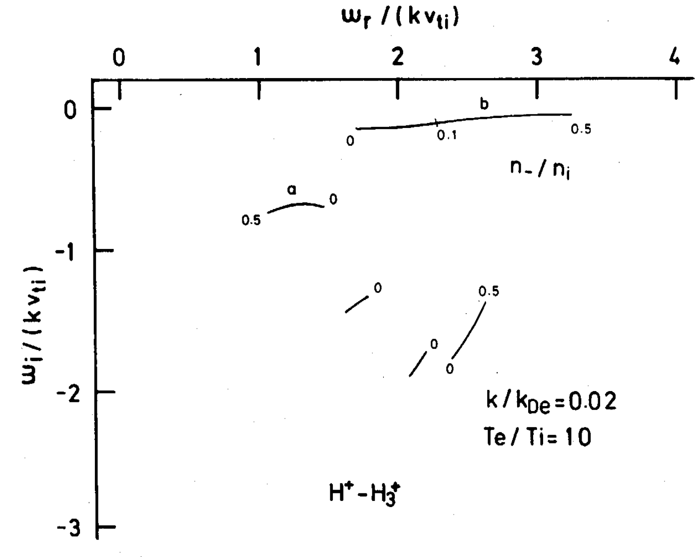

Fig. 15 Behaviors of solutions for the dispersion equation in $H^+-H_3^+-H^-$ plasma, *a*: principal heavy ion mode and *b*: principal light ion mode. Positive ion species ratio is $H^+:H_2^+:H_3^+ =$ 10 : 0 : 90. Parameter is the ratio of H^- ion density to total positive ion density, n_-/n_i. Plasma remains quasineutral, $n_1 + n_3 = n_i = n_e + n_-$. Temperature ratio and wave number are fixed.

The dispersion relation of the two modes (the principal light ion and heavy ion modes) are presented in Figs. 16 and 17. The damping per cycle of the two modes is also shown in Fig. 18. From these numerical results, we can conclude that only the light ion mode is observable.

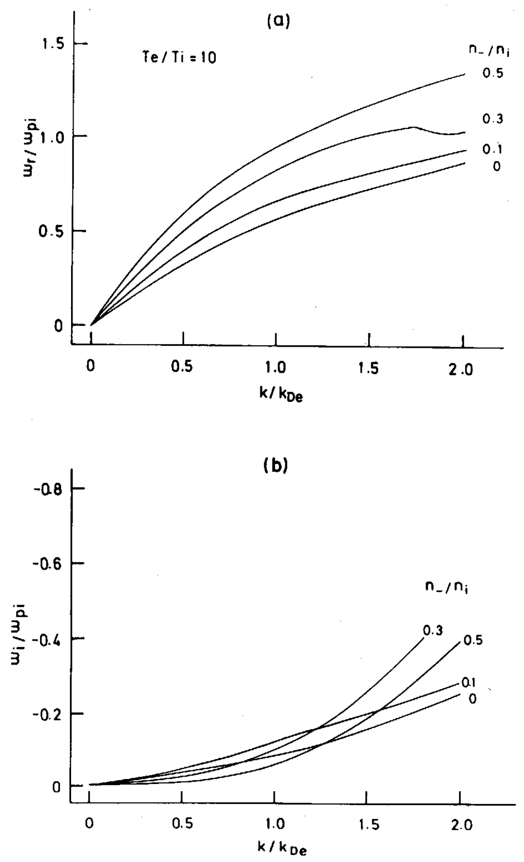

Fig. 16 Dispersion relation of principal light ion mode in $H^+ - H_3^+ - H^-$ plasma. Numerical conditions are the same as in Fig. 15.

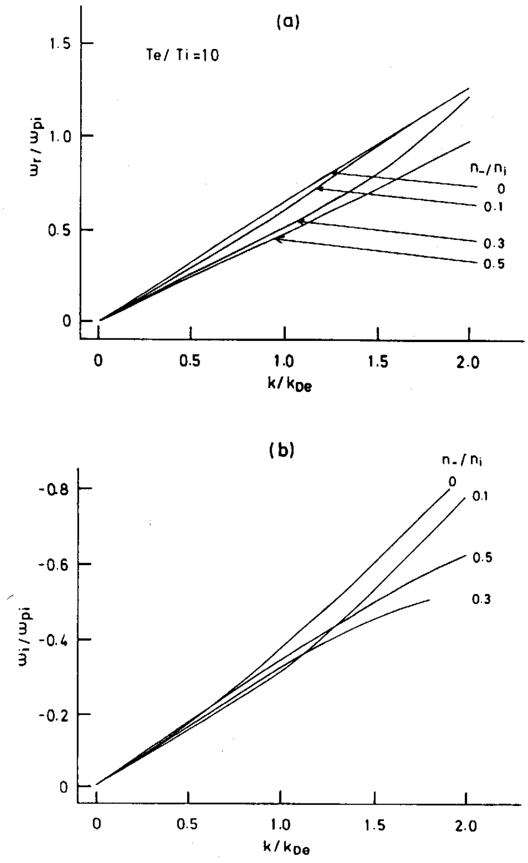

Fig. 17 Dispersion relation of principal heavy ion mode in $H^+ - H_3^+ - H^-$ plasma. Numerical conditions are the same in Fig. 15.

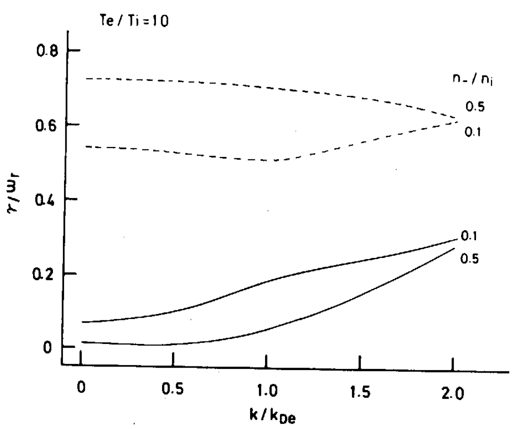

Fig. 18 Damping per cycle vs k/k_{De}. Solid and dashed lines are for principal light ion mode and principal heavy ion mode, respectively. Parameter is the density ratio of H^- ion, n_-/n_i.

3.4 Applicability of ion acoustic waves as an ion species diagnostic

According to the numerical results reported in section 3.3, there can be no conditions where two waves coexist[4,7], although the dispersion characteristics of the ion acoustic waves in hydrogen plasmas are modified by the effects of multi–ion species. This is mainly due to small mass ratio, i. e. $M_{H3}/M_H = 3$, $M_{H2}/M_H = 2$ and $M_{H3}/M_{H2} = 1.5$. However, the phase velocity of the light ion mode is increased remarkably by adding small amount of H^- ions. We can obtain the phase velocity of the wave by the method of time of flight, i. e. a relatively simple method. Because, whether H^- ions are present or not, only the principal light ion mode is observable. Therefore, we can use the ion waves as the density ratio diagnostic of H^- ions to the total positive ions n_-/n_i, but only if the positive ions species ratio is given.

So far, in experiments concerning the development of ion sources, H^- ion density and ion species ratios are obtained by photodetachment,[11] Langmuir probe[12] and mass spectrometer.[9] Although at present absolute negative ion density measurements using photodetachment are considered the more reliable, there are differences between the Langmuir probe measurements and the photodetachment technique. Therefore, it is very important and useful if the method of time of flight can estimate the density of H^- ions or the ratio n_-/n_i.

Finally, we estimate this method numerically by using typical experimental data and the results of the present calculation.

In a multicusp ion source, a following plasma is easily produced by hydrogen discharge,[9] : Hydrogen gas pressure $p = (4-5) \times 10^{-3}$ Torr, discharge voltage $V_d = 50V$, discharge current $I_d = 5A$, $n_e = 5.5 \times 10^{10}\ cm^{-3}$, $\kappa T_e = 1.3$ eV and ion species ratios $H^+ : H_2^+ : H_3^+ = 9 : 0 : 91$. This ion species ratio is nearly the same as the numerical condition in Fig. 15.

On the other hand, according to the results in Fig. 15, the phase velocity ratio between v_{ph} (10%) at 10% H^- ions and v_{ph} (0%) at no H^- ions is about 1.36. So, the time difference Δt between the transit time of the wave without H^- ions and that with 10% H^- ions is given by

$$\Delta t = \frac{L}{v_{ph}(0)} - \frac{L}{v_{ph}(10)} = \frac{L}{v_{ph}(0)} \cdot \frac{\dfrac{v_{ph}(10)}{v_{ph}(0)} - 1}{\dfrac{v_{ph}(10)}{v_{ph}(0)}}, \tag{9}$$

where L is the distance between the exciter grid and receiving grid of the ion acoustic wave. We assume that $v_{ph}(0)$ is the sound velocity for H^+ ions at $\kappa T_e = 1.3$ eV. Then, $v_{ph}(0) = 1.12 \times 10^6$ cm/sec. Substituting $L = 5$cm, $v_{ph}(10)/v_{ph}(0) = 1.36$ and $v_{ph}(0) = 1.12 \times 10^6$ cm/sec into eq. (9), we obtain the time difference $\Delta t = 1.18\ \mu$sec. We can easily measure this time difference precisely by using usual oscilloscopes.

4. Conclusions

From a view point of wave propagation in a multi–ion plasma, we have studied theoretically the characteristics of dispersion relation of ion acoustic waves in a hydrogen plasma. Findings are as follows :

1) In $H^+ - H_3^+$ plasma, with introducing H^+ ions, both the light ion mode and the heavy ion mode appear.

2) Without reference to H^+ ion densities, only the light ion mode is the most dominant mode. Namely, there is no condition where two waves coexist as they do in $H_e^+ - A_r^+$ plasma.

3) Introduction of a small amount of H^- ions significantly affects the phase velocity of the light ion mode. Thus, the measurement of the wave phase velocity can be used as a diagnostic of the H^- ratio n_-/n_i, but only if positive ion species ratios are obtained by some other method.

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