Turbulence Measurements in a Two-dimensional Turbulent Wake

By   Hidemi YAMADA*, Yoshimasa KUWATA**,
Hideo OSAKA* and Yoshiro KAGEYAMA*

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Abstract

Detailed measurements of turbulent quantities in the two-dimensional wake have been made in a wide range $30.5 \leq x/d \leq 475$. The measurements comprise profiles of mean velocity, turbulent kinetic energy, Reynolds shear stress, velocity diffusion and dissipation. An attempt has been made to evaluate the asymptotic process to self-preservation, and comparison between the simple semi-empirical models and measured profiles have been done.

The individual terms of the turbulent kinetic energy equation have been evaluated to present these balance.

1. Introduction

The two-dimensional wake in a uniform flow without pressure gradient is the most fundamental flow field as one of the self-preserving shear flows. So far there have been numerous studies for this flow field[1]~[3]. The recent years, Townsend[4] (1976), Tennekes and Lumley[5] (1974) have done the superior review of the prediction and the structure of the mean and turbulence field with respect to the wake. It was, however, pointed out that the turbulence field in the two-dimensional wake was one of the most difficult flows to properly predict[6]. There still have been large difference between the prediction and the development of flows.

Detailed measurements of turbulence quantities of the two-dimensional wake is investigated by Townsend 1949, and none of the investigations followed are comparable in detail this work. Especially with the shear stress, there does not almost exist the data about them. Thus, the shear stress calculated from the mean velocity measured by Townsend have been quoted widely. In addition, with the turbulent kinetic energy balance, the value obtained by integrating the diffusion term estimated by Townsend across the section is not zero. Therefore, it is well known[7] that for Townsend's experiment there exist the question due to the accuracy of the measurement.

So in the present work, the detailed measurement of the mean velocity, turbulent intensities, and Reynolds shear stress have done in a wide range of flows. An attempt has been made to evaluate the asymptotic process to self-preservation and the profile in the self-preservation region. In addition the applicability of various simple semi-

* Department of Mechanical Engineering
** Graduate student, Mechanical Engineering

empirical models has been discussed. Finally the various terms of the turbulent kinetic energy equation and the mean flow kinetic energy equation were estimated respectively.

Nomenclature

X, Y, Z	:	Cartesian coordinates axis, X in the direction of the free stream, Y perpendicular to axis of cylinder, Z along axis
U, V	:	mean velocity components in the X and Y directions, respectively
u', v', w'	:	fluctuating turbulent velocity components in the X, Y and Z directions, respectively
U_1	:	free stream velocity
u	:	mean velocity defect $= U_1 - U$
u_0	:	maximum mean velocity defect at a fixed X
d	:	diameter of cylinder
b	:	half-width
$\overline{q^2}$	:	turbulent kinetic energy $= \overline{u'^2} + \overline{v'^2} + \overline{w'^2}$
p'	:	pressure fluctuation
$-\overline{u'v'}$	:	Reynolds shear stress
ε	:	dissipation rate
ν_T	:	eddy viscosity
ν	:	kinematic viscosity
l	:	mixing length
l^*	:	length scale
u^*	:	velocity scale
a_1	:	structure parameter $= -\overline{u'v'}/\overline{q^2}$
R_d	:	Reynolds number $= U_1 d/\nu$
R_T	:	turbulent Reynolds number $= u_0 b/\nu_T$
R_T^*	:	turbulent Reynolds number $= u^* l^*/\nu_T$

2. Basic equations and quantities measured

The equations which describe the flow field in a two-dimensional wake can be derived from the Navier-Stokes equation applied boundary layer approximations. The basic equations are as follows:

Continuity equation;

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{1}$$

Momentum equation;

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} + \frac{\partial}{\partial Y}(\overline{u'v'}) - \nu\left(\frac{\partial^2 U}{\partial Y^2}\right) = 0, \tag{2}$$

Mean flow kinetic energy equation;

$$\frac{U}{2}\frac{\partial U^2}{\partial X}+\frac{V}{2}\frac{\partial U^2}{\partial Y}+\frac{\partial}{\partial Y}(U\overline{u'v'})-\overline{u'v'}\frac{\partial U}{\partial Y}+\nu\left(\frac{\partial U}{\partial Y}\right)^2=0. \qquad (3)$$

Each term is conveniently written in non-dimensional form with the maximum velocity defect u_0 and the half-width b, as follows respectively;

Advection : $\dfrac{b}{u_0^3}\left[\dfrac{U}{2}\dfrac{\partial U^2}{\partial X}+\dfrac{V}{2}\dfrac{\partial U^2}{\partial Y}\right]$,

Work   : $\dfrac{b}{u_0^3}\left[\dfrac{\partial}{\partial Y}(U\overline{u'v'})\right]$,

Production: $\dfrac{b}{u_0^3}\left[-\overline{u'v'}\dfrac{\partial U}{\partial Y}\right]$, $\qquad (4)$

Dissipation: $\dfrac{b}{u_0^3}\left[\nu\left(\dfrac{\partial U}{\partial Y}\right)^2\right]$.

Turbulent kinetic energy equation;

$$\frac{U}{2}\frac{\partial \overline{q^2}}{\partial X}+\frac{V}{2}\frac{\partial \overline{q^2}}{\partial X}+\overline{u'v'}\frac{\partial U}{\partial Y}+\frac{\partial}{\partial Y}\overline{v'\left(\frac{q^2}{2}+\frac{p'}{\rho}\right)}+\varepsilon=0. \qquad (5)$$

Each term is written in non-dimensional form similar to eq. (4) as follows;

Advection : $-\dfrac{b}{u_0^3}\left[\dfrac{U}{2}\dfrac{\partial \overline{q^2}}{\partial X}+\dfrac{V}{2}\dfrac{\partial \overline{q^2}}{\partial Y}\right]$,

Production: $\dfrac{b}{u_0^3}\left[-\overline{u'v'}\dfrac{\partial U}{\partial Y}\right]$,

Diffusion   : $-\dfrac{b}{u_0^3}\left[\dfrac{\partial}{\partial Y}\overline{v'\left(\dfrac{q^2}{2}+\dfrac{p'}{\rho}\right)}\right]$, $\qquad (6)$

Dissipation: $-\dfrac{b}{u_0^3}\varepsilon$.

The physical interpretation of the various terms is well known. The following quantities were measured:

(a) U component of mean velocity
(b)   Turbulent intensity of three components in the Cartesian coordinate $(\sqrt{\overline{u'^2}}, \sqrt{\overline{v'^2}}, \sqrt{\overline{w'^2}})$
(c)   Reynolds shear stress $(-\overline{u'v'})$
(d)   Dissipation $(\overline{(\partial u'/\partial x)^2}, \overline{(\partial v'/\partial x)^2}, \overline{(\partial w'/\partial x)^2})$
(e)   Triple velocity correlations $(\overline{u'^2v'}, \overline{v'^3})$.

3.   Description of experiment

The wind tunnel used in this experiment is an open-circuit type with centrifugal fan driven by a variable-speed motor. The contraction ratio is 6.25 : 1 and the turbu-

lence level at the contraction exit is about 0.35%. The test section is 0.4 m square and about 4 m long. The side wall is variable to be able to adjust a pressure gradient in free stream. In the present experiment the pressure gradient in the flow field is set in about zero. The two-dimensional circular cylinder as a wake generator is a brass one, of which the diameter d is 5 mm. Measurements are made, keeping a constant value of Reynolds number; $R_d = 4 \times 10^3$. Mean velocity is measured by means of a Pitot tube (outer diameter 2.35 mm). Turbulent quantities are measured by means of constant temperature hot-wire anemometers with X-wire probe (5 μm Tungsten). Higher order correlation and dissipation measurements are performed on a sum-difference-product circuit and analogue correlator. The measurements are made in the streamwise region from $X/d = 30.5$ to 475.

4. Results and Discussion

4-1　Mean Velocity

The profile of mean velocity non-dimensionalized with a half-width b and a maximum velocity defect u_0 is shown in Fig. 1. A solid line in the figure shows the Townsend's empirical formula as below;
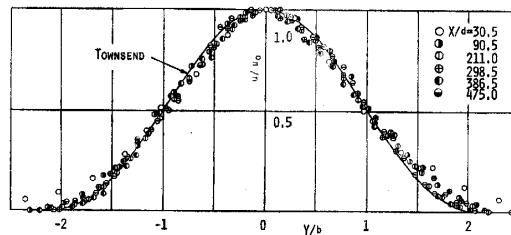

Fig. 1　The mean velocity profile.

$$\frac{u}{u_0} = \exp\left[-0.6619\left(\frac{Y}{b}\right)^2 \left\{ 1 + 0.0465\left(\frac{Y}{b}\right)^4 \right\} \right]. \tag{7}$$

At all of the section beyond $X/d = 90.5$, the mean velocity distribution becomes similar within the experimental errors and approximately agrees with Townsend's empirical formula. This trend corresponds to Tennekes and Lumley's analogy which mean velocity distribution exhibits similarity beyond about 80 times cylinder diameter.

4-2　Turbulent intensity

The fluctuating components u', v' and w' are shown in non-dimensionalized form in Figs. 2, 3 and 4 respectively. A solid line in the figures shows the Townsend's experimental result. These results agree with Tennekes and Lumley's analogy which the turbulent intensity distributions exhibit similarity beyond about 200 times cylinder diameter. The general forms of $\sqrt{\overline{u'^2}}/u_0$ and $\sqrt{\overline{w'^2}}/u_0$ exhibit the same tendency as Townsend's distribution, but in quantities they exhibit larger value than those measured
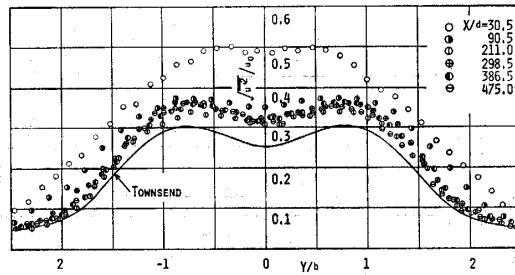
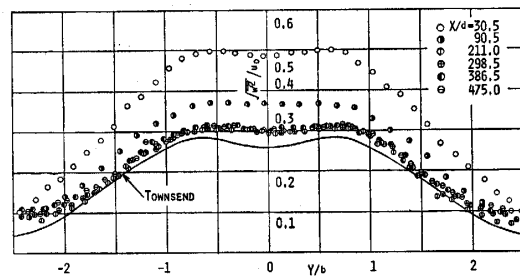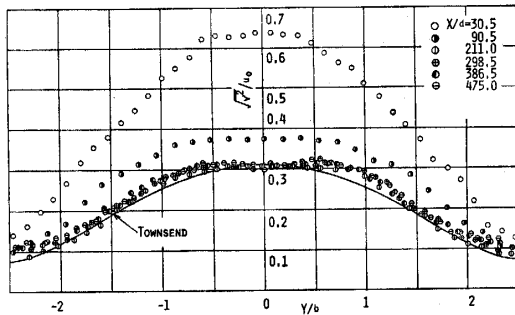Fig. 2 X component of turbulent intensity profile.

Fig. 3 Y component of turbulent intensity profile.   Fig. 4 Z component of turbulent intensity profile.

by Townsend by about 20% and 10% respectively.   On the other hand, the profile of $\sqrt{\overline{v'^2}}/u_0$ differs from Townsend's distribution which exhibits a maximum value at the center of wake, and exhibits a hollow which is the same profile as $\sqrt{\overline{u'^2}}/u_0$ and $\sqrt{\overline{w'^2}}/u_0$. This tendency is also shown in the distribution of a flat plate wake measured by Kovasznay[8].   In addition, it is one of the most interesting points that the distributions of $\sqrt{\overline{v'^2}}/u_0$ and $\sqrt{\overline{w'^2}}/u_0$ are nearly the same in tendency and magnitude, and there is no obvious difference as Townsend's profile between them.

4-3   Shear stress

The distributions of shear stress are plotted in Fig. 5 together with results calculated from the mean velocity measured by Townsend and Narasimha's[9] experimental result.   The distributions of shear stress also exhibit similarity at all of the section beyond $X/d = 211.0$ and are found to fit the Townsend's distribution extremely well. It's maximum value takes the position of $Y/b \fallingdotseq 0.85$.   Thus, both turbulent intensities and the shear stress exhibit self-similar nearly in the same region.   This result is in excellent agreement with Tennekes and Lumley's analogy.

4-4   Turbulence characteristics

For turbulent free shear flows, some predictions by semi-empirical model have been attempted earlier.   But it seems that there are various problems about their models which represented turbulent structure by only one rough parameter.   By investigating the behavior of their parameter in a wake in detail, the propriety of applying their models for a two-dimensional wake is examined.
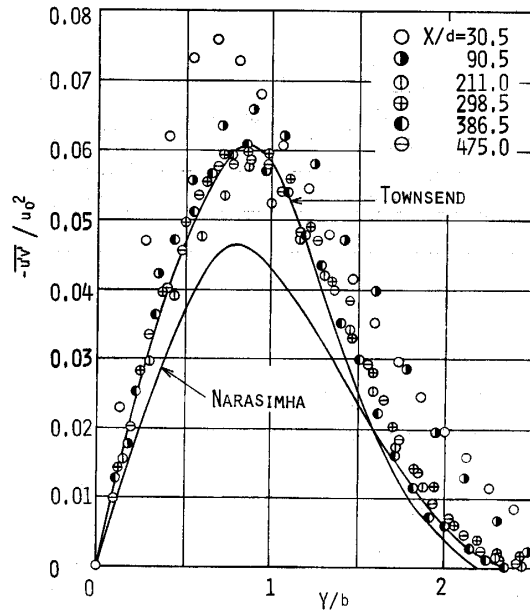
Fig. 5   Reynolds shear stress distribution.

Fig. 6 shows the distributions of the eddy viscosity v_T, where v_T is defined as follow;

$$-\overline{u'v'} = v_T(\partial U/\partial Y).\qquad(8)$$

The values of the eddy viscosity take a nearly constant value in the region of $Y/b<1.0$, but they decrease slowly as Y increases.   Thus a prediction using a eddy viscosity model overestimates the mean velocity at the edge of wake.   Fig. 7 shows the distributions of the mixing length l, where l is defined as follow;

$$-\overline{u'v'} = l^2(\partial U/\partial Y)^2.\qquad(9)$$

The values of the mixing length take a constant value about 0.35 in the region of $0.5<Y/b<1.5$.   This value is evidently smaller than Townsend's experimental results 0.41. In the region of $Y/b<0.5$, mixing length increases abruptly.   Thus a model of calcu-
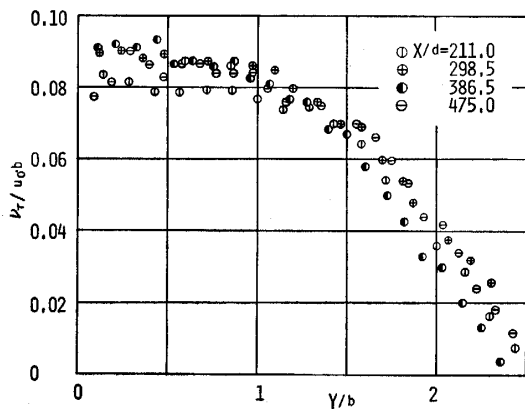

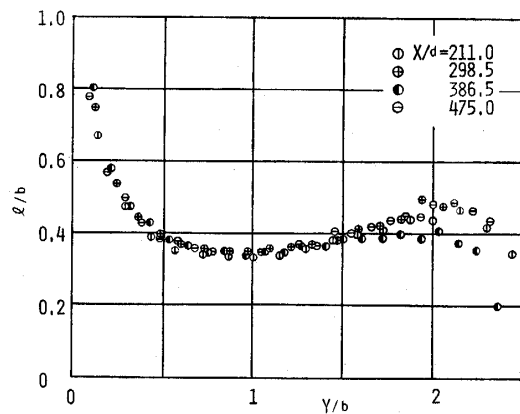Fig. 6   Eddy viscosity distribution.

Fig. 7   Mixing length distribution.

lation by mixing length model underestimates the mean velocity near wake center. Fig. 8 shows the distributions of the structure parameter a_1, where a_1 is a ratio between turbulent shear stress and turbulent kinetic energy, and defined as follow;

$$a_1 = -\overline{u'v'}/q^2. \tag{10}$$

The structure parameter a_1 increases slowly as Y increases. Thus a constant value of a_1 which is shown in turbulent boundary layer (Bradshaw[10] $a_1 = 0.15$) does not exist in Fig. 8. Therefore in the present flow field, the linearity between the shear stress and the turbulent kinetic energy can not be confirmed. Fig. 9 shows the correlation coefficient of shear stress compared with Bradbury's[11] plane jet. The value of the correlation coefficient is larger than that of the plane jet except near wake center, and a constant value as it is in turbulent boundary layer, pipe flow does not exist.
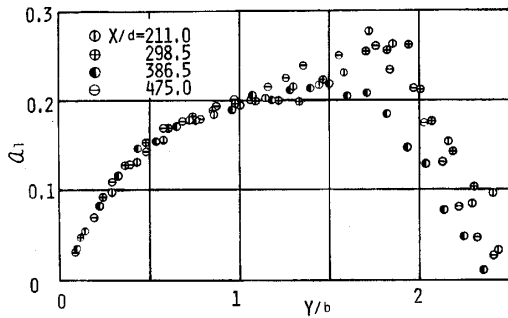

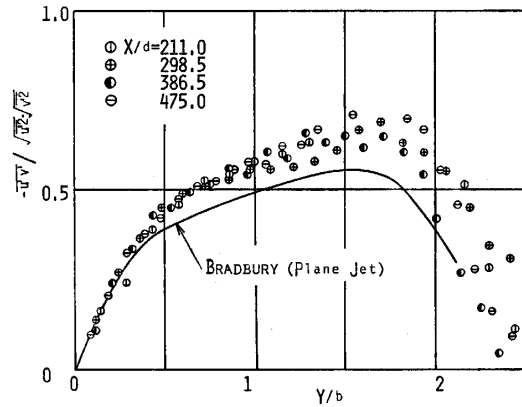Fig. 8   Structure parameter distribution.

Fig. 9   The shear stress correlation coefficient distribution.

In the following, as a representative length scale of wake, we now consider the half-width b and l^* proposed by Tennekes and Lumley defined as follow:

$$l^* = u_0\, e^{-1/2}/(\partial U/\partial Y)_{max}. \tag{11}$$

Fig. 10 shows the development of the characteristic length scale in the downstream direction. The slope of b and l^* are represented as

$$b \propto X^{0.49},$$

$$l^* \propto X^{0.51},$$

respectively. Both exponents of development is in excellent agreement with 0.5 estimated from the self-preservation hypothesis. Therefore it is found that l^* is appropriate as a length scale. As a velocity scale of wake, we consider the maximum velocity defect u_0, and u^* proposed by Tennekes and Lumley defined as follow:

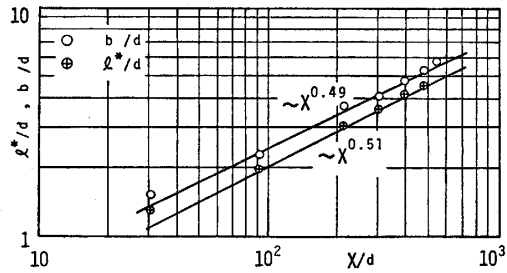$$u^* = (-\overline{u'v'})^{\frac{1}{2}}_{max}. \tag{12}$$

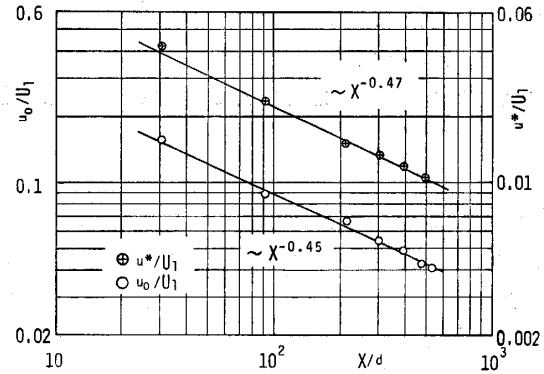Fig. 10   The development of characteristic length scales.

Fig. 11   The decay of characteristic velocity scales.

Fig. 11 shows the decay of the characteristic velocity scale in the downstream direction. The results are

$$u_0 \propto X^{-0.45},$$

$$u^* \propto X^{-0.47},$$

respectively. Both exponents of decay agree with -0.5 estimated from the self-preservation hypothesis. Therefore it is found that u^* is suitable as a velocity scale.

Fig. 12 shows a downstream variation of turbulent Reynolds number R_T, which is constructed by maximum velocity defect u_0, half-width b and eddy viscosity v_T. Beyond $X/d = 211.0$, the turbulent Reynolds number R_T is a constant value ($R_T = 12.4$). It almost coincides with Townsend's result ($R_T = 12.5$) but Narasimha's results ($R_T = 15.4$) is considerably larger than those. It can be considered that this difference is due to the measured value of Reynolds shear stress. Furthermore, Tennekes and Lumley proposed R_T^*, as a more meaningful turbulent shear flows, defined as follow:

$$R_T^* = \frac{u^* l^*}{v_T} .\qquad (13)$$

Fig. 13 shows a downstream variation of R_T^*. Tennekes and Lumley indicated that this turbulent Reynolds number was clearly separated into two groups; one group for wakes ($R_T^* \simeq 2.8$), the other group for jets and mixing layers ($R_T^* \simeq 4.0 \sim 4.8$). Present wake measurements indicate $R_T^* = 2.60$. This result supports Tennekes and Lumley's criterion.
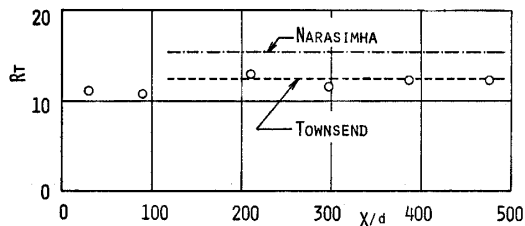

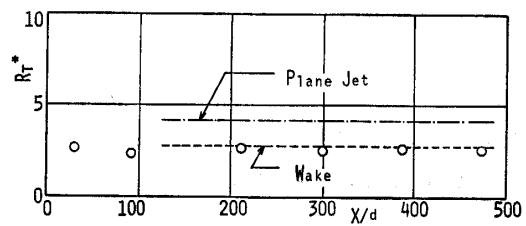Fig. 12   The downstream variation of the conventional turbulent Reynolds number R_T.

Fig. 13   The downstream variation of the turbulent Reynolds number R_T^*.

4-5   Energy balance

The mean flow kinetic energy balance can be written as:

$$\frac{b}{u_0^3}\left[\frac{U}{2}\frac{\partial U^2}{\partial X}+\frac{V}{2}\frac{\partial U^2}{\partial Y}\right]+\frac{b}{u_0^3}\left[\frac{\partial}{\partial Y}(U\overline{u'v'})\right]+\frac{b}{u_0^3}\left[-\overline{u'v'}\frac{\partial U}{\partial Y}\right]+\frac{b}{u_0^3}\left[\nu\left(\frac{\partial U}{\partial Y}\right)^2\right]=0.$$

$$(14)$$

An estimation of the various terms have been made at the section of $X/d=298.5$. This is shown in Fig. 14. Through the wake, the advection by the mean motion and the work by the turbulent shear stress are significant and nearly in balance each other. The production of turbulence energy and direct dissipation terms are negligible compared with the advection and work terms. The advection term and the work term show the reversal of sign near the position of maximum shear stress. In the inner region of that position, the loss due to the advection by the mean motion is equal to the gain due to the work by the turbulent shear stress. Reversely, in the outer region, the gain due to advection is equal to the loss due to the work. The behavior of each term is consistent with the explanation by Townsend and Tennekes and Lumley. The production of turbulence energy by the mean motion is very small. From this result it may be considered that the structure of the mean motion and the turbulence are independent.

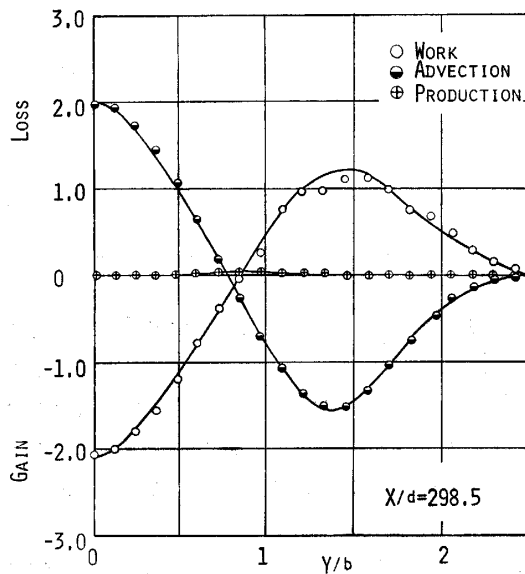In the following, the turbulent kinetic energy balance can be written as follow;

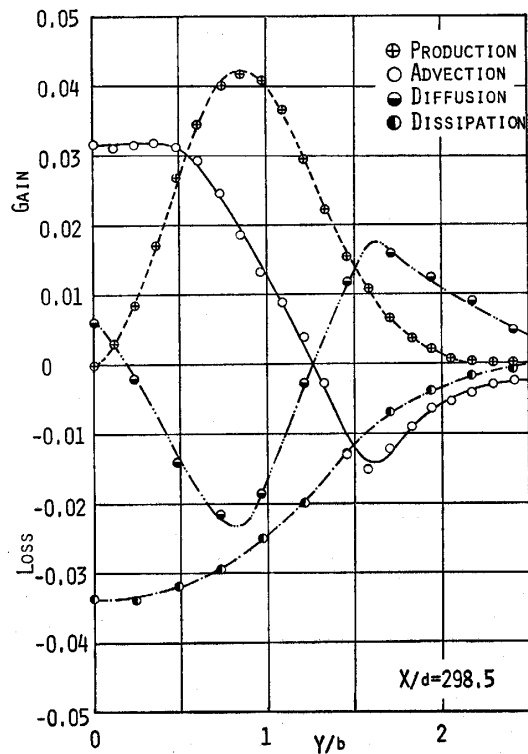
Fig. 14   The mean flow kinetic energy balance.

Fig. 15   The turbulent kinetic energy balance.

$$-\frac{b}{u_0^3}\left[\frac{U}{2}\frac{\partial\overline{q^2}}{\partial X}+\frac{V}{2}\frac{\partial\overline{q^2}}{\partial Y}\right]+\frac{b}{u_0^3}\left[-\overline{u'v'}\frac{\partial U}{\partial Y}\right]-\frac{b}{u_0^3}\left[\frac{\partial}{\partial Y}\overline{v'\left(\frac{q^2}{2}+\frac{p'}{\rho}\right)}\right]-\frac{b}{u_0^3}\varepsilon=0.$$

$$(15)$$

An estimation of the various terms have been made at the section of $X/d=298.5$. This is shown in Fig. 15. The dissipation term is inferred as follow;

$$\varepsilon=3\nu\left\{\overline{\left(\frac{\partial u'}{\partial x}\right)^2}+\overline{\left(\frac{\partial v'}{\partial x}\right)^2}+\overline{\left(\frac{\partial w'}{\partial x}\right)^2}\right\}.$$

$$(16)$$

For the diffusion term, pressure diffusion does not measured and velocity diffusion is inferred as follow;

$$\frac{\partial}{\partial Y}\left(\overline{v'\frac{q^2}{2}}\right)=\frac{\partial}{\partial Y}\left[\frac{\overline{v'(u'^2+v'^2+w'^2)}}{2}\right]\doteqdot\frac{3}{4}\frac{\partial}{\partial Y}(\overline{u'^2v'}+\overline{v'^3}).$$

$$(17)$$

The production, advection, diffusion and dissipation terms have the same order through the wake, but each term is significant in the different parts of the wake. When the diffusion term is integrated across the wake, it's value is approximately zero. Near the wake center, production is nearly absent and the main gain is due to advection in the axial direction, and this gain is balanced by dissipation. At $Y/b \simeq 0.85$ the production term indicates the maximum value. This location corresponds to that of the maximum shear stress. Comparing the behavior of each term with Townsend's distribution, the contribution due to production and diffusion are markedly different. The maximum value of the production term is about three times as large as Townsend's value. For the diffusion term at the wake center, the present experiment shows gain but Townsend's shows loss. If the good balance of the turbulent kinetic energy at the local position is desired, the dissipation term must be multiplied by a factor of about 1.4. (Because a low pass filter was used to provide a sharp cut-off at 4 kHz[12] in the present measurements.)

5.   Conclusions

The mean flow and the turbulent flow fields of the two-dimensional turbulent wake were measured in a wide range in full detail. From these results the following have been revealed.

1) The mean velocity distribution becomes similar beyond $X/d=90.5$, and approximately agrees with Townsend's empirical formula. The turbulent intensity distribution becomes similar beyond $X/d=211.0$, and three components of them have a hollow at the center of wake. The shear stress also becomes similar beyond $X/d=211.0$, and is found to fit the result calculated from the mean velocity measured by Townsend extremely well.

2) The maximum velocity defect u_0 and the half-width b as a representative scale of a wake agree well with exponent estimated from the self-preservation hypothesis.

l^* and u^* also have the same exponent as u_0 and b have, respectively, therefore these may be considered as suitable scale of a wake.

3) The values of the eddy viscosity and the mixing length take a constant value in the region $Y/b < 1.0$ and $0.5 < Y/b < 1.5$, respectively. A constant value of the structure parameter a_1 does not exist. Therefore in the present flow field, the linearity between the shear stress and the turbulent kinetic energy can not be confirmed.

4) In the mean flow kinetic energy balance, the contribution of the advection by the mean motion and the work by the turbulence shear stress have a significant meaning, and the loss due to production of turbulence energy is extremely small compared with the contribution of both terms.

5) In the turbulent kinetic energy balance, the production, advection, diffusion and dissipation terms have the same order, but each terms have a significant meaning in the different parts of the wake. The location of the maximum production of turbulence energy corresponds to the one of the maximum shear stress. The balance in the present experiment considerably differs from that of Townsend, but the budget is quiet well. And it is supposed that this estimation is appropriate.

Acknowledgement

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