Unsteady Heat Transfer Combined with Thermal Radiation along with Consideration of Temperature-dependent Properties and Turbulent Flow through a Thermal Entrance Region of a Concentric Annulus and a Circular Tube

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Abstract

The turbulent flow of steam moving through the thermal entrance region of a concentric annulus and a circular tube, is heated by an inner pipe wall of the annulus and the circular tube wall, respectively. An analysis of this was made so as to study unsteady heat transfer combined with thermal radiation and turbulent flow. At the time this study was carried out, we took into consideration the influence of temperature-dependent thermophysical properties, and restricted ourselves to the optically thin limit, using the Planck mean coefficient, $K_p(T)$.

The influence from the radius ratio of the annulus and the temperature ratio were examined through numerical calculations of the finite difference methods. In particular, the unsteady states with consideration directed to the temperature-dependent properties of steam are emphasized in this paper.

Nomenclature

a	thermal diffusivity
C_p	specific heat
D	diameter of circular tube
D_e	equivalent diameter of annulus
E_b	$\alpha \cdot T^4$
f	friction factor
F_0	Fourier number
$K_p(T)$	Planck mean coefficient
L	pipe length
N_R	radiation to conduction ratio
Nu_r	radiative Nusselt number
Nu_c	convective Nusselt number
Pr	Prandtl number
q_r	radiative heat flux
r	radial length
r_1	inner radius
r_2	outer radius
Re	Reynolds number
Rt	turbulent Reynolds number
T	temperature
T_1	inner temperature
T_2	outer temperature
T_i	inlet temperature
T_w	wall temperature
t	$F_0 \cdot Pr$
u	velocity

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u^+	nondimensional velocity
u_1^+	inner nondimensional velocity
u_2^+	outer nondimensional velocity
$\bar{u}$	mean velocity
X	$(r-r_1)/(r_2-r_1)$ (annulus), y/r_0 (circular tube)
y	r_0-r
Z	axial length
z	Z/L

Greek symbols

α	r_2/r_1
θ	$(T-T_2)/(T_1-T_2)$ (annulus), $(T-T_i)/(T_w-T_i)$ (circular tube)
ε_H	eddy diffusivity for heat
ε_M	eddy diffusivity for momentum
σ	Stefan Boltzmann constant
ν	kinematic viscosity
τ	time
τ_0	$K_p\cdot(r_2-r_1)$ (annulus), $K_p\cdot r_0$ (circular tube)
λ	thermal conductivity
ρ	density

Suffix

1	inner	2	outer
b	bulk	i	inlet
w	wall		

1. Introduction

Determination of energy transferred to a flowing gas is a basic consideration in many engineering problems, and in high temperature applications a detailed study of the radiative contribution is frequently required. In addition, considerable practical interest in the unsteady heat transfer characteristics of a flow system has been stimulated by the needs of modern technology, especially in relation to the temperature-dependent properties. However, because of the trend toward increasing temperature in modern technological systems, heat transfer by simultaneous thermal radiation and other modes of energy transfer is very important in engineering systems of high temperature heat transfer, including thermal radiation absorbing and emitting media (steam, carbon dioxide, etc.). This study is frequently required in many engineering problems. But the study of these problems is difficult due to both the highly nonlinear nature of thermal radiation and the interaction of thermal radiation with convection and conduction. Thus, basic equations which describe these phenomena are quite complex.

Unsteady heat transfer with temperature-dependent properties by combined thermal radiation and turbulent flow through thermal entrance region in a concentric annulus and a circular tube has not yet been studied.

Concentric annular geometry, limited to cases of parallel plates and circular pipe, is of considerable importance in heat exchanger and nuclear reactor design. In this respect, many research workers [1] [2] [3] have investigated turbulent heat transfer in an annulus. The experimental and analytical study in an annulus was carried out by Nichols [1].

In the present papeı, Cess and Mighdoll's equations, modified by Kawachi, one of the authors are employed as radiative terms in an annulus, in the case which is optically thin.

A thermal radiation term in a circular tube is approximated for a case under an optically thin limit condition. The flow is a fully developed turbulent gas flow in a circular tube subjected to a constant wall temperature.

2. Energy Equation

The energy equation is shown by the following equation in cylindrical coordinates at the unsteady state.

$$\frac{\partial T}{\partial \tau}+u\cdot\frac{\partial T}{\partial Z}=\frac{1}{r}\cdot\frac{\partial}{\partial r}\left[r\cdot(\varepsilon_M+a)\frac{\partial T}{\partial r}\right]-\frac{1}{\rho\cdot C_p}\operatorname{div} q_r. \tag{1}$$

The case in which is optically thin is treated by the authors and so, as the thermal radiation term, $\operatorname{div} q_r$, the second term in the right side of equation (1), in a circular tube, Cess' formula [2] is used, according to Landram [3], but in a concentric annulus, as the thermal radiation term, $\operatorname{div} q_r$, Cess and Mighdoll formula [2] is used and has been modified by Kawachi in consideration of the circumferential extension of a concentric annulus.

Annulus:

$$-\operatorname{div} q_r=-4K_pE_b(T)+2\frac{r_1}{r}K_m(T,\ T_1)E_b(T_1)$$

$$+2\frac{r_2}{r}K_m(T,\ T_2)E_b(T_2)$$

$$K_m(T,\ T_n)=K_p(T_n)\frac{T_n}{T} \qquad E_b(T)=\sigma T^4$$

Circular tube:

$$-\operatorname{div} q_r=-4K_p(T)E_b(T)+4K_p(T_w)\frac{T_w}{T}E_b(T_w).$$

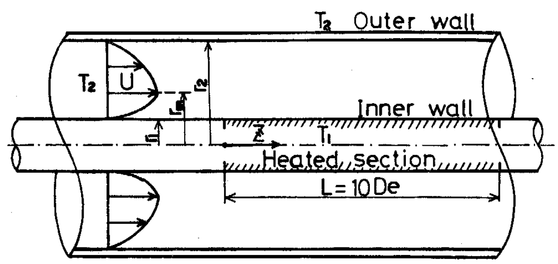

Fig. 1 Coordinate of the numerical analysis.

To reduce (1) to a nondimensional form, the following nondimensional variables are introduced.

Annulus: $z=Z/L \quad X=(r-r_1)/(r_2-r_1) \quad \alpha=r_2/r_1$

$$\theta=(T-T_2)/(T_1-T_2) \quad t=\tau\cdot\nu/(r_2-r_1)^2=F_0\cdot Pr \quad \tau_0=K_p(r_2-r_1)$$

$$N_R=\lambda(T_1-T_2)/\{(r_2-r_1)\sigma(T_1-T_2)^4\} \qquad Rt_1=Re\sqrt{f/2\cdot(r_m^2-r_1^2)/\{r_1(r_2-r_1)\}}$$

$$Rt_2=Re\sqrt{f/2\cdot(r_2^2-r_m^2)/\{r_2(r_2-r_1)\}} \qquad Re=\bar{u}\cdot(r_2-r_1)/\nu.$$

From (1), the nondimensional energy equation for the annulus takes the following form,

$$\frac{\partial\theta}{\partial t}+\frac{Rt}{2}\cdot\frac{D_e}{L}\cdot u^+\cdot\frac{\partial\theta}{\partial z}=\frac{1}{\left(X+\frac{1}{\alpha-1}\right)}\cdot\frac{\partial}{\partial X}\left[\left(X+\frac{1}{\alpha-1}\right)\left(\frac{\varepsilon_M}{\nu}+\frac{1}{P_r}\right)\cdot\frac{\partial\theta}{\partial X}\right]$$

$$-4\frac{\tau_0}{N_R\cdot Pr}\cdot\left[\frac{K_p(\theta)}{K_p(\theta_w)}\cdot\left(\theta+\frac{T_2}{T_1-T_2}\right)^4-\frac{1}{2}\cdot\frac{1}{\{(\alpha-1)X+1\}}\cdot\frac{\left(\frac{T_1}{T_1-T_2}\right)^5}{\left(\theta+\frac{T_2}{T_1-T_2}\right)}\right.$$

$$\left.-\frac{1}{2}\cdot\frac{\alpha}{\{(\alpha-1)X+1\}}\cdot\frac{K_p(\theta_2)}{K_p(\theta_w)}\cdot\frac{\left(\frac{T_2}{T_1-T_2}\right)^5}{\left(\theta+\frac{T_2}{T_1-T_2}\right)}\right]. \qquad (2)$$

Boundary conditions;

$$(\theta)_{t=0}=0,\quad (\theta)_{X=0}=1 \text{ (inner wall)},\quad (\theta)_{X=1}=0 \text{ (outer wall)} \qquad (3)$$

$$(\theta)_{z=0}=0 \text{ (inlet)}$$

$$f\left(\frac{\alpha-1}{\alpha}\right)^{-0.1}=0.055(Re)^{-0.2}$$

$$r_1\leqq r\leqq r_m;\quad u_1^*=\sqrt{\frac{\tau_{w1}}{\rho}}=\bar{u}\sqrt{\frac{f}{2}\cdot\left\{\frac{r_m^2-r_1^2}{r_1(r_2-r_1)}\right\}} \qquad \text{(Knudsen, Katz) [4]}$$

$$r_m\leqq r\leqq r_2;\quad u_2^*=\sqrt{\frac{\tau_{w2}}{\rho}}=\bar{u}\sqrt{\frac{f}{2}\cdot\left\{\frac{r_2^2-r_m^2}{r_2(r_2-r_1)}\right\}}$$

$$r_m=r_1\left[\frac{1+\alpha^n}{1+\left(\frac{1}{\alpha}\right)^n}\right] \quad n=0.343 \qquad \text{(Roberts) [5]}$$

inner velocity profile:

$$u_1^+=\frac{1}{K_1}\ln y_1^+ + B_1 \qquad \text{(Roberts) [5]}$$

outer velocity profile:

$$u_2^+=2.5\ln\eta_0^+ + 5.5$$

$$\eta_0^+=1.5y_2^+(1+\eta_0)/(1+2\eta_0^2) \qquad \eta_0=(r_m-r)/(r_m-r_2) \quad \text{(Kays, Leung) [6]}$$

$r_1\leqq r\leqq r_m$;

$$\frac{\varepsilon_M}{\nu}=\frac{K}{6}\cdot\delta_2^+\left[1-\left(\frac{y_1^+-\delta_1^+}{\delta_1^+}\right)^2\right]\left[1+b\left(\frac{y_1^+-\delta_1^+}{\delta_1^+}\right)^2\right]\left[1+\exp\left(-\frac{y_1^+}{A^+}\right)\right]$$

$r_m\leqq r\leqq r_2$;

$$\frac{\varepsilon_M}{\nu}=\frac{K}{6}\cdot\delta_2^+\left[1-\left(\frac{y_2^+-\delta_2^+}{\delta_2^+}\right)^2\right]\left[1+b\left(\frac{y_2^+-\delta_2^+}{\delta_2^+}\right)^2\right]\left[1+\exp\left(-\frac{y_2^+}{A^+}\right)\right]$$

$\delta_1^+=(r_m-r_1)u_1^*/\nu \qquad \delta_2^+=(r_2-r_m)u_2^*/\nu$

$b_1=2 \qquad b_2=2.5 \qquad A^+=42 \qquad K=0.4$ (Wilson, Medwel) [7]

$Prt=\varepsilon_M/\varepsilon_H \qquad 1/Prt=1.5\phi[1-\exp(-1/\phi)]$

$$\phi=\frac{(\varepsilon_M/\nu)\cdot Pr}{[4.13+0.743(\varepsilon_M/\nu)^{1/2}\cdot Pr^{1/3}]}$$

(Mizushina) [8], (Kawamura) [9]

Circular tube:

$\theta=(T-T_i)/(T_w-T_i) \qquad t=\tau\cdot\nu/(r_0)^2=F_0\cdot Pr \qquad X=y/r_0$

$y=r_0-r \qquad z=Z/L \qquad N_R=\lambda/[r_0\cdot\sigma\cdot(T_w-T_i)^3]$

$r_0^+=Re\cdot(f/8)^{1/2}=Rt \qquad f=0.046Re^{-0.2}.$

From (1), the nondimensional energy equation for circular tube takes the next form.

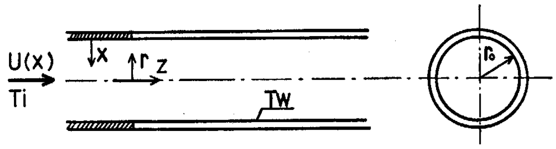

Fig. 2 Physical model and coordinate.

$$\frac{\partial\theta}{\partial t}+\frac{D}{2L}Rt\cdot u^+\cdot\frac{\partial\theta}{\partial z}=\frac{1}{1-X}\frac{\partial}{\partial X}\left\{(1-X)\left(\frac{\varepsilon_M}{\nu}+\frac{1}{Pr}\right)\cdot\frac{\partial\theta}{\partial X}\right\}$$

$$-\frac{4\tau_0}{PrN_R}\left[\frac{K_p(T)}{K_p(T_w)}\cdot\left(\theta+\frac{T_i}{T_w-T_i}\right)^4-\frac{\left(\frac{T_w}{T_w-T_i}\right)^5}{\left(\theta+\frac{T_i}{T_w-T_i}\right)}\right]. \qquad (4)$$

Boundary conditions:

$(\theta)_{t=0}=0 \qquad (\theta)_{X=0}=1$ (tube wall)

$\left(\frac{\partial\theta}{\partial X}\right)_{X=1}=0$ (center) $\qquad (\theta)_{z=0}=0$ (inlet)

$$\frac{\varepsilon_M}{\nu}=\frac{0.4}{3}r_0^+[0.5+(1-X)^2][1-(1-X)^2]\left[1-\exp\left(-\frac{r_0^+X}{A^+}\right)\right] \quad A^+=40$$

$1/Prt=1.5\phi[1-\exp(-1/\phi)] \qquad \phi=(\varepsilon_M/\nu)Pr/[4.13+0.743(\varepsilon_M/\nu)^{1/2}Pr^{1/3}]$

$u^+=5.5+2.5\ln[r_0^+\cdot X\cdot(3-1.5X)/\{1+2(1-X)^2\}].$

The temperature dependence of $K_p(T)$ [10], $\lambda(T)$ and $Pr(T)$ of steam is shown by the equations,

$$K_p(T)=77.427+0.2038\cdot T+2.406\times10^{-4}\cdot T^2-1.413\times10^{-7}\cdot T^3$$
$$+3.306\times10^{-11}\cdot T^4$$
$$\lambda(T)=0.01343-2.965\times10^{-5}\cdot T+1.614\times10^{-7}\cdot T^2$$
$$-6.911\times10^{-11}\cdot T^3+8.246\times10^{-15}\cdot T^4$$
$$Pr(T)=0.9203-7.195\times10^{-4}\cdot T-2.173\times10^{-6}\cdot T^2$$
$$+2.069\times10^{-9}\cdot T^3-6.584\times10^{-13}\cdot T^4.$$

Radiative Nusselt number for the annulus:

$$Nu_r=\frac{2(\alpha-1)}{(\theta_w-\theta_b)}\cdot\frac{\tau_0}{N_R}\int_0^1\left(X+\frac{1}{\alpha-1}\right)\left[-4\frac{K_p(\theta)}{K_p(\theta_w)}\left(\theta+\frac{T_2}{T_1-T_2}\right)^4\right.$$
$$\left.+\frac{2}{\{(\alpha-1)X+1\}}\frac{\left(\frac{T_1}{T_1-T_2}\right)^5}{\left(\theta+\frac{T_2}{T_1-T_2}\right)}+\frac{2\alpha}{\{(\alpha-1)X+1\}}\frac{K_p(\theta_2)}{K_p(\theta_w)}\cdot\frac{\left(\frac{T_2}{T_1-T_2}\right)^5}{\left(\theta+\frac{T_2}{T_1-T_2}\right)}\right]dX.$$

Convective Nusselt number for the annulus:

$$Nu_c=\frac{-2}{(\theta_w-\theta_b)}\cdot\frac{\lambda_w}{\lambda_b}\left(\frac{\partial\theta}{\partial X}\right)_{X=0},\quad \theta_b=\frac{\int_0^1\theta u^+\left(X+\frac{1}{\alpha-1}\right)dX}{\int_0^1 u^+\left(X+\frac{1}{\alpha-1}\right)dX}.$$

Total Nusselt number:

$$Nu_t=Nu_r+Nu_c.$$

Radiative Nusselt number for the circular tube:

$$Nu_r=\frac{8\tau_0}{N_R(\theta_w-\theta_b)}\int_0^1(1-X)\left[-\frac{K_p(\theta)}{K_p(\theta_w)}\left\{\theta+\frac{T_i}{T_w-T_i}\right\}^4+\frac{\left\{\frac{T_w}{T_w-T_i}\right\}^5}{\left\{\theta+\frac{T_i}{T_w-T}\right\}}\right]dX.$$

Convective Nusselt number for the circular tube:

$$Nu_c=\frac{-2}{(\theta_w-\theta_b)}\cdot\frac{\lambda_w}{\lambda_b}\left(\frac{\partial\theta}{\partial X}\right)_{X=0}\qquad \theta_b=\frac{\int_0^1\theta u^+(1-X)dX}{\int_0^1 u^+(1-X)dX}.$$

3. Results of numerical calculations and summary

A great many numerical calculations in detail were performed by Kawachi and Koshin. The energy equation and boundary conditions (2), (3) for the concentric annulus, and (4), (5) for the circular tube were solved numerically by finite difference methods, 0 (10^{-5}).

A fee examples of the results of these many numerical calculations are shown as follows:

Summary in circular tube ($Re = 5 \times 10^4 \sim 10^6$):

The influence of the temperature dependence of K_p, λ and Pr in a circular tube are great at the center side in the steady state. The influence of the large values of T_w/T_i, τ_0, z and t for the thermal radiation in a circular tube is great.

Summary of the annulus ($Re = 5 \times 10^4 \sim 10^6$):

The profile of the temperature difference between radiation and nonradiation, and the absorption of radiation in an annulus tend to approach the results of Nichols [1]. This suggests that the above calculation results for the annulus mentioned may be accurate.

The development of temperature distributions combined with thermal radiation in an annulus are notable for large z, large $\tau_0/N_R \cdot Pr$, long time periods and small ratios of radius in the annulus.

The radiative Nusselt number Nu_r and the development of temperature distributions are notable for small ratios of wall temperatures in the annulus.

The radiative Nusselt number Nu_r and the mixed mean temperature in an annulus is the case of temperature-dependent properties of K_p, λ and Pr are smaller than those in the case of constant properties.

The numerical solutions of transient energy equations combined with thermal radiation and turbulent flow in the annulus with temperature-dependent properties are given for the thermal entrance region with heating at the inner surface the annulus. The effects of radiation on the temperature profile and Nusselt number are calculated for radius ratios, 1.25, 5 and 80 in the annulus.

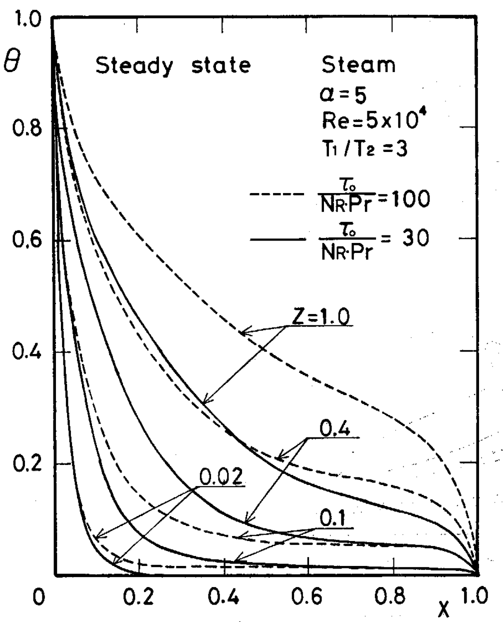

Fig. 3 Temperature profile in steady state in an annulus.

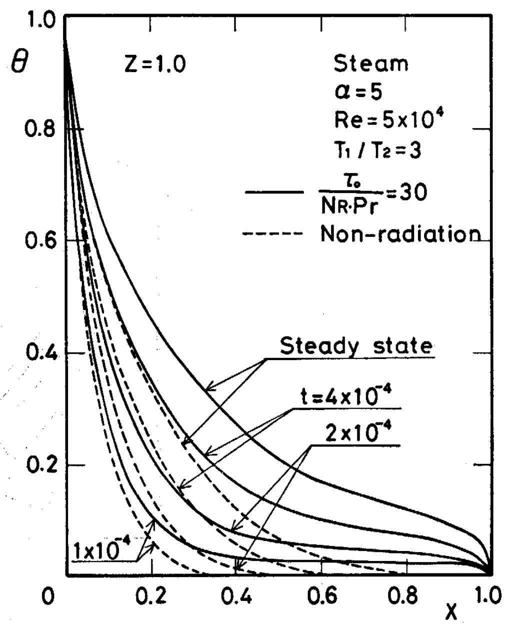

Fig. 4 Temperature profile in transient state in an annulus.

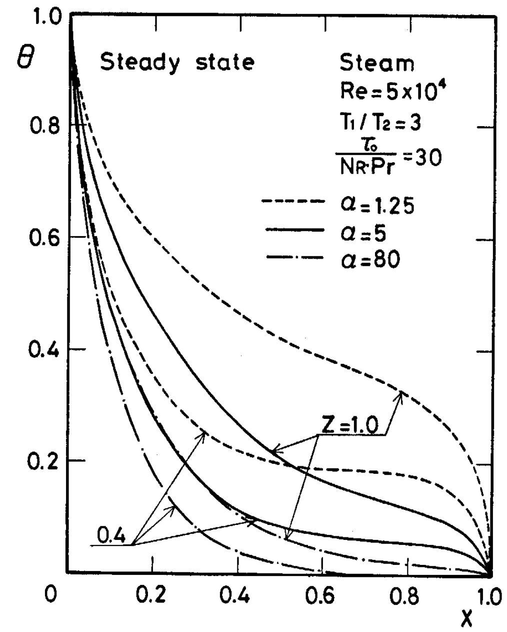

Fig. 5 Variation of temperature profile with radius ratio in an annulus.

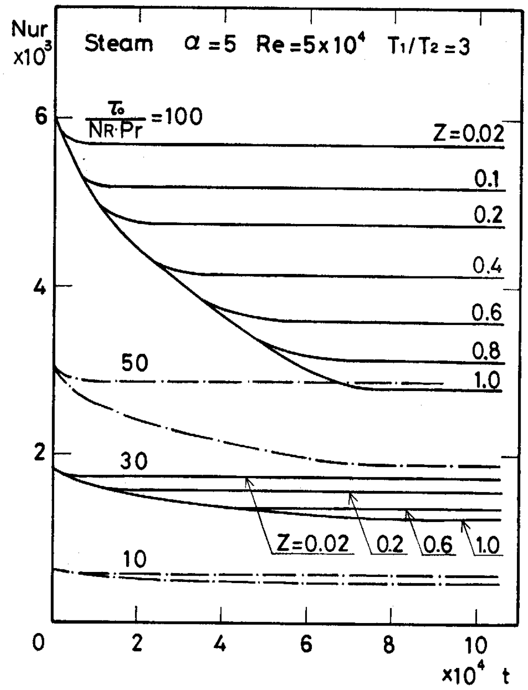

Fig. 6 Variation of radiative Nusselt number in transient state in an annulus.

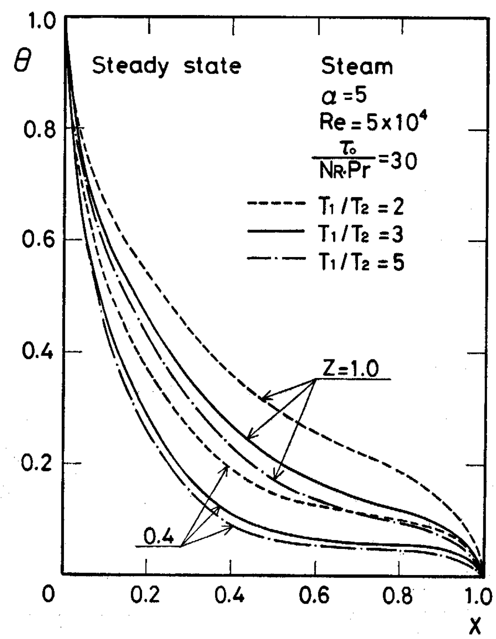

Fig. 7 Variation of temperature profile with T_1/T_2 in an annulus.

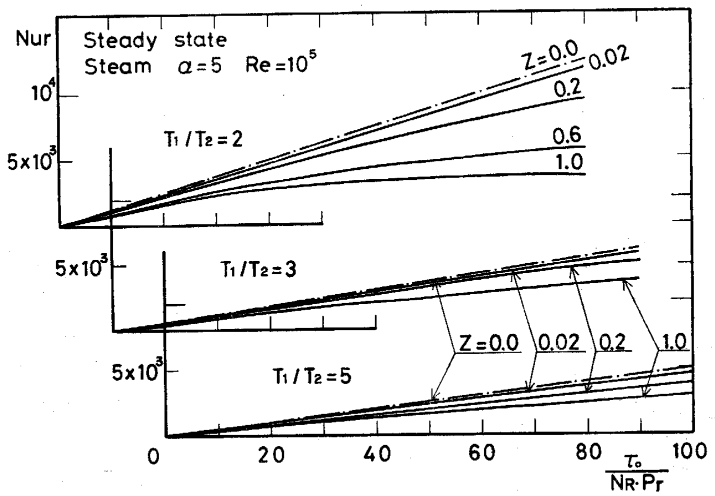

Fig. 8 Variation of radiative Nusselt number with radiative parameter $\frac{\tau_0}{N_R \cdot Pr}$ in an annulus.

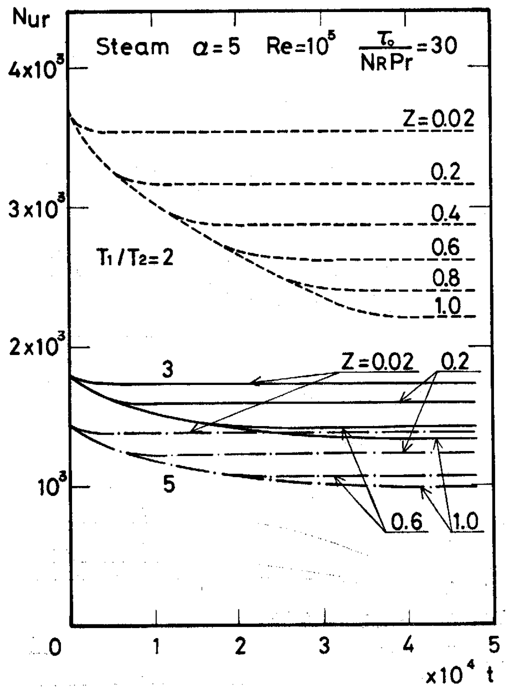

Fig. 9 Variation of radiative Nusselt number in transient state in an annulus.

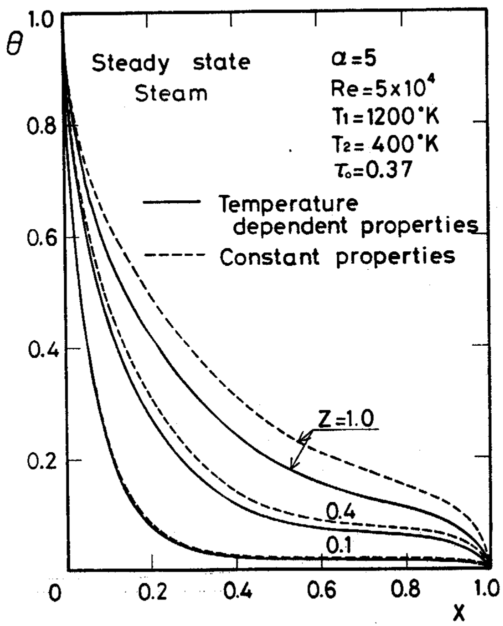

Fig. 10 Comparison of temperature profile for constant properties and for temperature dependent properties in an annulus.

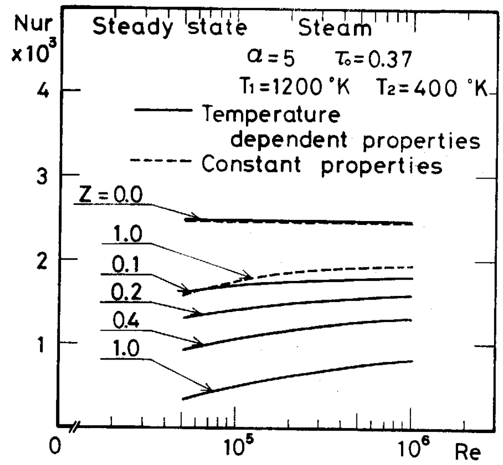

Fig. 11 Variation of radiative Nusselt number with Reynolds number in an annulus.

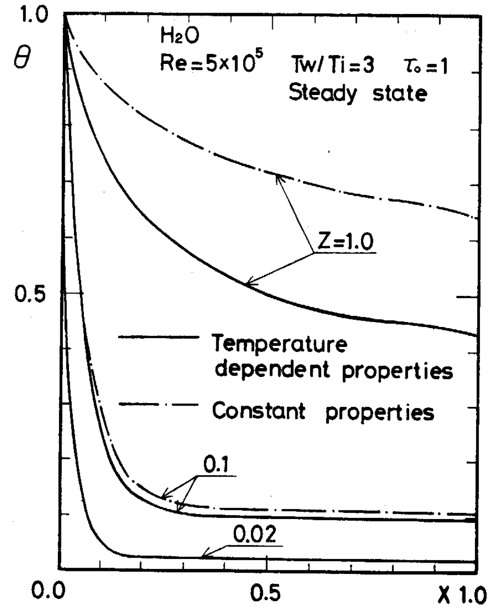

Fig. 12 Comparison of temperature distribution for constant properties and temperature dependent properties in a circular tube.

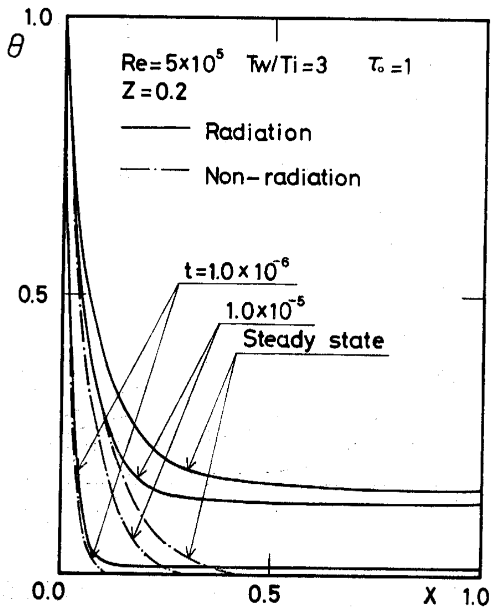

Fig. 13 Comparison of temperature distribution for radiation and non-radiation in a circular tube.

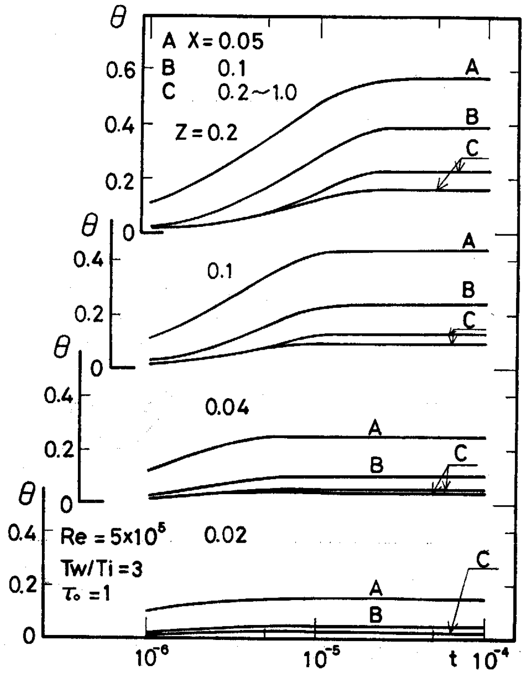

Fig. 14 Temperature dependent time in a circular tube.

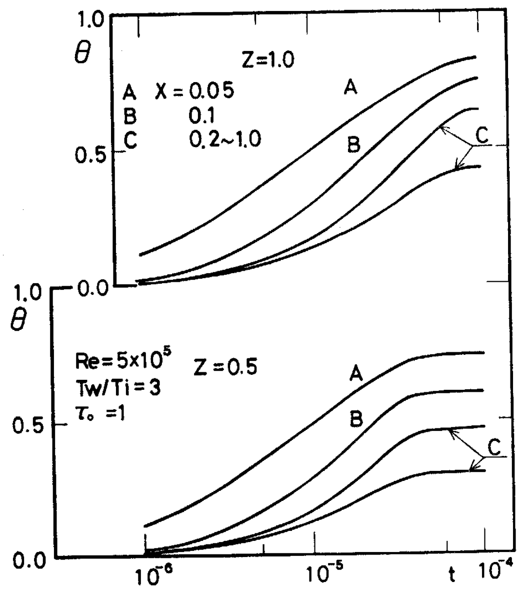

Fig. 15 Temperature dependent time in a circular tube.

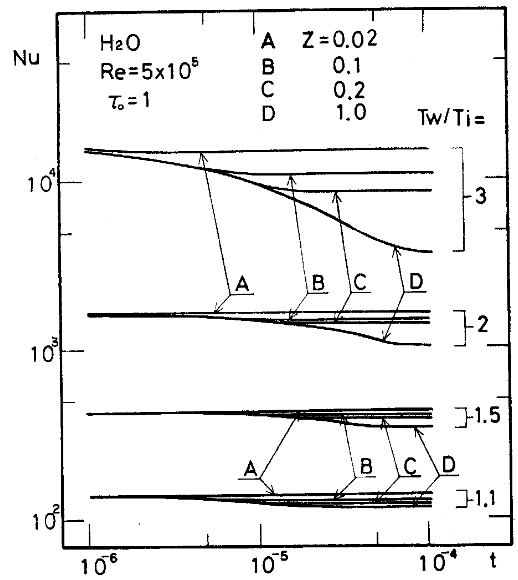

Fig. 16 Total Nusselt number dependent time in a circular tube.

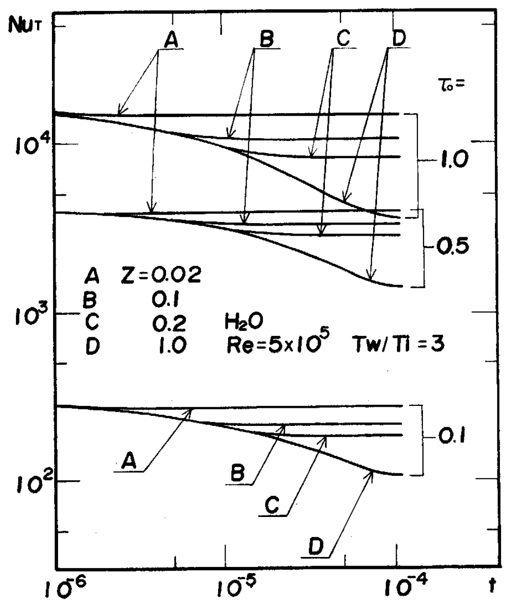

Fig. 17 Total Nusselt number dependent time in a circular tube.

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