The Drop-size Distributions in Well-developed Convective Rainclouds : Part 2

Numerical experiments on the generation of the terminal state of drop-size distribution

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Abstract

Generation processes of the characteristic raindrop-size spectra reported in Part 1 of this paper were investigated by numerical experiments. Intermittent supply of liquid water content with a certain size spectra was introduced in the well mixed cloud model under the action of drop breakup and coalescence, corresponding to the observation fact of combination of cell echoes by radar. The results represent well the terminal state of size distribution as observed at Hitoyoshi, which is characterized by the numerous space density of drops smaller than 1 mm diameter, the high population of large drops, and the flat shape in medium size range. Furthermore, the very flat distribution which is frequently observed at the beginning stage of shower rain was obtained by the one dimensional sedimentation model in which the rain with the stationary distribution was released at the upper level. The $Z-R$ relation derived from the above computational results may be well represented by $Z=383R^{1.6}$ which was obtained in Part 1.

1. Introduction

The size distributions of drops from the developed convective rainclouds, observed at Hitoyoshi, revealed the remarkable departure from the Marshall-Palmer distribution[6] as described in Part 1[8] of this report. They were characterized by the more abundant population of small and large size raindrops as compared to the M–P. Especially in the medium size range they appeared to be rather flat. Furthermore, the profile of the distribution seems to move in parallel according to its rainfall rate or liquid water content. The writer suggested that such a characteristic change of the size distribution might be the result of the combination of raincloud cells or bubbles that came through the different history of growth process with each other. The facts that the cell echoes coalesced and increased the radar reflectivity may support strongly this model.

Another important fact from RHI observations, that the high reflectivity used to be seen in the lowest part of the convective echo, suggested that the process of coalescence of raindrops might be the main role. This may be easily considered also from the characteristic Z–R relationship due to the high concentration of large drops. Srivastava[9] already reported the results of the numerical experiments on the stationary drop-size distributions under the action of drop breakup and coalescence by means of the stochastic process. The resulted distributions are quite similar to the observed at Hitoyoshi as shown in Part 1, while the computation was done under the spatially homogeneous condition. This condition would be easily admitted in the well mixed cloud by turbulence like as a convective cloud.

In this paper, furthermore, the effects of intermittent supply of liquid water content corresponding to the fact of the coalescence of rain-cells will be studied under the same procedure of Srivastava's experiment. And another purpose is to investigate the effects of sedimentation of drops and the vertical air current on the evolution of drop-size distribution in the falling rain-cell. This is related to the observation fact that the most flat distribution used to be found at the beginning of a shower. To this end, the transport equation, which has a sedimentation term in addition to the terms in the stochastic equation, will be used. The computational results from such a transport equation will give some information for the doppler radar system.

2. Combination effect of raincould cells

a) Method of calculation

The governing equation for this problem is applied by the same one as for the well-mixed could used in Srivastava's experiment. But where the combination effect is introduced as the intermittent collision of two rain cells. Thus, the combined or coalesced cell would gradually increase its liquid water content by colliding cells. Time interval of occurrence of collision might be due to a size of raincould cell considered. The smaller cells are considered in the system and the more frequency of collision will be expected.

In the following computations, some different cases of the time interval of collision and the liquid water content of a colliding cell are considered. Where the initial size distribution and liquid water content of the growing cell are assumed to be equal to those of a colliding cell which are unchangeable in time. And their dropsize distribution is given by the M–P.

The equation is written as Eq. (1), adopting the notation system of Srivastava.

$$\frac{\partial f(x,t)}{\partial t}=\int_{x_0}^{\frac{x}{2}} f(y,t)f(x-y,t)K(y,x-y)\,dy - f(x,t)\int_{x_0}^{x_{max}} f(y,t)K(x,y)\,dy - f(x,t)P(x) + \int_{x}^{x_{max}} f(y,t)Q(y,x)P(y)\,dy \qquad \cdots\cdots\cdots(1)$$

where x : mass of raindrop

$f(x, t)$: the number density of drops of mass x at time t
y : mass of dummy variable of integration
x_{max} : the largest mass drop allowed in the system
x_0 : the smallest mass drop considered in the system
$K(x, y)$: the collection kernel of x and y
$P(x)$: the probability of mass x disintegrating in unit time
$Q(y, x)$: the production rate of the drops of mass x formed by the breakup of one drop of mass y
$P(x)Q(y, x)$: the breakup kernel of y to form x.

All of the functions described above adopt the forms given by Srivastava, because this paper also concerns with the drops greater than 0.2 mm diameter. So that,

$$K(x, y)=\pi(r+r_0)^2|V-V_0| \qquad \text{.........(2)}$$

$$P(x)=2.94\times10^{-7}\exp(34r) \qquad \text{.........(3)}$$

$$Q(y, x)=\frac{ab}{3x}\left(\frac{r}{r_0}\right)\exp\left(-b\frac{r}{r_0}\right) \qquad \text{.........(4)}$$

$$a=62.3,\ b=7$$

are used. Eq. (2) is the so-called geometric sweepout function, where r, r_0 and V, V_0 are the radii and the terminal velocities corresponding to the drops of mass x and y. The fall velocity was given by the values of Gunn and Kinzer[3]. Eq. (3) was given by Komabayasi et al.[4] and Eq. (4) was found by Srivastava by reanalizing the data reported by Komabayasi et al. to satisfy the water conservation, that is $y=\int_{x_0}^{y} xQ(y, x)dx$.

Without requiring a ridiculous amount of computer time, numerical procedure to solve the stochastic collection equation was first found by Berry[1] transforming x to J to represent mass on a logarithmic scale. Applying this system for Eq. (1) was then developed by Nelson[7] and Srivastava. The accuracy of the computations was checked by the degree to which the total water content is conserved. The water content did not change by more than 5% in any of the calculations in this paper.

In the following cases of computation, the drop diameter range 0.2–10 mm is considered, and the fall velocity of drops greater than 6.4 mm diameter is set equal to that of 6.4 mm. The breakup of drops smaller than 5 mm diameter is not considered in the system, because ordinarily the drops are unstable in larger size than 5 mm diameter. The calculation was done with time step of 40 sec.

b) Results

Fig. 1 shows the results of the computations on the combination effect of cells, in the cases of a) the size distribution of both of the initial growing cell and the colliding cell was given by the M–P curve of $R=14$ mm/hr, and the colliding cell was allowed to come across the growing cell at every 400 seconds, b) similarly the M–P 6 mm/hr and at every 200 seconds, c) also similarly the M–P 1 mm/hr and at every 80 seconds. It is fairly seen that the more the growing cell has the liquid water content and the faster the size distribution becomes to be the stationary of Srivastava, in the size range larger than 1 mm diameter, as shown at the time of 1240 sec., 2040 sec., and 2440 sec. corresponding to the case A, B and C, respectively. The concentration of drops smaller than 1 mm diameter remains higher in the case that the colliding cell has less liquid water content, for example, at 2040 sec. and 2440 sec. in the case C, which seem to be very similar to the observed distributions at Hitoyoshi as shown by Fig. 5 (b) and (g) in Part 1. The former was brought from the large scale coalescence of two line type echoes and the latter was from the coalescence of two medium convective echoes. Therefore, the frequent coalescences of smaller bubbles with less liquid water content ("bubble" was proposed by Ludlum[5]) in cumulonimbus mecha-

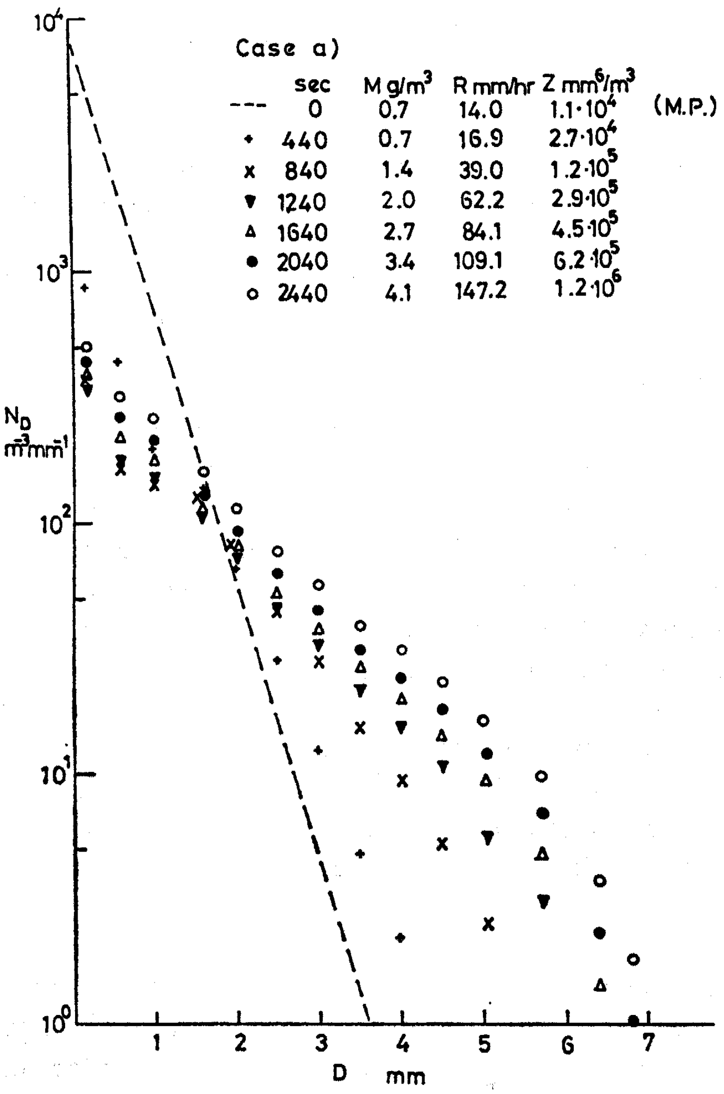

Fig. 1-a

nism) may be more important in the coalescence phenomena of radar echoes.

Thus it might be suggested the developing bubble has a great appetite to the supplied liquid water content from another bubble, and then will be a larger bubble containing a great amount of liquid water content, on the falling way to the earth surface. This is suitable for the fact that the highest radar reflectivity used to be seen in the lowest part of the convective echo at Hitoyoshi. Furthermore, to the numerous population of drops smaller than 1 mm diameter in the heavy rainfall at Hitoyoshi, the existence of the warm layer cloud will be much responsible when those developed bubbles rush down, by the large amount of supply of small drops in it.

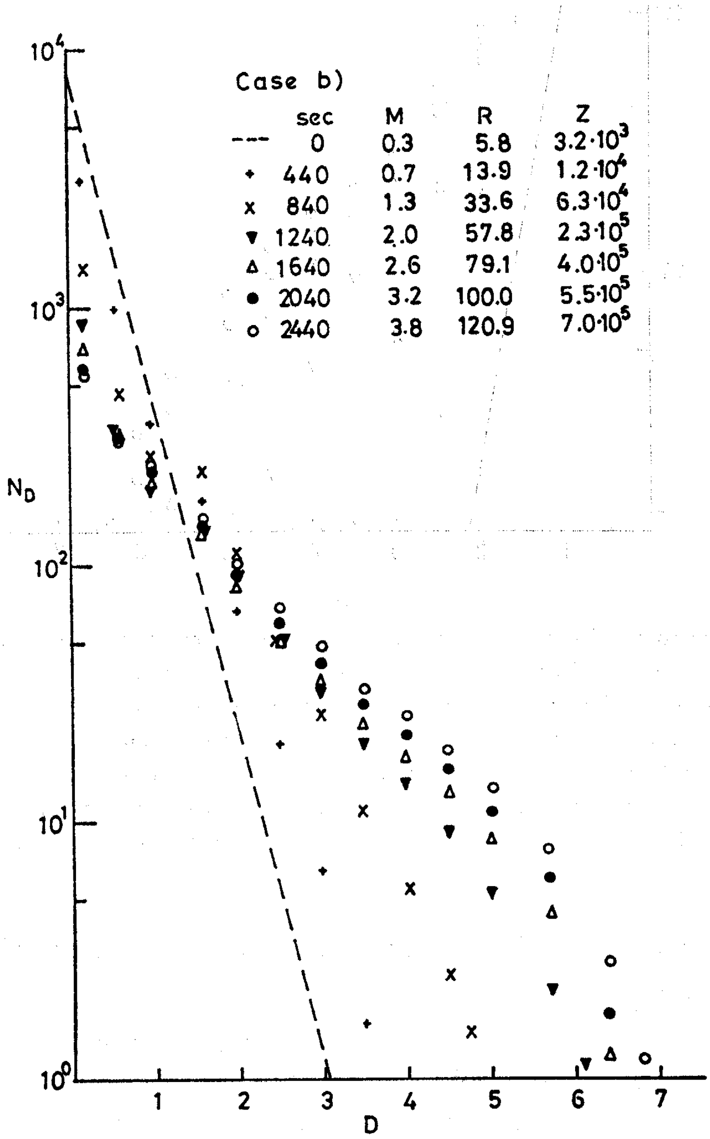

Fig. 1-b

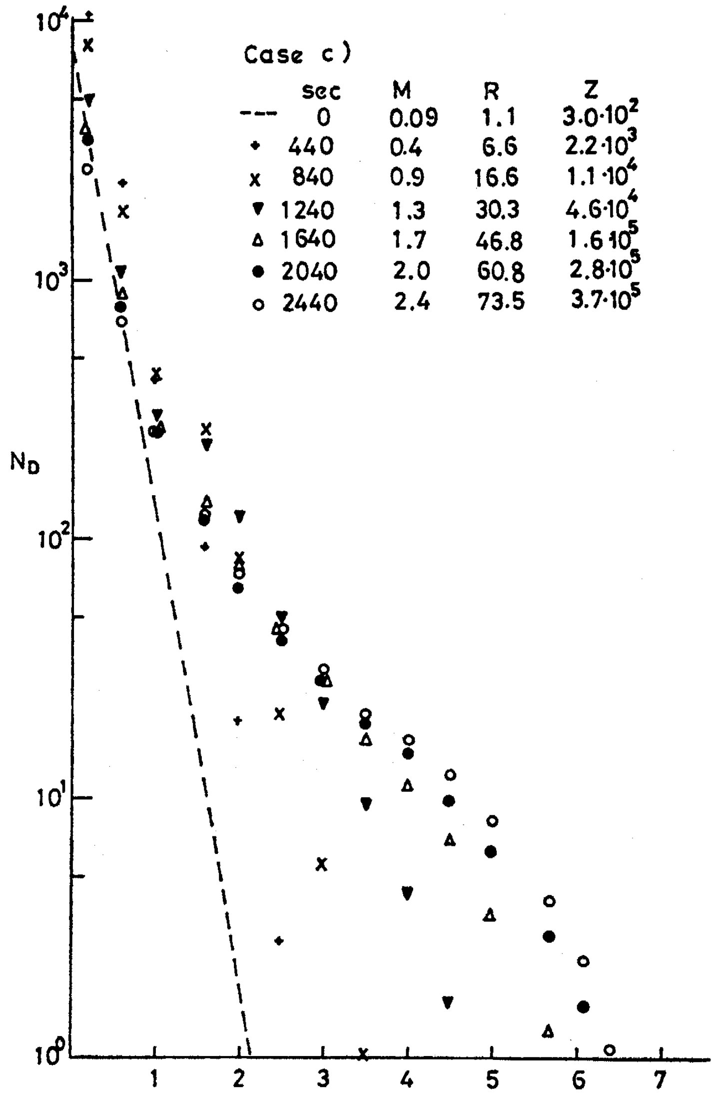

Fig. 1-c

Fig. 1 Evolution of drop-size spectra in the intermittent supply of liquid water content of colliding rain bubble which has a certain drop size distribution, under the action of drop breakup and colescence in the one dimensional model.
a) The initial spectra of the growing cell and the colliding bubble was given by M.P.14 mm/hr distribution. Colliding was made at every 400 sec.
b) Samely as a), by M.P.6 mm/hr distribution and at every 200 sec.
c) Samely as a), by M.P.1 mm/hr distribution and at every 80 sec.

3. Effect of vertical drop transport in one-dimensional model

a) Computational procedure

Spatial size distribution of raindrops is one of the most important information for the doppler radar system. As described in Part 1, in RHI scope we can frequently see the most intensity part of radar reflectivity near the bottom and at the downwind side of a developed convective echo. Those intensified echoes bring the drops of the beginning stage of the rainfall, and the size distribution is characterized with many large drops. In the section 2, the growth process of larges drops were discussed under the drop breakup and coalescence in the developing bubble of which liquid water content was increasing by the colliding with the other bubble. In this section, it will be discussed that an influence of vertical transport of drops on the characteristic distribution at the beginning of rainfall.

Recent development of the numerical studies for the sedimentation or transport equation was made by Warshaw[11] and Nelson[7]. Sedimentation or transport is accounted for by simply adding a term $-\frac{\partial}{\partial Z}\{(U-V)f(x)\}$ to the right hand side of Eq. (1), where U is the air updraft velocity in the cloud. A time step of 10 sec. and a height interval of 500 m were used in the following computation, in which the drop size range was considered with diameter from 200 μ to 8 mm. Stability of computation was checked by computing the total amount of liquid water in the system. Actually the total redistributed mass never exdeeded 5%. The vertical dimension in the model is represented by 9 or 10 space levels with a ΔZ of 500 m; the level 1 represents the layer from the ground surface to the level of 500 m height; and similarly the level 10 for the layer from the level of 4500 m to one of 5000 m. And the level 1 was retained as a rain accumulator to conserve total mass within the system.

b) Results

Fig. 2 shows the altitude change of drop-size distribution at 300 sec. after the onset of rain at level 9 and 10. The intial size distribution of rain was given by the M–P 100 mm/hr at the levels 9 and 10, and by the M–P 1 mm/hr for the layer between the level 1 and 8. In the figure are also shown the values of L.W.C. (g/m^3), R(mm/hr), Z(mm^6/m^3) and D_0 (mm) at each level 1∼10. If the D_0 represents the average fall speed of rain roughly, the product of the fall speed of D_0 drop and time of 300 sec. might give the fall distance of a center of rain. From Fig. 2, as almost of D_0 exceed 3 mm of which fall speed is about 8 m/s, the fall distance may be about 2.4 km at 300 sec. As shown in the figure, as the center of rain (initially at 4.5 km height) is seen at level 4∼5 or about 2 km height, the fall distance is resulted in 2.5 km. Thus, the agreement of the fall distance approves to adopt $\Delta t=300$ sec. and $\Delta z=500$ m reasonable.

It is distinctly noted from the figure that the obtained distribution reveals remarkable different to the corresponding M–P's, and quite similar to the observed at Hitoyoshi. Especially, the distribution of level 1 at 300 sec. shown by the symbols of (●) (accumulated distribution of this level are shown by the symbols of (○)) appears flat in medium size range and resembles to the instantaneous distribution at Hitoyoshi that was frequently observed at the biginning stage of shower. The examples of those observed flat distributions were described in the figure in Part 1, and here again another example which has high rain intensity is shown in Fig. 3. In Part 1 those instantaneous flat distributions

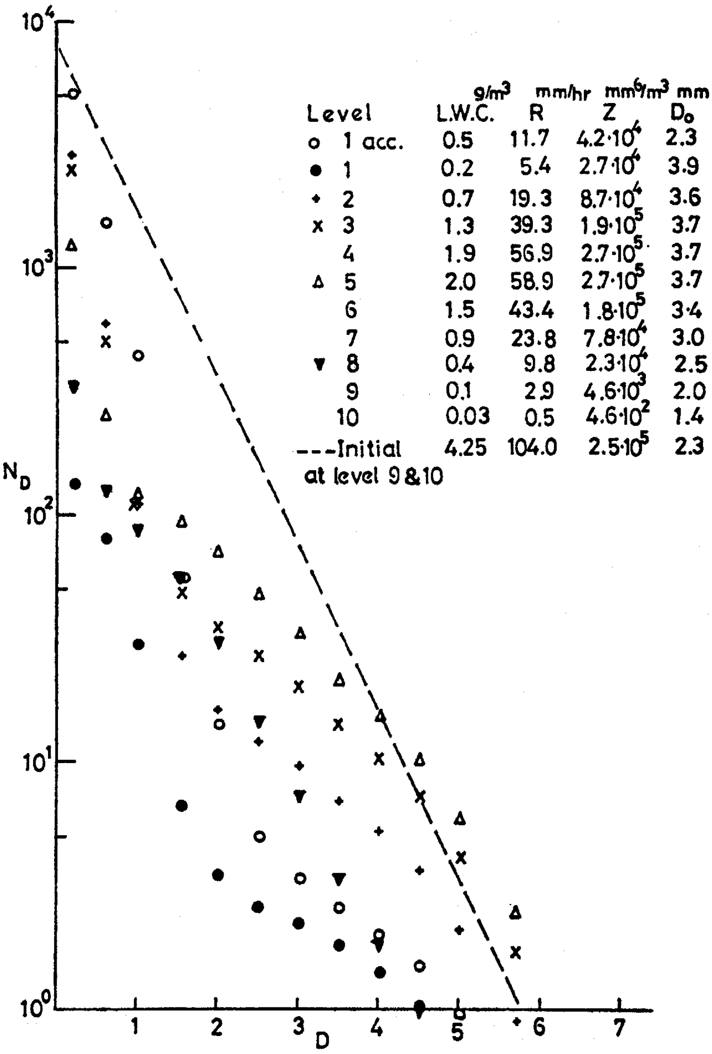

Fig. 2 Altitide change of drop-size spectra computed by the one dimensional sedimentation model under the action of drop breakup and coalescence. The rain of M–P 100 mm/hr distribution was released at the level 9 and 10 (4500~5000 m), and the figure show the results at the time of 300 sec after it.

were discussed little, because their sampling volume were not statistically enough. But such a characteristic feature of distribution might be fairly expected by the sedimentation effect.

The more flat distribution can be obtained by using the e^{-AD^2} distribution for the initail rain, instead of the M–P distribution in Fig. 2. The result is shown in Fig. 4, where the numerical procedures are same as in the case of Fig. 2, except that the initial size distribution was given by $N_D=N_0\ e^{-AD^2}$ ($N_0=200$, $A=1.2\,R^{-0.5}$, $R=100$ mm/hr). (a) is for 240 sec. after the onset of rain level 9 and 10, and (b) is for 300 sec. There we can find the more flat distribution in the lower levels. The distribution at the level 1 in Fig. 4, (b) seems to be completely flat in the range of 200 μ~5 mm diameter. And at the level 1 in

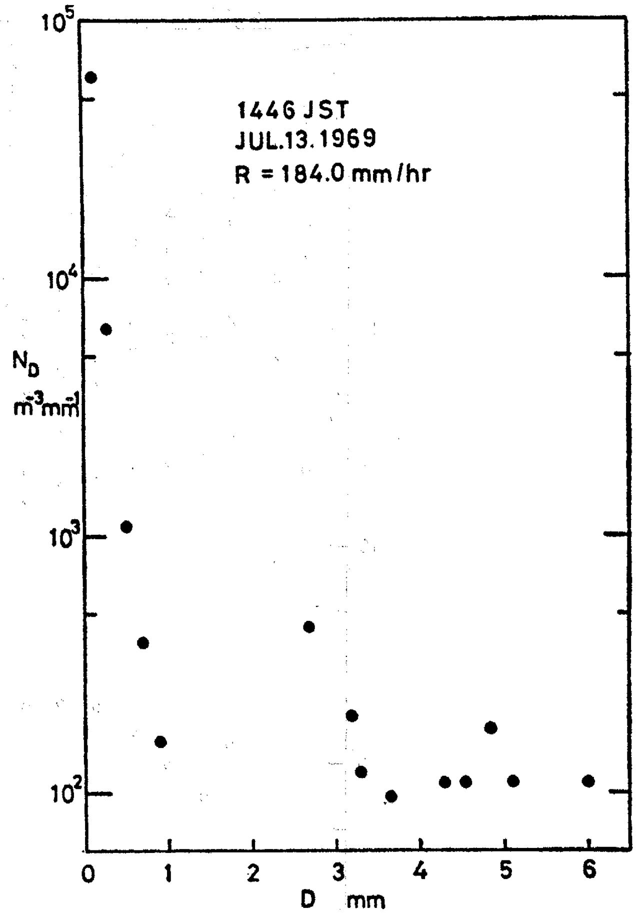

Fig. 3 Example of the flat size distribution of drops from developed convective rain in summer, sampled in the instantaneous exposure 1 sec.

Fig. 4, (a), the calculated distribution shows a great dip in the range between $1 \sim 2$ mm diameter, and a rather symmetrical mount in larger size range than 2 mm diameter. Neglecting the rain intensity and the order of N_D values, this feature seems to correspond to Fig. 3, where the observed distribution has no drops between $1 \sim 2.5$ mm diameter.

Next, we see the effect of vertical air current on the evolution of size distribution. Fig. 5 shows the result in thecase that the initial size distribution was given by the M–P 100 mm/hr at the level 8 and 9, and no clouds between the level 1 and 8. And in order to evaluate the effect of drop sorting

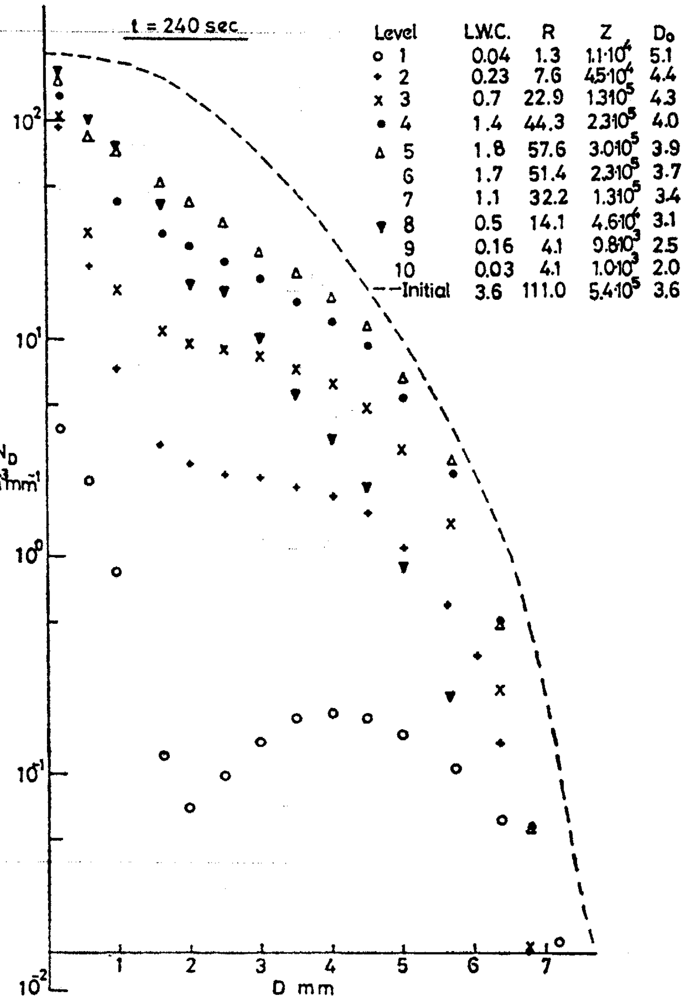

Fig. 4-a

by vertical air current, the rather strong fictitious downdraft below levle 7 and updraft at level 9 were given as shown in the figure; where, for easiness of comparison to each other, the rain intensity at each level was not given by the rainfall flux including the effect of vertical air current, but by the rainfall rate calculated with only the terminal velocity of drop itself. The obtained distribution at each level, except level 9, appeares to be similar to each other because of the rapid disperse of rain by the strong downdraft. Although they are remark-

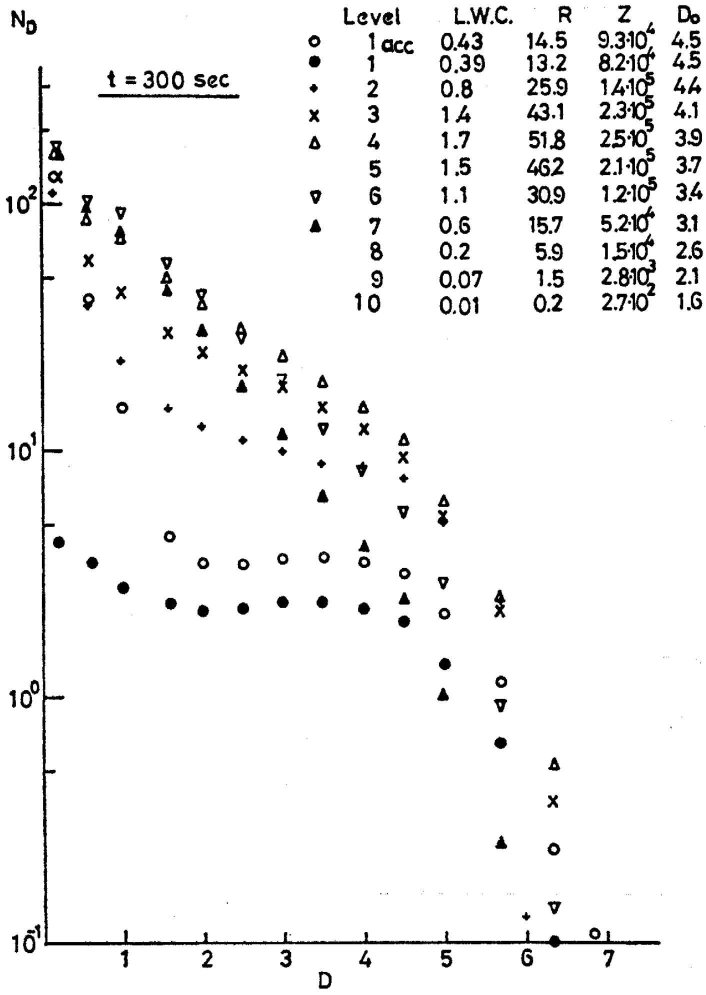

Fig. 4-b

Fig. 4 The results under the same procedures of Fig. 2, but here the drop-size spectra of the bubble at initial state was given by the Hitoyoshi 100 mm/hr distribution ($N_D = N_0\, e^{-AD^2}$).

a) The results at 240 sec after the release of rain at level 9 and 10.

b) Samely, at 300 sec.

edly different from the M–P type, they are not so flat and do not attain to the stationary so much as like Fig. 2. They are much characteristic in the size range smaller than 1 mm diameter; there is found the considerable decrement of number density, conversely to Fig. 2. Such mountlike distributions are rather similar to the monomodal distribution of shower rain which was reported by Fujiwara[2].

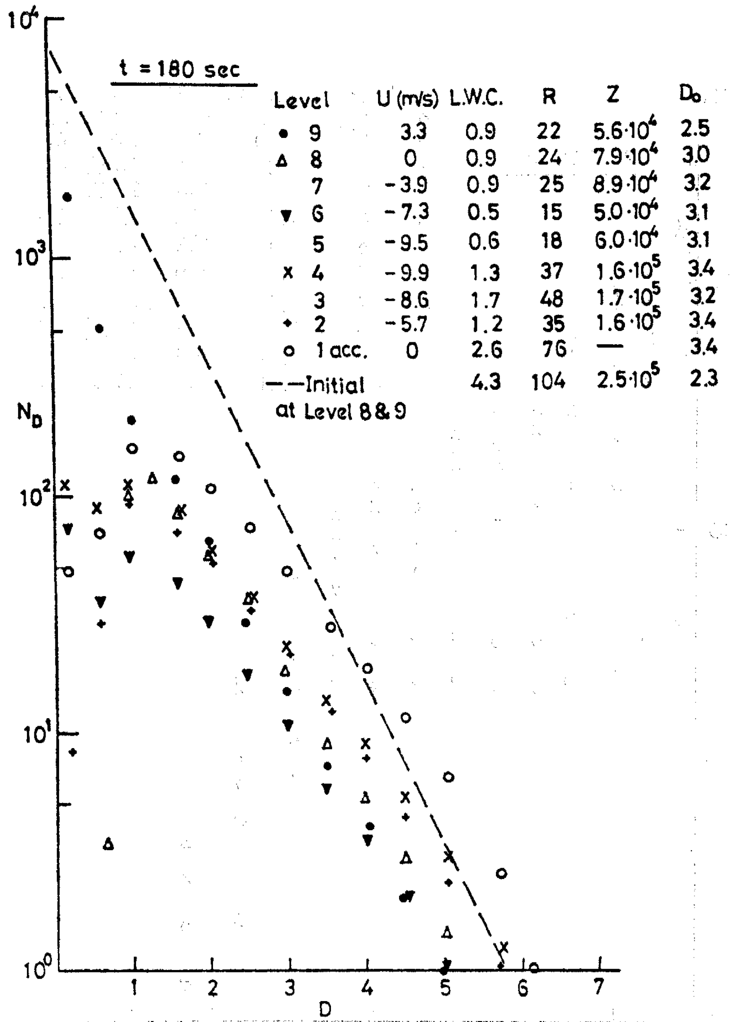

Fig. 5 Effects of the vertical current on the drop sorting by use of the transport equation. The initial rain bubble was set at level 8 and 9 (4000∼4500 m), and was given by the M–P 100 mm/hr distribution. The figure shows the results 180 sec after the release of rain.

4. Considerations and concluding remarks

As described in the preceding sections, the characteristic dropsize distribution observed at Hitoyoshi would be expected from some numerical experiments. In the section 1, the coalescence of bubbles was introduced to the stochastic equation under the processes of drop breakup and coalescence, according to the facts of coalescence of cell echoes at Hitoyoshi and Ludlum's model for the cumulonimbus mechanism. The results might suggest that the coalescence of bubbles, that is, the intermittent supply of liquid water content of a colliding bubble to a growing bubble in a convective cloud, is much responsible to form the terminal state of size distribution with high population of small and large drop, and with nearly flat shape in medium size range. And the frequent coalescence of the smaller rain bubbles may be more important.

Next, in the case that such a stationary distribution is developed at the middle height of atmosphere and then transported to the ground surface in no vertical current, the sedimentation equation was used for studying the time and height change of size distribution. The very flat distribution as like the observed at Hitoyoshi is fairy obtained as in Fig. 4 from the computation. Thus, we can say that an appearance of the flat distribution might propose much important information for the forcasting of heavy rainfall, because the upper air must have prepared the suitable conditions for clouds to make most rain intensity by given liquid water content. This may be also the important information for the radar with other observations of a echo height, a coalescence of cell echoes, and a strong intensity of reflectivity at the lower layer of convective echo.

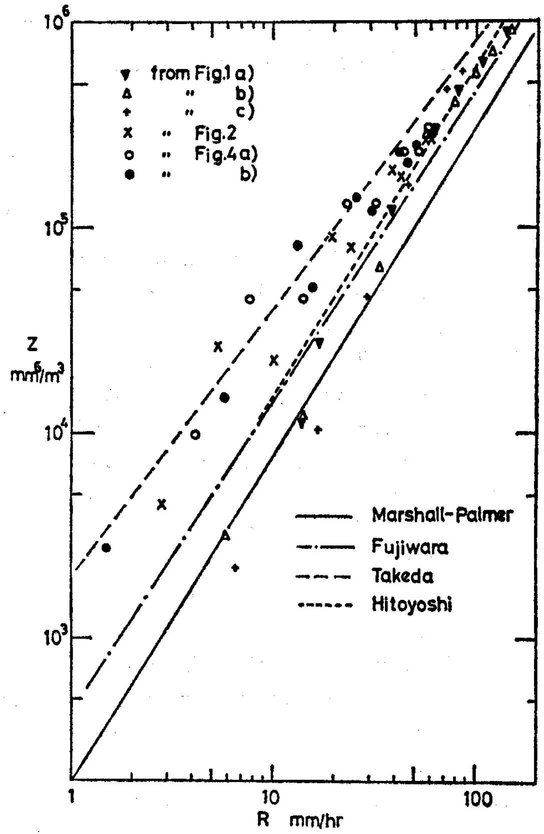

Fig. 6 Correlation of Z-R values in the computational results in this paper, as compared with the various types of Z-R relation

Finally, Fig. 6 is illustrated for the RHI and the doppler system. The figure may show a vertical Z-R relation from the data of Fig. 1, 2 and 4. The plots seem to converge well in the region of medium rainfall rate more than 20~30 mm/hr. The wide scatter of plots in a weak rainfall region is because of the difference of dropsize distribution between upper and lower region out of the center of the falling bubble (as shown in Fig. 4), and also because of the data of initial stage of which distribution does not yet grow to the stationary as shown in Fig. 1. Four Z-R relations are illustrated in the figure for comparison.

They are $Z=200\ R^{1.6}$ from Marshall and Palmer, $Z=450\ R^{1.46}$ for the thunderstorm rain by Fujiwara, $Z=2000\ R^{1.3}$ obtained by Takeda[10] in his numerical experiment on the long-lasting rain coulds, and $Z=383\ R^{1.6}$ at Hitoyoshi described in Part 1. It is seen that almost of the plots in the region of rainfall rate more than 20 mm/hr are within the region between two lines of Fujiwara and Takeda, and the line of Hitoyoshi may be nearly representative for the whole region of plots. Thus, the $Z=383\ R^{1.6}$ obtained from $N_D=N_0e^{-AD^2}$ distribution observed at Hitoyoshi may be well applicable for the vertical Z–R relation of convective echo.

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References

1) Berry, E. X., 1967: Cloud droplet growth by collection. J. Atmos. Sci., **24,** 688–701.
2) Fujiwara, M., 1965: Raindrop-size distribution from individual storms. J. Atmos. Sci., **22,** 585–591.
3) Gunn, R., and G. D. Kinzer, 1949: The terminal velocity of fall for water droplets in stagnant air. J. Meteor., **6,** 243–248.
4) Komabayashi, M., T. Gonda and K. Isono, 1964: Life time of water drops before breaking and size distribution of fragment droplets. J. Meteor. Soc. Japan, **42,** 330–340.
5) Ludlum, F. H., 1966: Cumulus and cumulonimbus convection. Tellus, **XVIII,** 678–698.
6) Marshall, J. S., and W. M. Palmer, 1948: The distribution of raindrops with size. J. Meteor., **5,** 165–166.
7) Nelson, L. D., 1971: A numerical study on the initiation of warm rain. J. Atmos. Sci., **28,** 752–762.
8) Shiotsuki, Y., 1973: Drop-size distributions in well-developed convective rainclouds: Part 1. Technology Rep. of the Yamaguchi Univ., Vol. 1, No. 2, 211–232.
9) Srivastava, R. C., 1971: Size distribution of raindrops generated by their breakup and coalescence. J. Atmos. Sci., **28,** 410–415.
10) Takeda, T., 1971: Numerical simulation of a precipitating convective cloud: The formation of a "long-lasting" cloud. J. Atmos. Sci., **28,** 350–375.
11) Warshaw, M., 1967: Cloud droplet coalescence: Statistical foundations and a one-dimensional sedimentation model. J. Atmos. Sci., **24,** 278–286.