The Talbot Type Formula and N_D Distributoin

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Abstract

The raindrop-size distribution (N_D distribution) is one of the most important meteorological factors because by using it we can obtain many other rain parameters. Recently Shiotsuki proposed the new method which determines the N_D distribution by using 10 minutes rainfall intensity data only. So the authors inspected this method by using the data of Western-Japan heavy rainfall, Nagasaki heavy rainfall, Ube, and seven locations in the Philippines.

So the authors found the regional characteristics of a, b values included in Talbot type formula, which are the most basic in the Shiotsuki's method . And also found some ralationships between the values of a, b and N_D distribution.

1. Introduction

The raindrop-size distribution (N_D distribution) is one of the most basic meteomoelogical factors by which it is possible to determine not only the rainfall intensity but also the other rain parameters. As the rainfall intensity is the most popular meteorological factor, the observations have been taken at many places. Therefore if we can determine the N_D distribution by using the data of rainfall intensity, we can obtain various rain parameters not only for the present but also for the past. Another methods of determining the N_D distribution have been proposed. Recently, Shiotsuki proposed the new method (Shiotsuki : 1985) which use 10 minutes rainfall intensity data only. In this paper, the authors inspected Shiotsuki's method by using the data of heavy rainfall in western-Japan (1967), Nagasaki (1982), Ube (1980), and seven locations in the Pillippines. The authors also invesigated the regional characteristics of a, b value included in Talbot type formula, which is the most basic in Shiotsuki's calculation method.

2. The method of determining the N_D distribution.

The following is the method by which N_D distribution is determining by using 10 minutes rainfall intensity data only.

2. 1 The basical characteristics of the model.

2. 1. 1 When we consider the actual survey data of N_D distribution, it is unnatural to suppose that the rain comes from one type of raindrop group. So we assume that a raincloud is made up from three different raindrop groups which have their own growth

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process.

2. 1. 2 Each raindrop group has its own maximum rainfall intensity curve (R_T curve), and the maximum rainfall water content curve (M_T curve) which has a close relationship with R_T. A R_T curve is shown in Fig. 1.

2. 1. 3 Because of the running out of the raindrop from each raindrop group, the water content of each group must be decreased with time, but we can supposed that the average diameter of the raindrops never change in one rainfall. Fig. 2 is the model by which this is shown. As we mentioned, the three rainfall groups have respectively water content (M), average diameter of raindrops ($\bar{D}$), decrement of water content (λ) which is related to the duration.

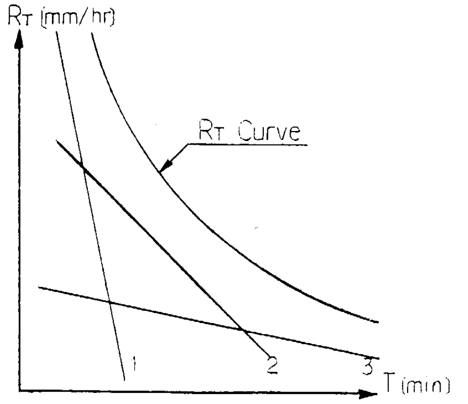

Fig. 1 R_T curve and 3 raindrops groups.

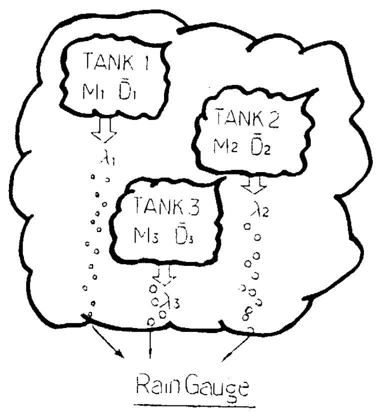

Fig. 2 Model of the raindrops groups.

2. 2 The type of R_T curve.

Some types of R_T curve have been suggested, but in this paper, we use Talbot type R_T curve because we found it was best when we analyzed the heavy rainfall data of Nagasaki and Western-Japan.

$$R_T = \frac{a}{T+b}$$

R_T : T minute rainfall intensity (mm/hr)

T : time (min)

a, b : parameter

2. 3. The determination of decrement.

We decide that the decrement of water content is shown by the index decrement type ($e^{-\lambda T}$). For example, we suppose that the second raindrop group governs the characteristics of R_T curve and gives decrement λ_2 which is related to the rainfall's life. The life of one rainfall (TT min) is so defined that it starts when 1 minute rainfall intensity is over 1mm/hr and ends under 1mm/hr.

$$r_{TT} = TT \times \frac{a}{TT+b} - (TT-1)\frac{a}{TT+b-1}$$

$$= 1 \text{ mm/hr}$$

$$TT = \frac{1-2b \pm \sqrt{4ab+1}}{2} \quad \cdots\cdots (1)$$

The value of equation (1) is never mimns because generally the value of a is 100～40000, b is 1～200. It can be said, in the R_T curve, when T=0 min, the maximum momentary rainfall intensity R_O is a/b and when T=TT min, the maximum TTmin rainfall intensity, R_{TT} is a/ (TT+b). So the ratio of the two is follwing.

$$\frac{R_{TT}}{R_0}=\frac{b}{TT+b} \qquad (2)$$

And this is same ratio of M_{TT}/M_O and $M_T=M_Oe^{-\lambda T}$, so we can obtain λ_2 in the following equation.

$$\frac{R_{TT}}{R_0}\fallingdotseq\frac{M_{TT}}{M_0}=e^{-\lambda_2 TT}$$

$$\lambda_2=\frac{-\ln\left(\frac{\mathrm{a}}{\mathrm{TT}+\mathrm{b}}\right)}{\mathrm{TT}} \qquad (3)$$

As for λ_1, λ_3, it is ideal to select the best value by trial and error, but in this paper we use following equation which was computed by the actual raindrop filter paper observation of 16 locations.

$$\lambda_1=0.8310/b^{0.7523} \qquad (4)$$

$$\lambda_3=\lambda_2/(\lambda_1/\lambda_2) \qquad (5)$$

2. 4. Determination of the water content, M and the average of raindrops $\overline{D}$.

N_D distribution is determind by the following equation which was shown by Shiotsuki.

$$N_D=\frac{6000}{\rho\pi}MD^{-3}\frac{1}{\sqrt{2\pi}\sigma}e^{-(D-\overline{D})^2/2\sigma^2} \qquad (6)$$

N_D : The spatial density of raindrop ($m^{-3}mm^{-1}$)
D : The diameter of the raindrop (mm)
M : The spatial water content of the raindrops (g/m^3)
$\overline{D}$: average of the diameter (mm)
σ : The standard deviation from $\overline{D}$ (mm)

$\frac{M}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(D-\overline{D})^2}{2\sigma^2}\right\}$ in the equation (6) shows the regular distribution of the water content. We determine that $M_{\overline{D}}$ is the water content of $\overline{D}$～$\overline{D}+\Delta D$ (ΔD is the span of the size)

$$M_{\overline{D}}=\frac{M}{\sqrt{2\pi}\sigma}e^{-(D-\overline{D})^2/2\sigma^2}\Delta D=\frac{M}{\sqrt{2\pi}\sigma}\Delta D$$

$$\therefore M=\sqrt{2\pi}\sigma M_{\overline{D}}/\Delta D$$

If we use k instead of σ/D, it is known that k≒0.25 at the momentary N_D distribution, and we give ΔD=0.1mm, we will get the following equation.

$$M=6.267M_{\overline{D}}\overline{D} \qquad (7)$$

So the relationship initial the beginning water content and the average diameter is simple function. Generally, $\overline{D}$ becomes larger and $M_{\overline{D}}$ smaller. Because $M_{\overline{D}}$ depends on the type of rainfall, so we can select the realistic value of $M_{\overline{D}}$. The $M_{\overline{D}}$ of each rainfall group must have its own value but in this paper we suppose that $M_{\overline{D}1}=M_{\overline{D}2}=M_{\overline{D}3}$. The determination method of M and $\overline{D}$ are as follows.

The approximation equation of rainfall intensity R from equation (6).

$$R=15.95M\sqrt{\overline{D}}\left(1-\frac{k^2}{8}\right)$$

$$R_T=\sum_{n=1}^{3}15.95\times M_n\times e^{-\lambda nT}\times\sqrt{\overline{D}_n}\times\left(1-\frac{k_n^{\,2}}{8}\right)$$

$$=99.18\sum_{n=1}^{3}M_{\overline{D}n}e^{-\lambda nT}D_n^{1.5} \quad\cdots\cdots(8)$$

"n" shows the number of the raindop group.

As we mentioned, in order to get $\overline{D}_1,\overline{D}_2,\overline{D}_3$ we must select the realistic $M_{\overline{D}}$ and apply R_1 (the maximum 1 ninute rainfall intensity), R_5 and R_{10} to equation (8) and solve three dimentional simultaneous equations about $\overline{D}_n$. But about $M_{\overline{D}}$ we found the relationship between $M_{\overline{D}}$ and a, b, so we will use the following equation. First we determine that $COF=6.267\times M_D$

$$b \geqq 0.1825a^{0.4954}$$

$$COF=0.009982\ (a/b)^{0.8475}\ (r=0.9825) \quad\cdots\cdots(9)$$

$$b<0.1825^{0.4954}$$

$$COF=0.003509\ (a/b)^{0.9183}\ (r=0.9908) \quad\cdots\cdots(10)$$

By using the equation (9), (10) and (7) we'll get Mn of each raindrop group.

2. 5 The determination of N_D distribution

We will get N_D distribution by applying the solution of 2. 1～2. 4 to equation (6). And we'll show the point of this method by using the flowchart. (Fig. 3).

3. Data

The data which are used in this paper are the records of 10 min railfall amounts measured at the weather bureau, meteological observatory, city office, airport, dam cite, etc. We must pick up six values of r_{10} (the maximum 10 min rainfall amounts), r_{20}, r_{30}, r_{40}, r_{50}, r_{60}, and by applying these values to the method of least squares, we'll get the

Table 1 List of the data.

Location	Date
Ube cite - Yamaguchi pref city	1980
Nagasaki heavy rainfall	July 23 1982
Western - Japan heavy rainfall	July 8 1967
Davao (philippines)	1949～1971
Camarines (philippines)	1980～1984
Gabaldon (philippines)	1970～1974
Cagayan (philippines)	1976～1969
Catbalogan (philippines)	1971～1975
Zamboange (philippines)	1965～1975
Tayabas (philippines)	1976～1980

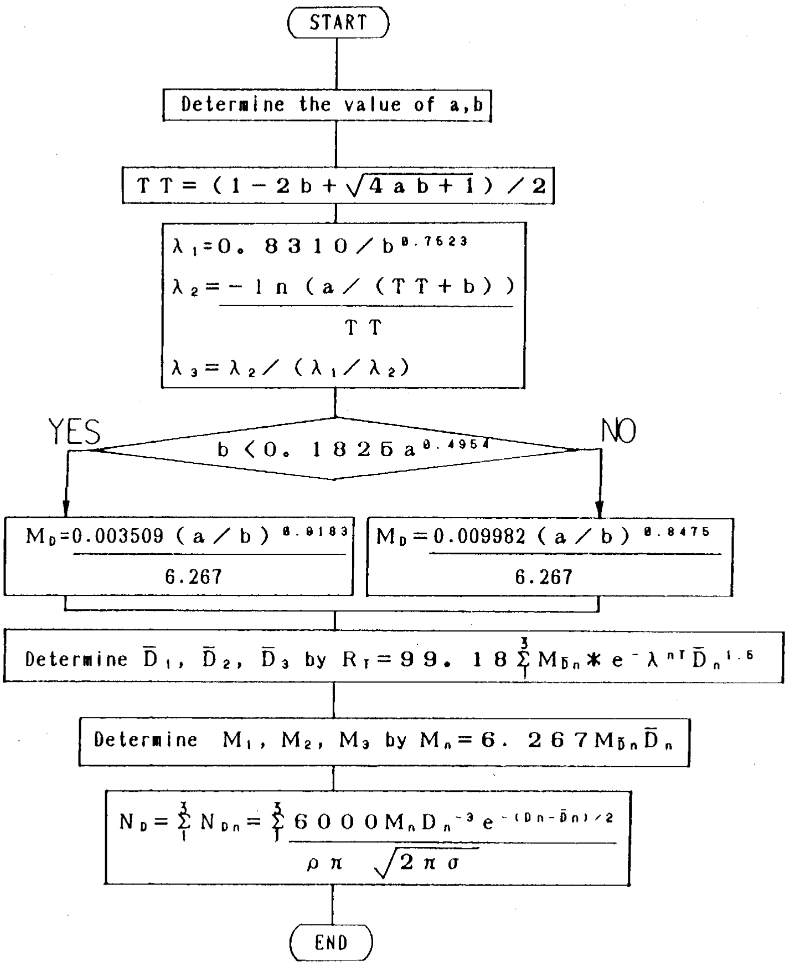

Fig. 3 Flowchart of the Shiotsuki's method.

parameter a, b of Talbot's equation. Table. 1 is list of the data.

4. The relationship between a, b and rainfall type.

In this paper, the type of rainfall is represented by the two values of a, b. In order to understand the relationship, we selected some values of a, b and applied them to the Talbot equation (R_T=a/T+b) and drew the R_T curve.

First, to understand the relationship between a and the rainfall type, we fixed the

Table 2 The relationship between a and the maximum T minutes rainfall intensity.

$RT = \frac{a}{T+100}$	(b = 100)			
	a = 1000	a = 2000	a = 10000	a = 20000
10 min	9.0	18.0	90.0	180.0
20 min	8.3	16.6	83.0	166.0
30 min	7.6	15.2	76.0	152.0
40 min	7.1	14.2	71.0	142.0
50 min	6.6	13.2	66.0	132.0
60 min	6.2	12.4	62.0	124.0

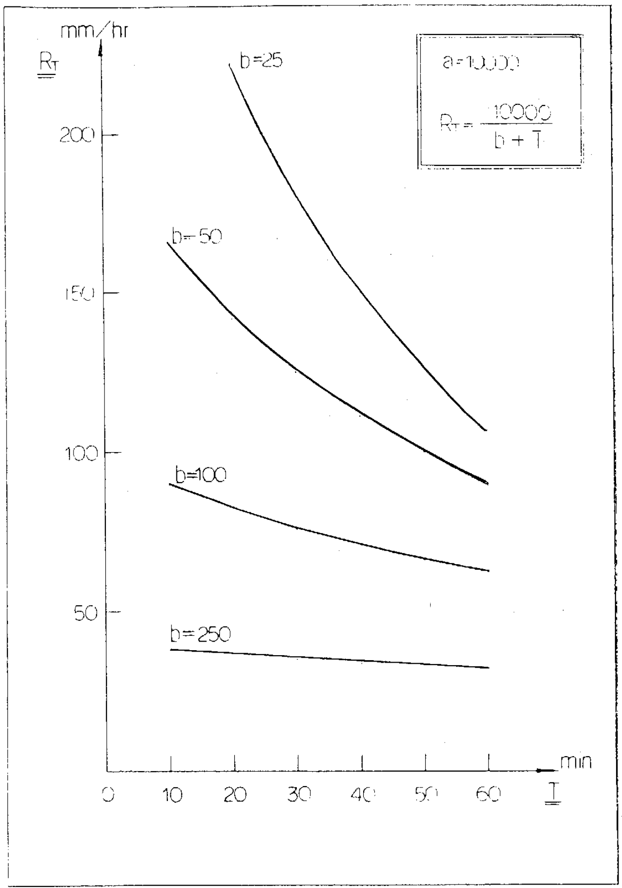

Fig. 4 The relationship between R_T curve and b.
(a=10000)

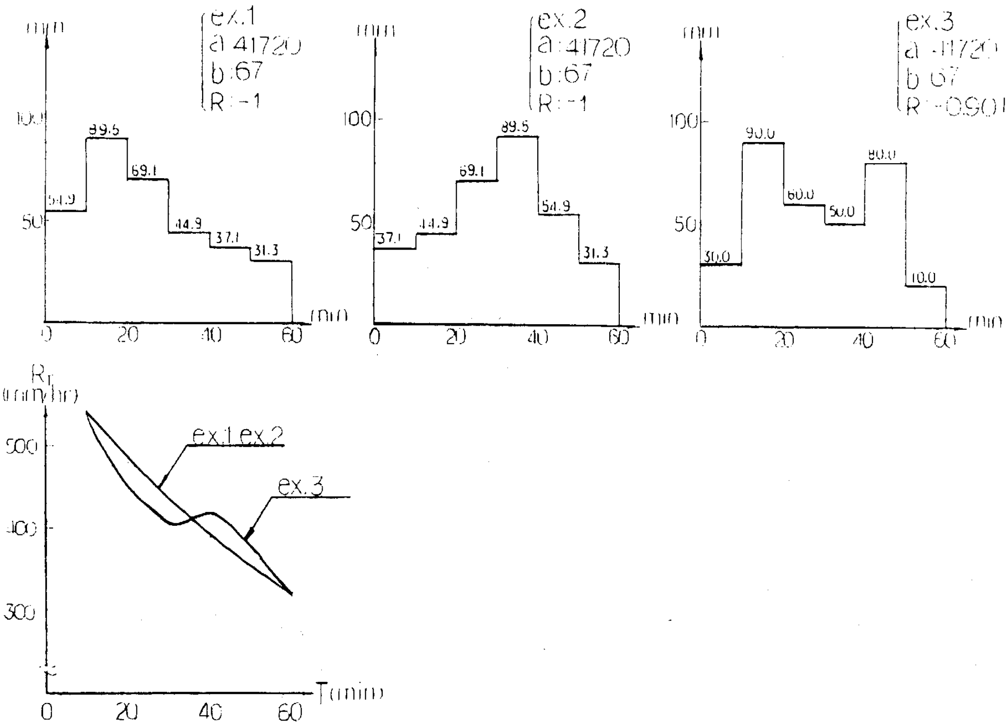

Fig. 5 3 types of the rainfalls and their R_T curves.
(which have the same value of a, b, a=41720, b=67)

value of b (=100) and changed the value of a. So we can get the result in Table 2.

It is easily understood that the rainfall amounts are closely connected with a and it is the parameter which shows the scale of rainfall. Secondly, we settled the value of a and changed b. So we can get the R_T curve in Fig. 4.

Considering this figure, we can understand that when the value of b is small, the slope of the R_T curve is steep, the rainfall intensity is considerable high and the decrement is active, but when b is big the rainfall intensity and decrement are small. So we found that the value of b is ralated to the duration of rainfall.

5. The limit of Talbot's equation.

Only one pair of a, b is determined by the rainfall type, but conversely it is impossible to determine the rainfall type by the value of a, b. In order to explain these things, we'll show three different types of rainfall in Fig. 5 which have same value of a, b. (They are not actual rainfall)
Ex. 1, Ex. 2, Ex. 3 have the same value of a, b (a=41720, b=67) but the type of rainfall are different. Ex. 1 and Ex. 2 have the same R_{10}, R_{20},·········R_{60}, but Ex. 3 has quite diffe.ent value.

After all, it is impossible to use the Talbot's eqation for all types of rainfall especially fluctual rainfall. (which has a low coefficient of correlation : r)

6. The relationship between N_D and a, b

First, in order to understand the relationship between N_D and a. we must settle the value of b (b=10 and 30) and change the value of a. So we get Fig. 6, 7. As we mentioned in §4, a is the paramater which shows the scale of the rainfall. The characteristic is such that with the increase of a, the maximum diameter of the raindrop becomes

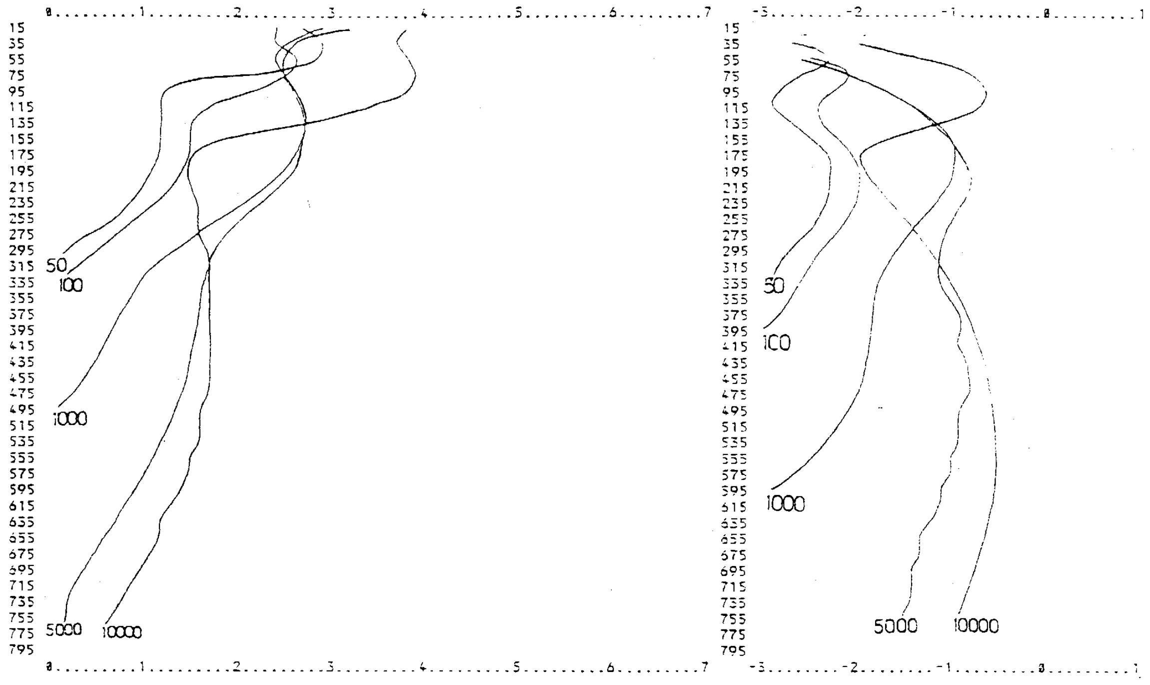

Fig. 6 The relationship between N_D and a (b=10)

Abscissa : Spatial raindrop concentration in number/m³/mm
Ordinate : Drop diameter in 10^{-2} mm

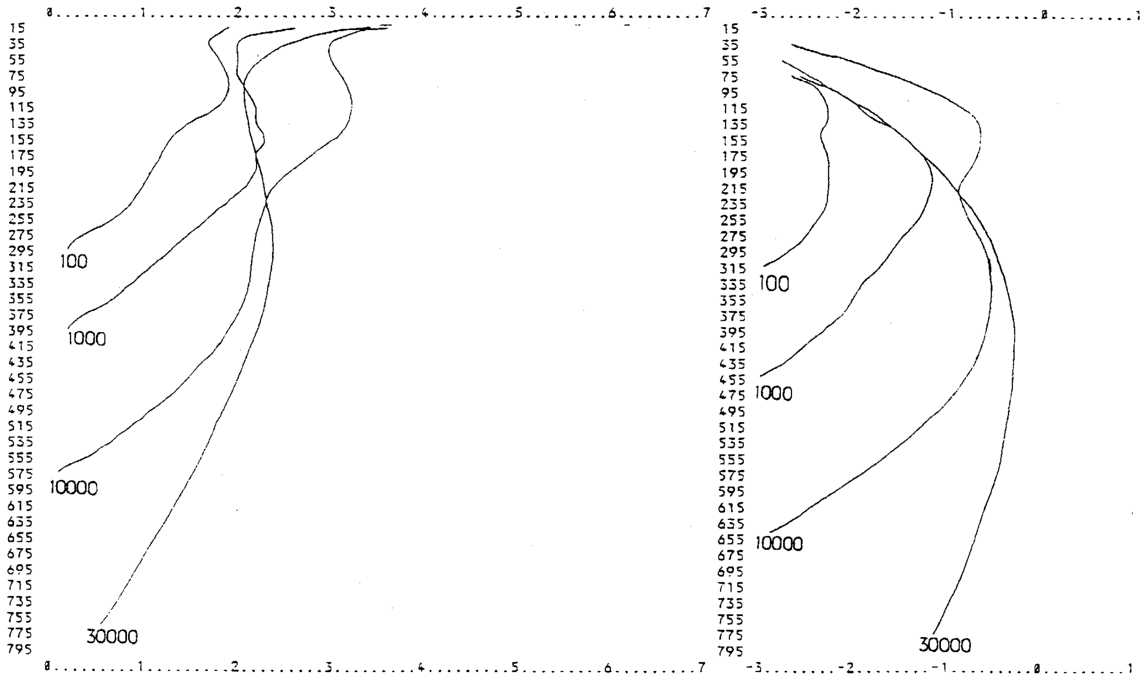

Fig. 7 The relationship between N_D and a (b=30)

bigger and thus the rainfall group contains bigger drops. On the other hand, without regard to a, the number of small size raindrops (about 0.15mm) are constant ($10^2 \sim 10^3$ drops/m^3) . After all, we suppose that by the increase of number of bigger size raindrops, the rainfall amount must increase.

Secondly, in order to understand the relationship between N_D and b, we must settle the value of a and change the value of b. So we can get Fig 8, 9. As we mentioned in §4, b is the paramater related to the duration. So, with the increase of b, the decrement

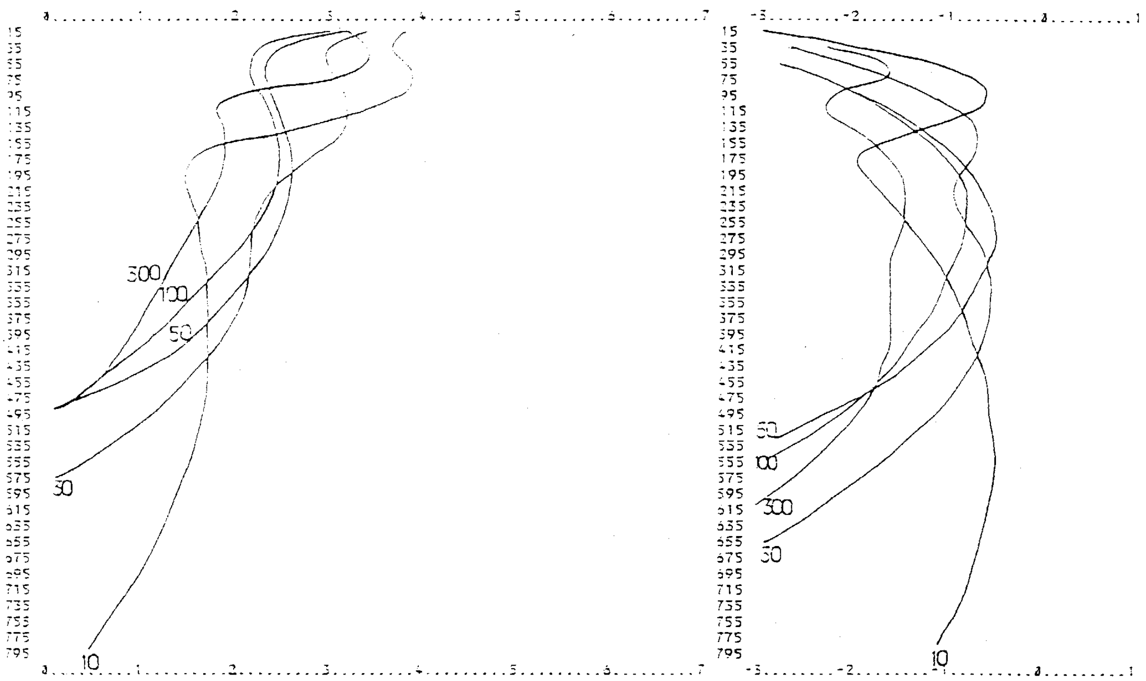

Fig. 8 The relationship between N_D and b (a=10000)

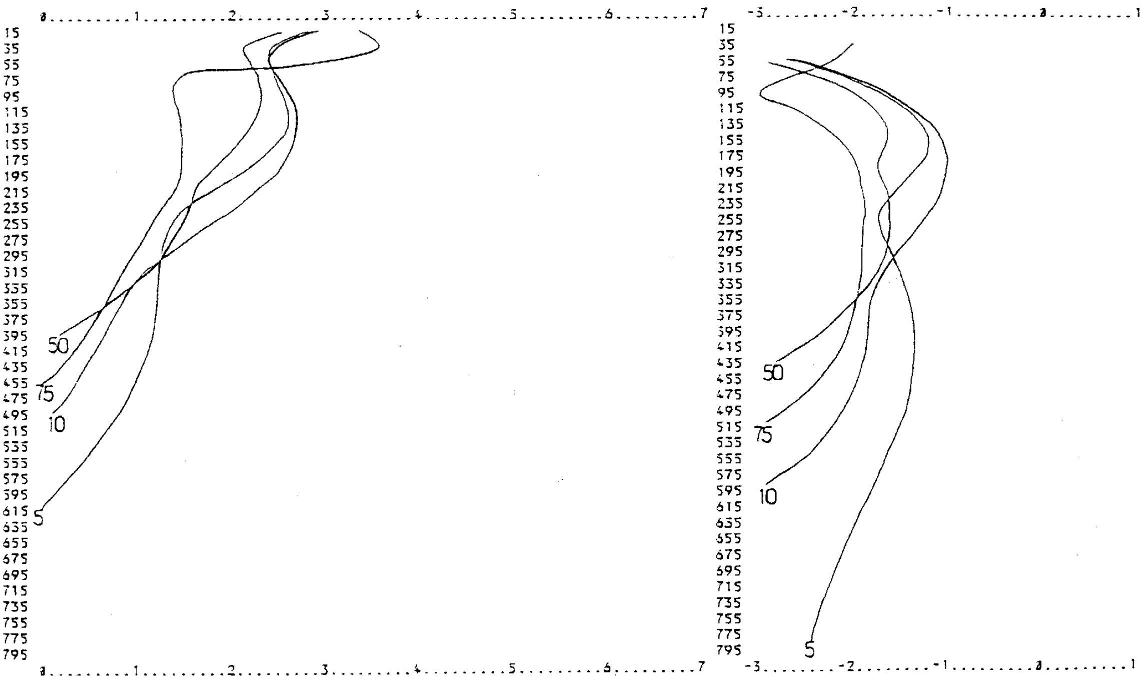

Fig. 9 The relationship between N_D and b (b=1000)

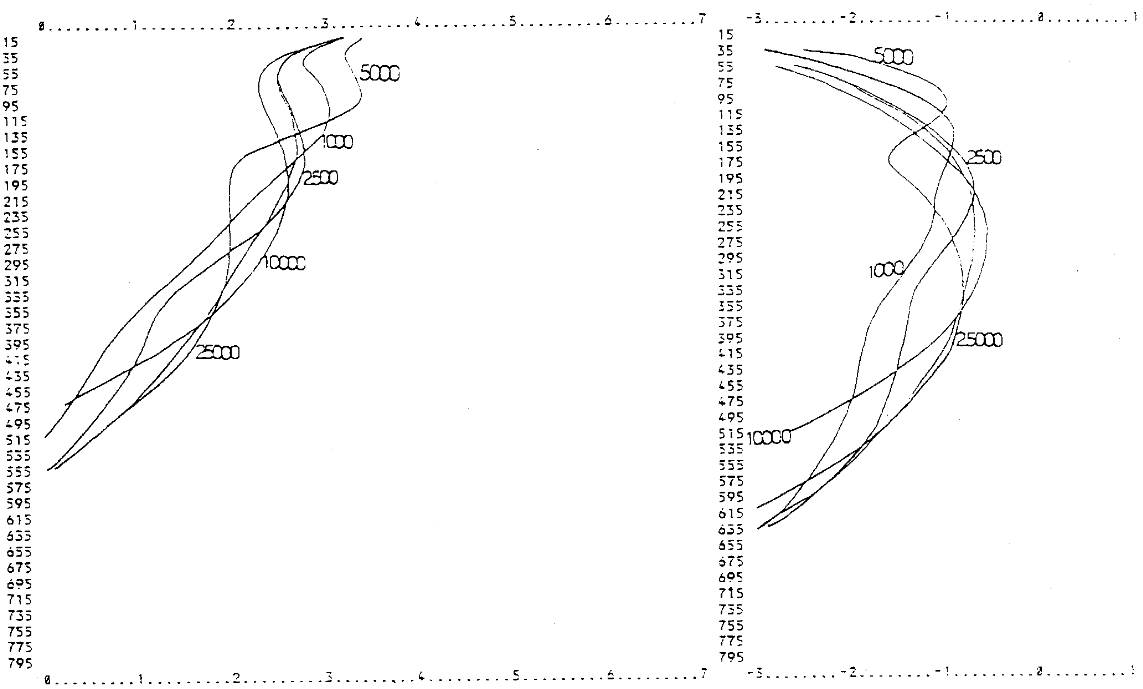

Fig. 10 The N_D distribution of a/b=171

of the water content becomes small, and also R_T becomes small. But the influence of b on the decrement is affected by a, so the ratio of the decrement is complicated. The relationship between N_D and b is such that when the value of b is small, especially less than 10, many bigger drops are contained generally. But the value of "less than 10" is valid when a is $10^3 \sim 10^4$. So, the value is connected with a. In short, big size drops have a tendency to be contained a lot when the value of a is large and b is small.

Third, the number of small size drops are constant ($10^2 \sim 10^3$ drops/m^3) without regard to a, b. In this casegs, we found that when a/b is constant, the N_D distributions are very similar. In order to undertand it, we settle the value of K to 171 and 34.2. So we can get the results in Fig. 11, 12.

And further, the value of 171 is the ratio of a/b when the filter paper used by the observation for the raindrops has been coverd by the drops for 1 second, and similarly

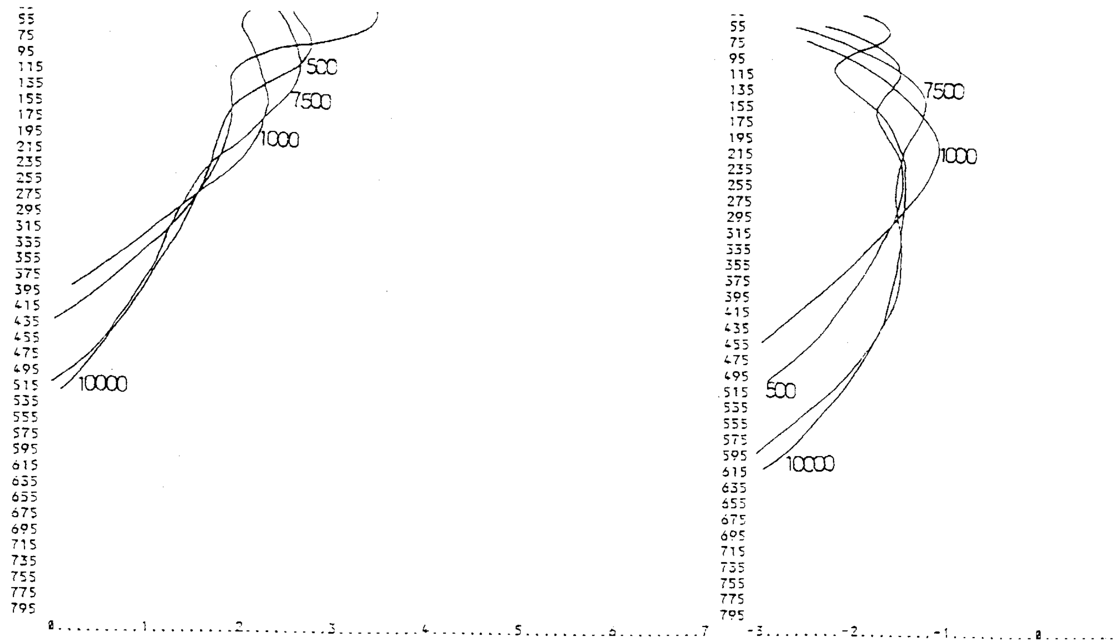

Fig. 11 The N_D distribution of a/b=34.2

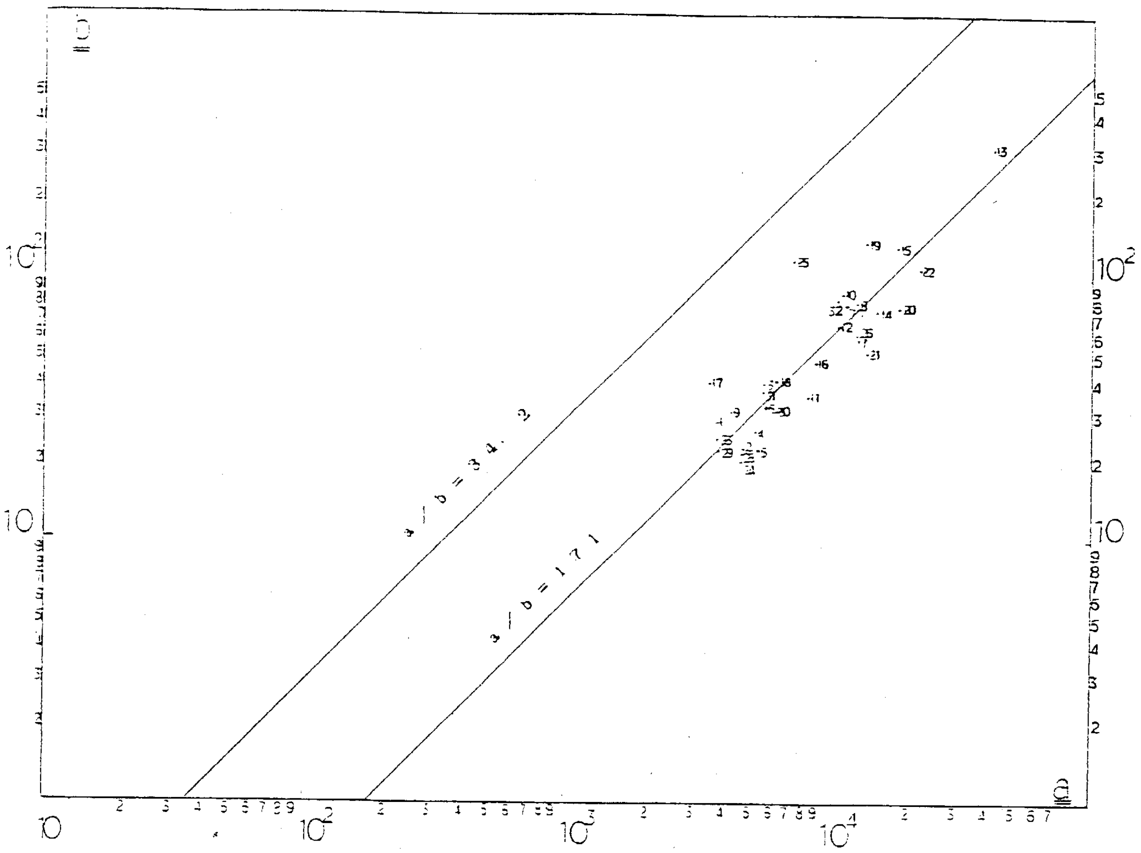

Fig. 12 a-b distribution of Western-Japan heavy rainfall

34.5 is for 5 sec. And generally, we call it heavy rainfall when the a, b is plotted on the right side of the straight line a/b=171 on the log-log paper.

It is easily understood from Fig 11,12 that when a/b is constant, those rainfalls have very similar N_D distributions. But the N_D distribution has a little tendency to contain big drops with the increase of K. Hereafter, it is necessary to study about this point in more detail.

7. The local characteristics of a, b

By using the method shown in the flow chart of Fig. 3, we get a, b from the 10 min rainfall amount data (Western-Japan heavy rainfall, Nagasaki heavy rainfall, Ube, and Philippines) and plot them on log-log paper. They are shown in Fig. 13~Fig. 22. And Ube's values of a, b were calculated every one rainfall. Here, one rainfall is defined that all 1 min rainfall intensities must be more than 1 mm/hr. Concerning the Western-Japan heavy rainfall and Nagasaki heavy rainfall, we plotted them maximum value of a,b of each day.

First, in order to study about the a- b distribution on the log-log paper of the heavy rainfall, we must consider the result of Western-Japan and Nagasaki heavy rainfall as shown in Fig. 13, 14. We found that the crowding of the dots is 400~4000 of a and 20 ~300 of b, and they lie on the line of a/b=171. Concerning those N_D distribution, many big drops (more than 4.5 mm) were contained. (As shown in Fig. 10) And N_D distribution of each point must be very similar because a/b in nearly equal to 171. Next, we pay attention to Fig. 15 of Ube. Ube city is in Yamaguchi Pref. and faces the Inland Sea of Japan. The climate is moderate and the rainfall amount is not so much. It is also

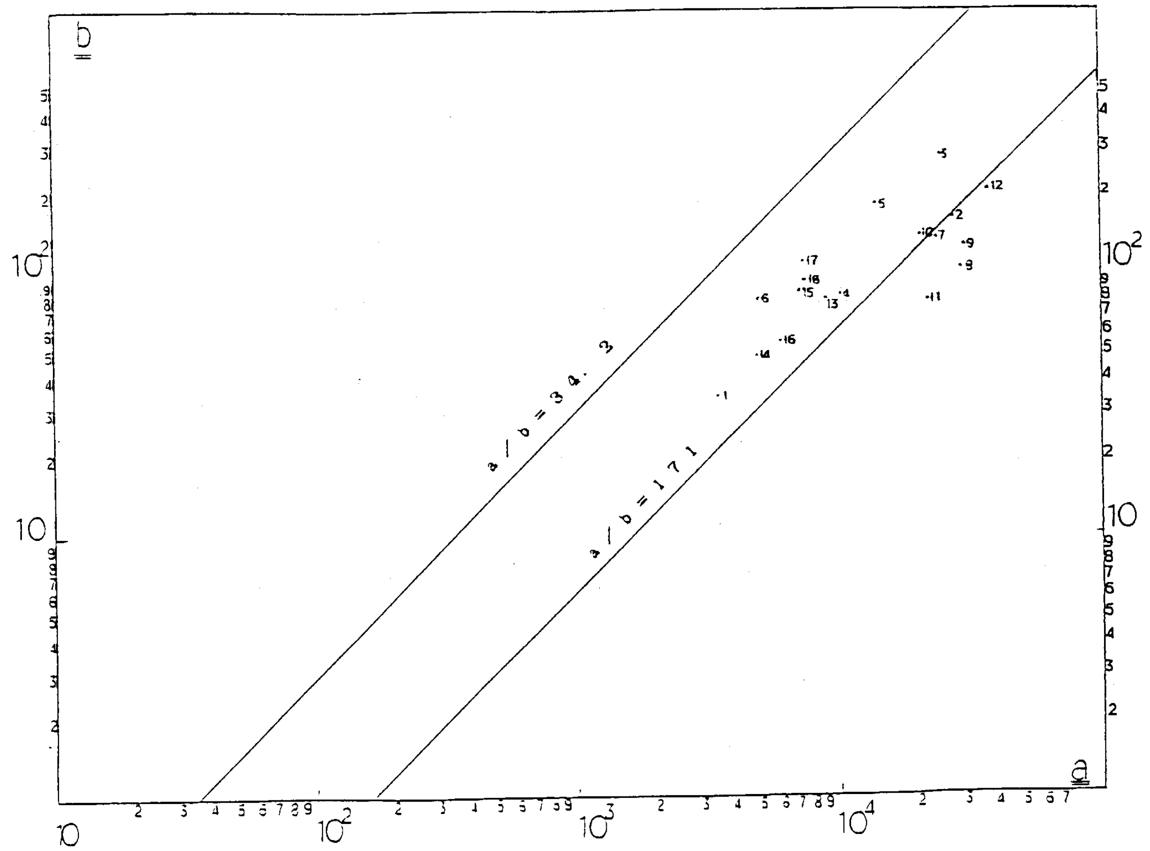

Fig. 13 a-b distribution of Nagasaki heavy rainfall

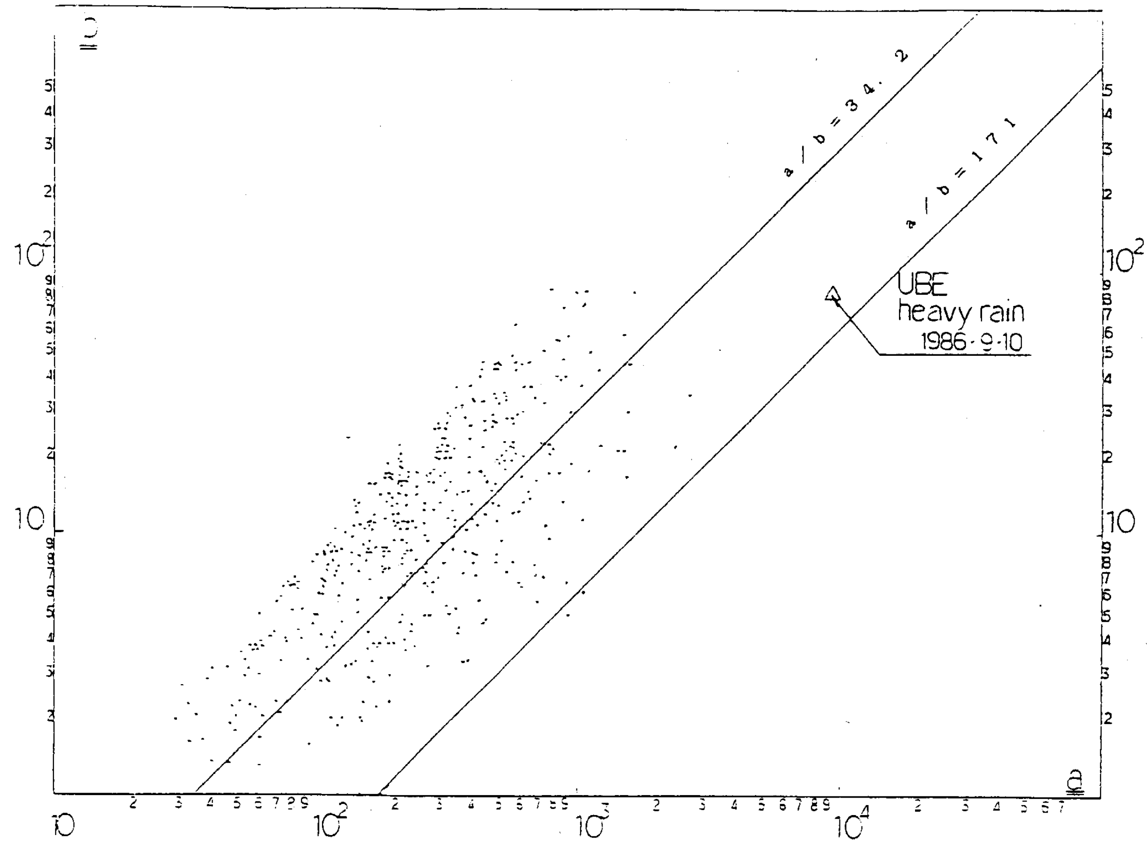

Fig. 14 a-b distribution of Ube

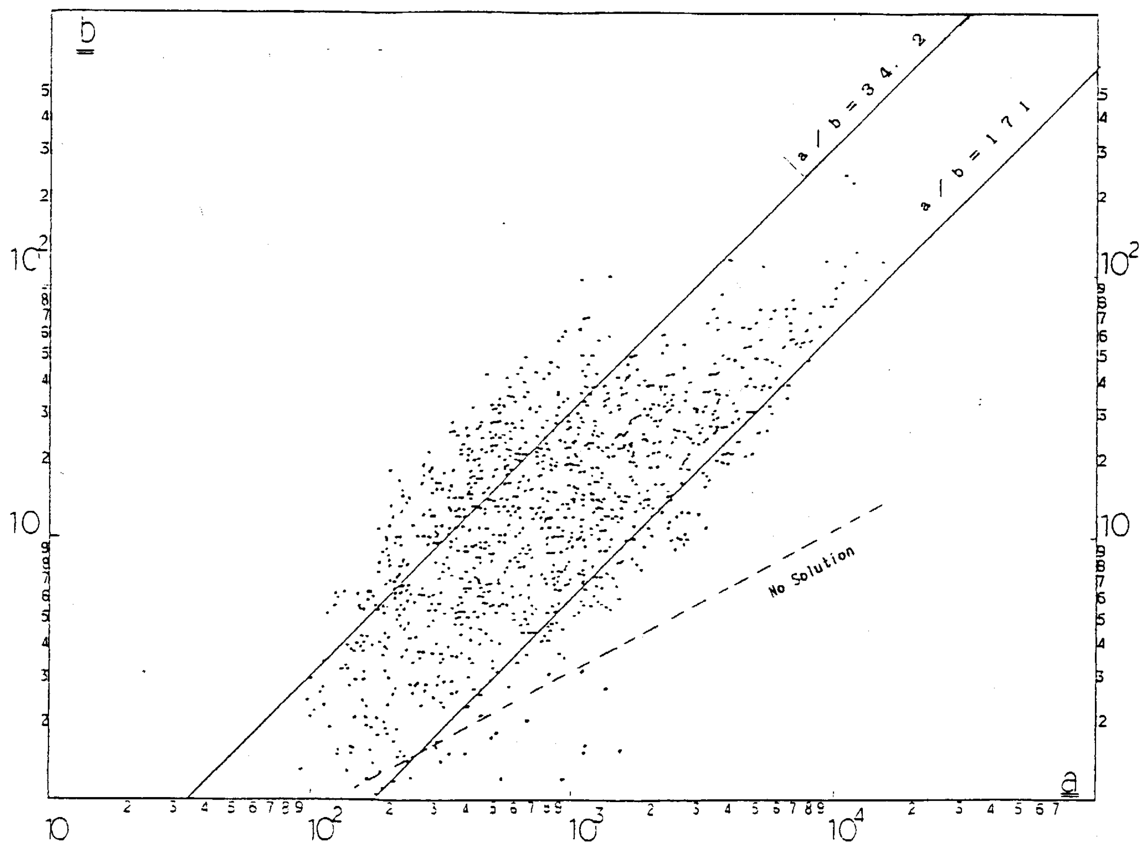

Fig. 15 a-b distribution of Davao (1949-1971) understood by examining Fig. 15. There are a few rainfalls whose value of a, b are more than 1000, and the center line of the crowd of the dots is on the 34.2 line. And there were three heavy rainfalls which were plotted on the right side of the 171 line in 1980, but the values of a, b were about (1000,5). It means the duration was not so long and we supposed it might be shower type rainfall. But last year, there was a heavy rainfall at Ube in Sep. 1986. The rainfall amount of 1 day was 115 mm/day and the maximum 1 hour rainfall amounts is 61 mm/hr. It is very rare to have this type of rainfall at Ube. According to the data of Yamaguchi-Ube Airport, the value of (a, b) is (9432, 81), and it is plotted in Fig. 15. So we can easily find that it is not ordinary rain. After all, in Ube there is seldom heavy rainfall and many of the rains are soft one as plotted on the 34.2 line.

Next, we mention about the data taken at seven points in the Philippines (As shown in the Fig. 16～22). We drew the 171 line that is the standard of Japanese heavy rainfall. When we compare them with the data taken in Ube (Fig.5) we will find that there are much more heavy rainfalls whose value of a is over 1000. The center line of the dots' crowding is a/b=80～100. And squall, characterisitic rain in the rainy season of these districts, they are plotted near the 171 line and their values of b are small. Namely the rain plotted near (1000,5) is the squall.

In order to compare with N_D distribution of Ube, we set up such values as the following.

First, as the typical rain's a/b in Ube, we set up a/b=34.2, and a<3000. Similary, concerning the Philippines we set a/b=100 and a<10000. So we calculate those N_D

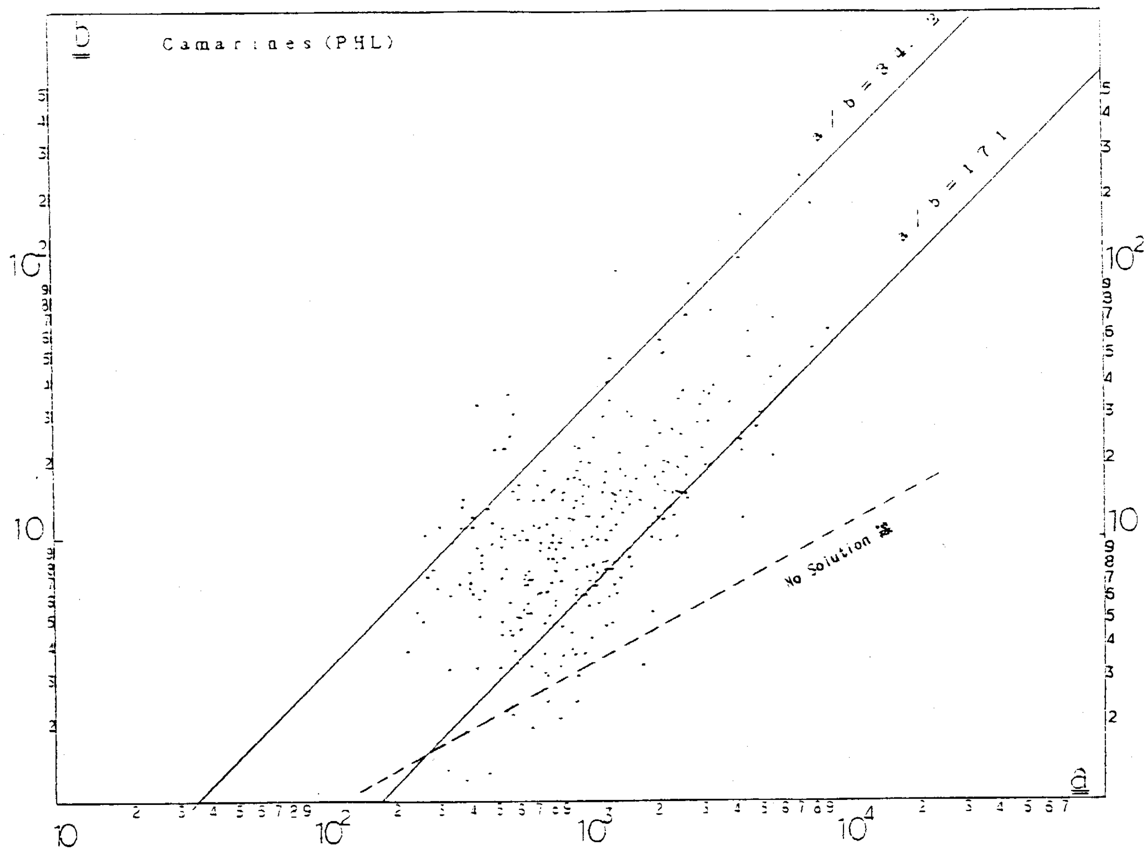

Fig. 16 a-b distribution of Camarines (1980-1984)

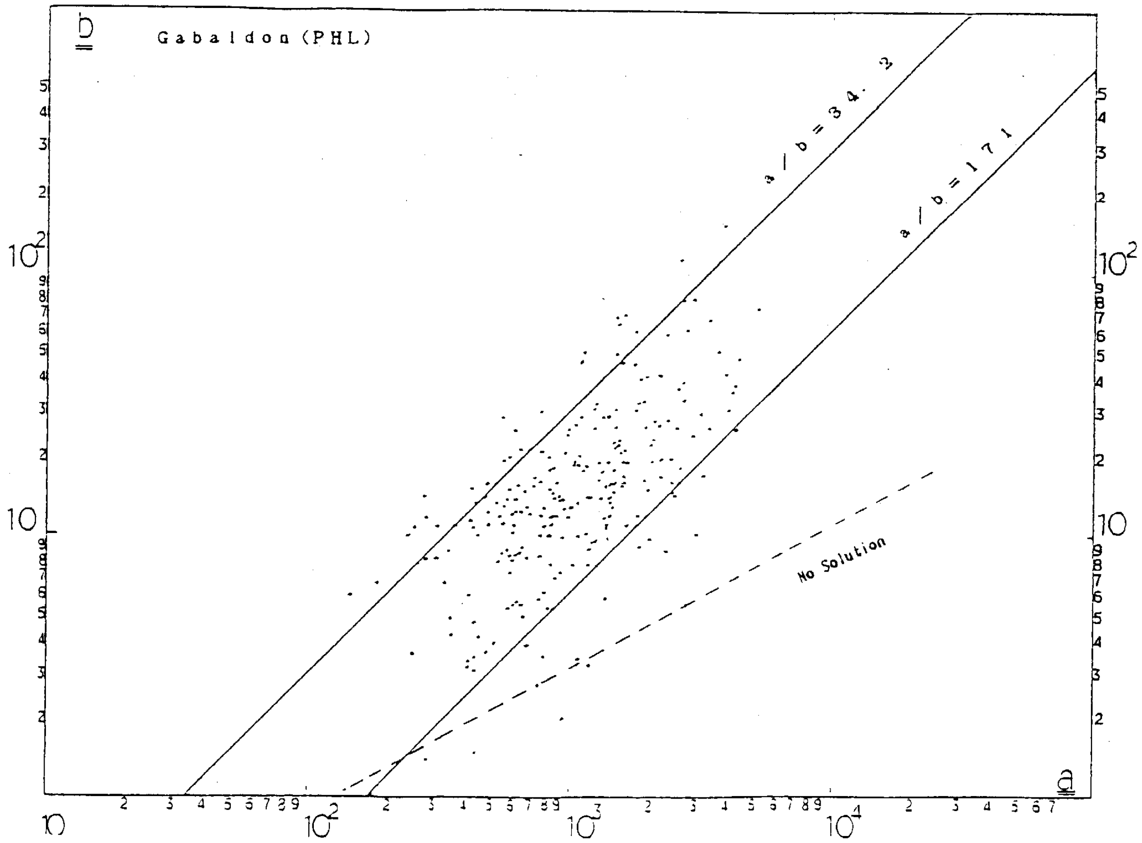

Fig. 17 a-b distribution of Gabaldon (1970-1974)

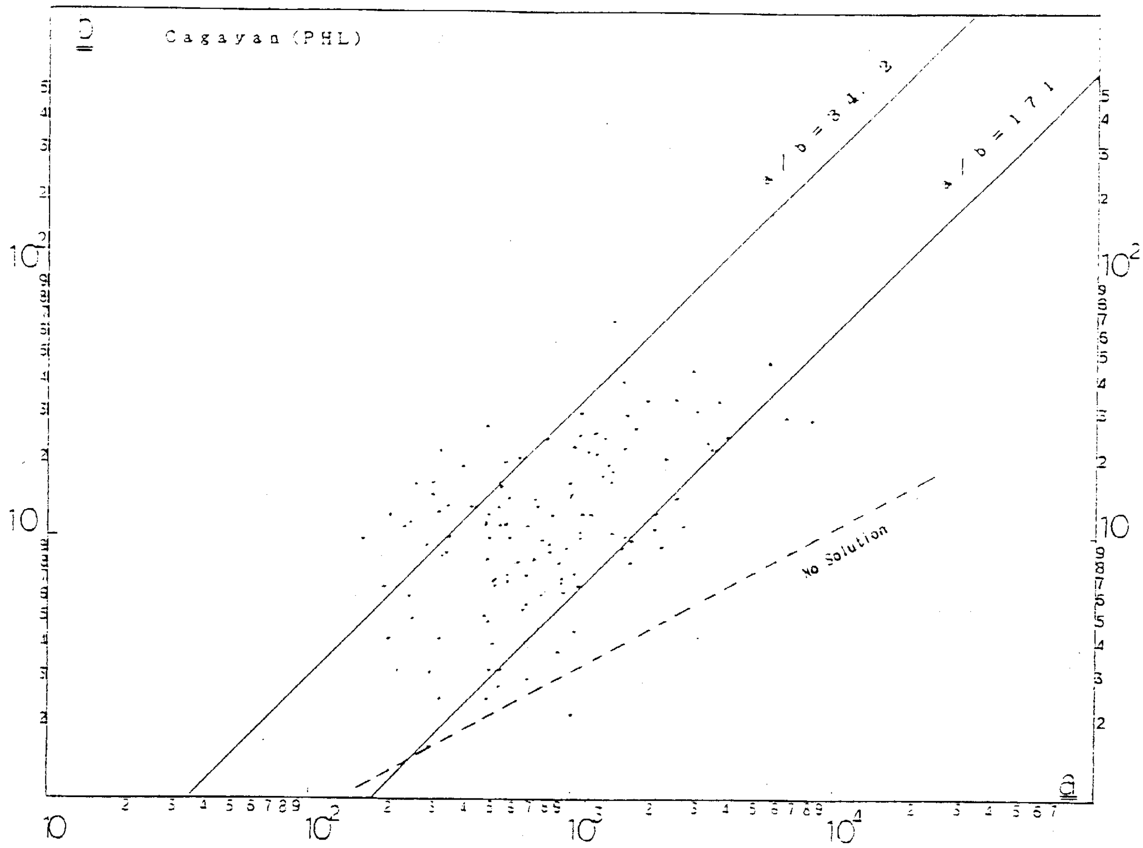

Fig. 18 a-b distribution of Cagayan (1976-1969)

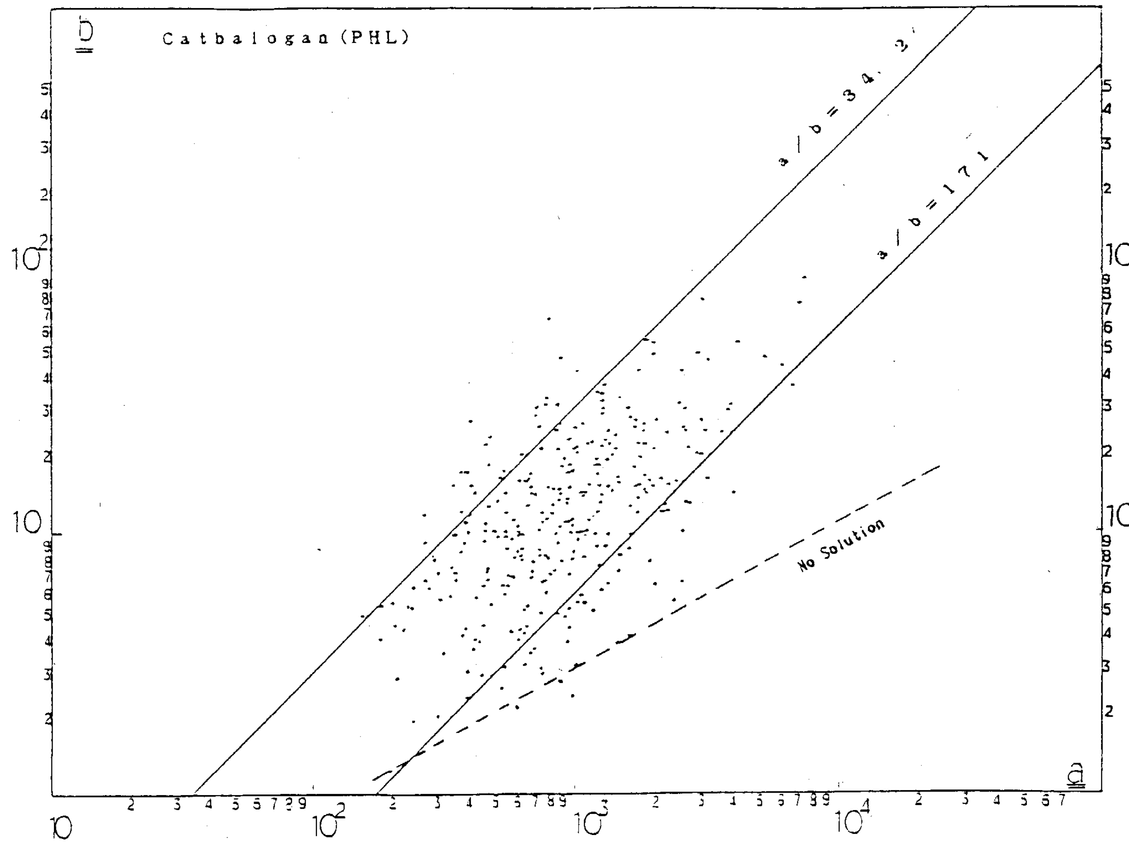

Fig. 19 a-b distribution of Catbalogan (1971-1975)

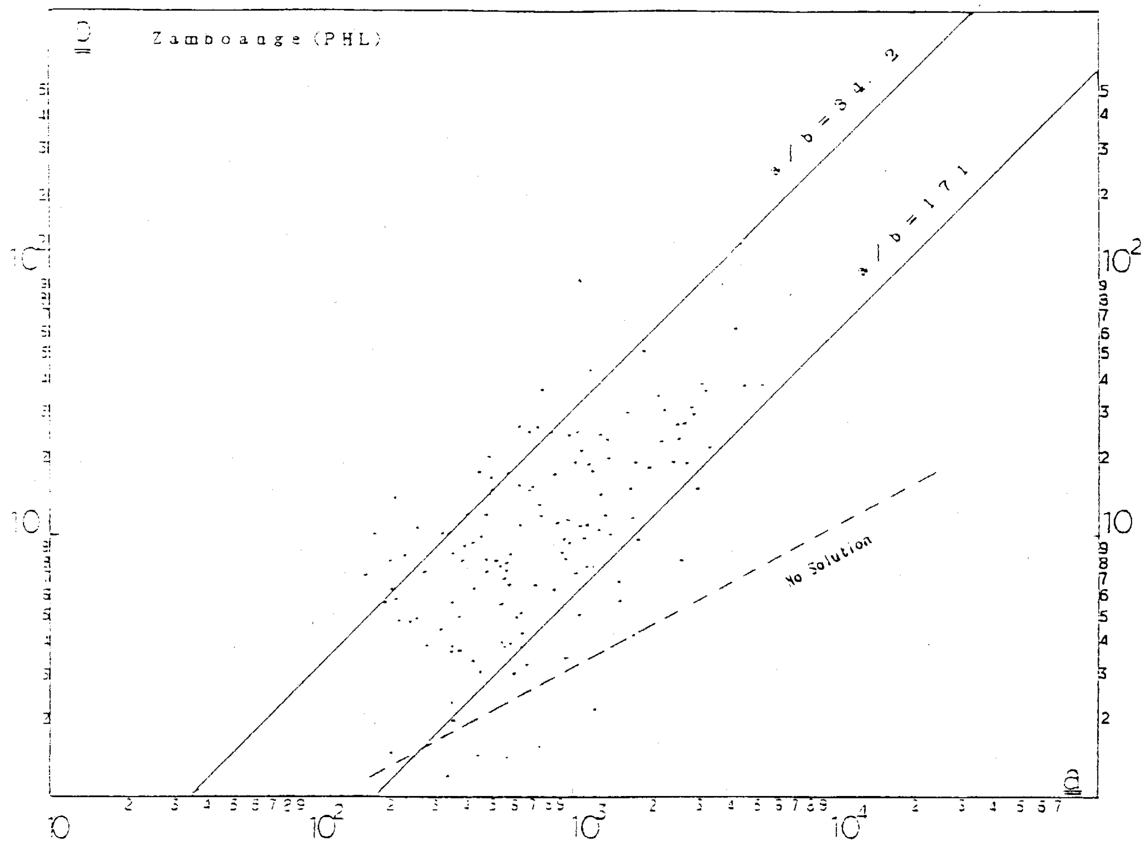

Fig. 20 a-b distribution of Zamboange (1965-1975)

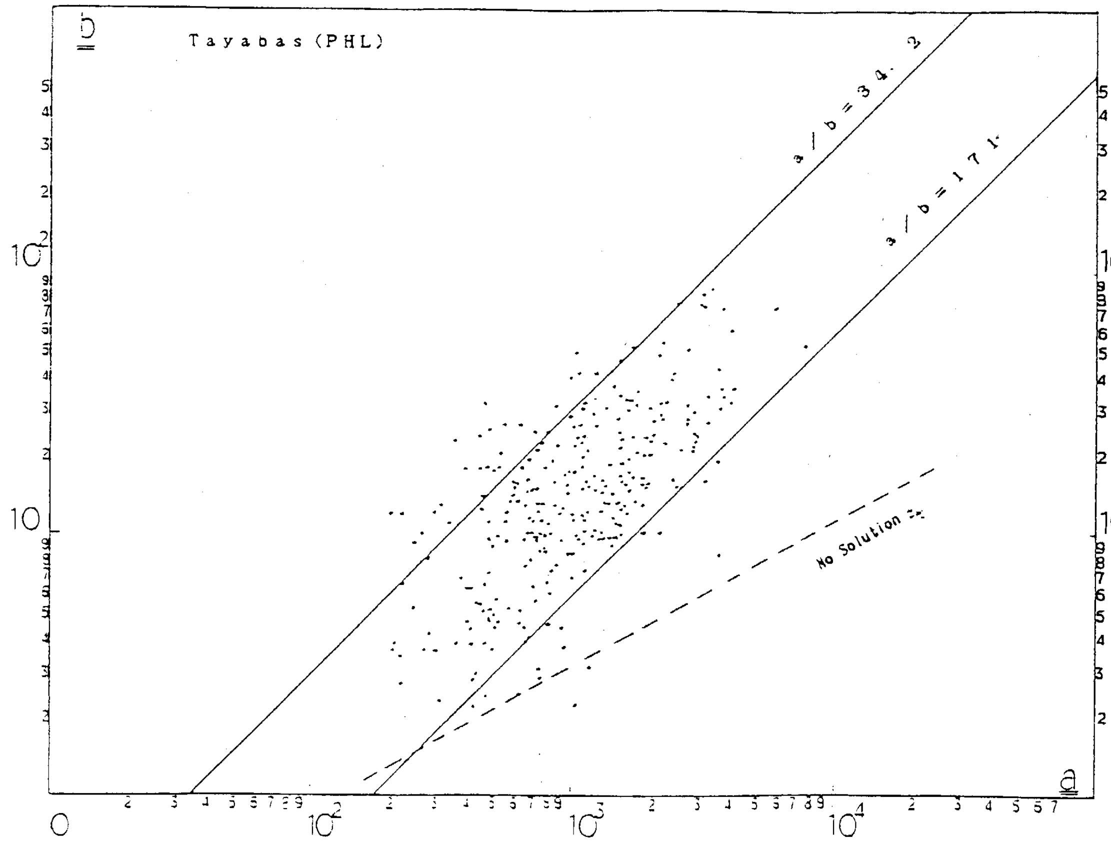

Fig. 21 a-b distribution of Tayabas (1976-1980)

distributions by using those values and compare them. The maxmum size of the drops in Ube are 5 mm and 6 mm in the Philippines, and the number of small drops less than 1 mm are $10^{2.5}$ either, but big drops more than 1 mm are much contained in the Philippines'rain. As we have seen, N_D distribution has local characters, so it is important to study it.

8. The examination of N_D distribution analysis method

We have calculated many N_D distributions by Shiotsuki's method, but it is impossible to compare with the actual N_D distribution. So all we can do is to compare with very limited actual data or to judge by experience.

And many of the results were imaginable one but there are some improper one. They are plotted in the rightside of "No Solution-Line" in Fig. 16~22 of the Philippines' data. It is impossible to calculate the N_D distribution by using those values, because if we solve the equation (8) shown in 2. 4 by using them we get $\overline{D}n$ of the minus value. $\overline{D}n$ is the average diameter so it should be plus. The type of those rains is that the value of b is much smaller than a, namely,they are squall type rain.

In Japan,the rain of this type has never existed but according to the studies of Dr. Isiguro and Dr. Iwai, we can't assert that it never exist. So it is necessary to reexamin this method. And following is the hint by which this can be done.

(1) M_Dn are given by the empirical formula led by very limited data only in Japan so it is impossible to apply the foreign data.

(2) It is impossible to apply the Talbot's equation to the squall type rain or changeable

rain.

(3) In this report we supposed three rainfall groups but in fact it is not always three.

9. Conclusion

As we have seen, the Shiotsuki's method is not perfect one, but it is important to become to solve the N_D distribution by using 10 min raifall date only, that is easy to get. And many N_D distributions can be solved by this method.

So it must be very useful for the future study of rain.

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