Variable Probability Circuits

By Takehiko HIRATA and Akio KIRIHARA

(Received July 4, 1979)

Abstract

This report aims to modulate the passage probability of a pulse by the instantaneous value of an analogue input signal. Up to now, some circuits where the status probabilities may be controlled by input from the outside have been found. However, it is considered that in order to freely vary the probability distribution by the instantaneous values of input signals, a still more complicated circuit construction is necessary. This paper describes that such circuits can be realized simply by using shiftregisters, random pulses, and analogue operation circuits.

1. Introduction

This report aims to modulate the passage probability of a pulse by an analogue input signal. Various machines, circuits, or systems that behave in accordance with the law of probability have been considered as hardware or software up to now. For example, von Neumann[1], Gill[2], Harrison[3], Gelenbe,[4] El-Ghoroury and Gupta[5][6], Lorenc[7], and others. The greater part of this hardware is realized by applying random input or random disturbances to sequential circuits. In particular, the circuits of Gelenbe[4] and El-Ghoroury and Gupta[6] can control the own state probability by other input. However, it is considered that in order to freely vary the probability distribution by the instantaneous values of input signals, a still more complicated circuit construction is necessary. On the other hand, the circuits proposed in this paper may vary the probability distribution by input signals in a relatively simple manner, by regulating the number of elements of the shiftregister which is shifting randomly with the output of the analogue operation circuit.

In the following, the fixed probability circuit that underlies the variable probability circuits will be described first, then the variable probability circuits and a few derivative circuits will be described.

2. Fundamental probability circuit

Suppose that a shiftregister which consists of many elements is supplied with only one input pulse, and that the pulses to shift the 1 state in the shiftregister are applied at random and at high speed. Moreover, suppose that the 1 state which has been shifted to the last element of the shiftregister is applied as input again in the form of a pulse at once, and random shift is repeated. At that time, it is assumed that the 1 state will not be lost or increased to two or more for some reason in shifting. Consequently, the sole 1 state keeps cycling always in the shiftregister at random and at high speed. Fig. 1 is a circuit as described above.

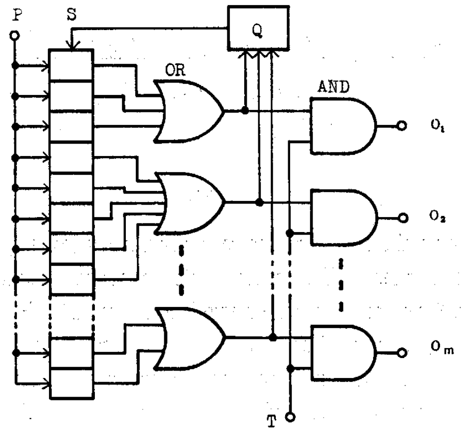

Fig. 1 Fixed probability circuit.

If the total number of elements of the shiftregister S is n, then it can be proved that the probability that every element is in the 1 state in one moment is $1/n$ (the proof will be given in the Appendix). Therefore, if n elements are divided into m groups, and the number of respective elements is $n_1, n_2,\ldots, n_m$, then the probability that the 1 state belongs to the 1st group in one moment is n_1/n, similarly, the probability that it belongs to the 2nd group is $n_2/n,\ldots$, the probability that it belongs to the mth group is n_m/n.

Thereupon, if the terminals that denote the states of the elements of each group are connected with one OR (logical sum) circuit each, the probability that the output of a respective OR circuit denotes 1 in one moment will be given by the value described above, respectively.

Therefore, if a narrow pulse corresponding to "trial" is applied to AND (logical product) gates connected with the respective outputs of the OR circuits described above, its pulse will come out of any one of m AND gates by the probability assigned respectively as described above. In this sense, let us call this pulse the "trial pulse".

Thereupon, if the number of events m and the respective probabilities $p_1, p_2,\ldots, p_m$ are designated, a shiftregister must be divided so that $n_1/n \rightarrow p_1$, $n_2/n \rightarrow p_2,\ldots$, $n_m/n \rightarrow p_m$, but the larger the total of elements n is, the more closely the probabilities will approach the designated values, of course.

Next, it is an important matter that the 1 state cycling in the shiftregister must be only one always. In case that the 1 state will be made to cycle by connection from the last output of the shiftregister to the input as described previously, it cannot be assumed that there is always only one state 1.

According to experiments, it is usual in many cases that the number of the 1 state will be increased by noise or other impacts. To solve this problem, the following is considered. The equipment Q shown in Fig. 1 detects that the 1 state in the shiftregister is gone, then and there, it sends a pulse into the top element of the shiftregister, without delay. For convenience, let us call this operation "quick-firing". The terminal P in Fig. 1 is the terminal to which the random shift pulses are applied, and the terminal T is the terminal to which the "trial pulse" is applied.

3. Variable probability circuits

In the fundamental probability circuit, the probability that a trial pulse applied to the terminal T comes out of the output terminal is a constant value which is decided by the number of elements of the shiftregister connected to the input of the OR circuit corresponding to the output terminal. But, if it is possible to vary the number of elements of the shiftregister by other signal voltages, it is possible to freely vary the probability. An example of a circuit which behaves as mentioned above is shown in Fig. 2.

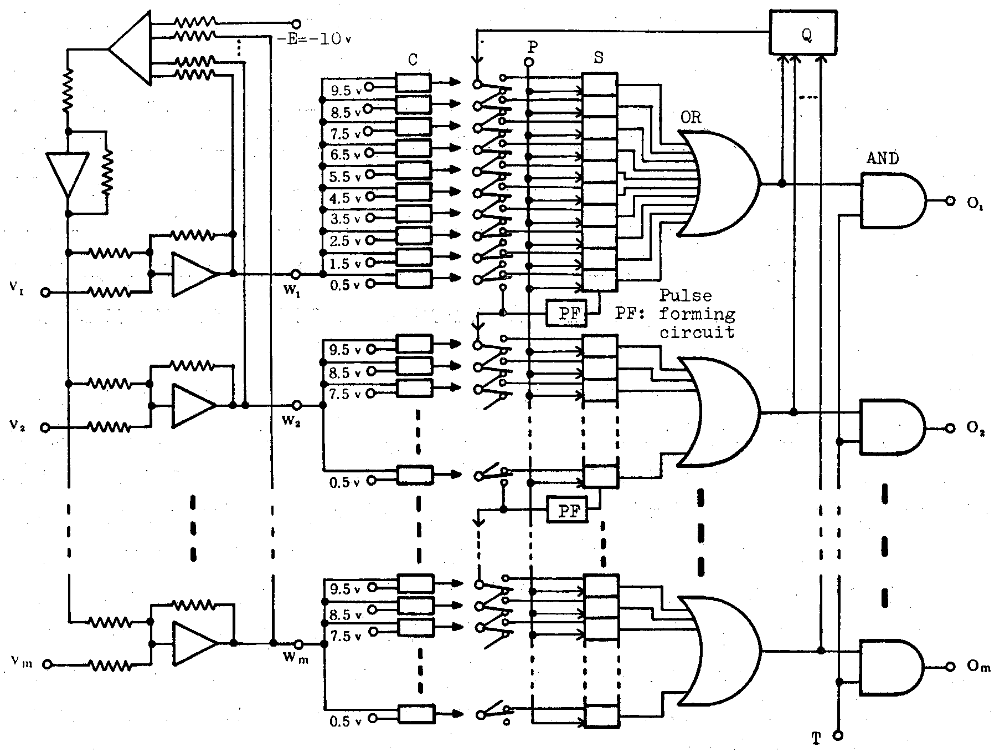

Fig. 2 Variable probability circuit.

The circuit of analog operation on the left of the figure is a circuit so as to automatically keep the sum of the output voltages $w_1, w_2, \ldots, w_m$ always at the standard voltage E, even if the regulation voltages $v_1, v_2, \ldots, v_m$ for varying the probabilities may vary widely. The standard voltage E equals the maximum limit value of the input voltage per set of the voltage comparator C. In Fig. 2, E is 10 (volt) and there are ten comparators per set. The output voltages of this analog circuit are respectively

$$w_1 = \frac{E}{m} - \frac{m-1}{m} v_1 + \frac{1}{m} v_2 + \cdots + \frac{1}{m} v_m$$

$$\left.\begin{aligned} w_2 &= \frac{E}{m}+\frac{1}{m}v_1-\frac{m-1}{m}v_2+\cdots+\frac{1}{m}v_m \\ &\vdots \\ w_m &= \frac{E}{m}+\frac{1}{m}v_1+\frac{1}{m}v_2+\cdots-\frac{m-1}{m}v_m \end{aligned}\right\} . \quad \cdots\cdots\cdots(1)$$

In this sense, let us call this circuit the intensive circuit.

As soon as the input voltage of a voltage comparator is over the comparative voltage, the output current of this comparator excites a relay so as to close its transfer contact upwards as shown in the middle of Fig. 2, and simultaneously the corresponding element of shiftregister S is activated. On the other hand, a transfer contact corresponding to a comparator with input voltage below the comparative voltage remains closed downwards, and simultaneously the corresponding element of the shiftregister maintains the inactive state. For example, for $w_1=3$ (volt), out of ten comparators the three from the bottom operate, the three transfer contacts corresponding to the active comparators close upwards, and the three bottom elements of the shiftregister out of ten maintain the function as shiftregister, while the upper seven elements have their functions suspended. All output of the ten elements of the shiftregister are applied as inputs to one OR circuit. The same applies for the following w_2 to w_m. Consequently, the elements of the shiftregister proportional to voltages quantized by the voltage comparators are connected to the inputs of respective OR circuits.

Because of $\sum_{i=1}^{m} w_i=10$ (volt), the sum of the elements of the shiftregisters that are connected to the inputs of all OR circuits amounts to about ten, for it is not always ten because of round-off error.

Thereupon, as soon as a pulse is put out from the equipment Q on the right upper side of Fig. 2, first the 3rd element from the bottom of the top shiftregister becomes 1. Then, every time random shift-pulses are applied to the shiftregisters from the terminal P, the 1 state shifts down and when it arrives at the bottom of its group, it transfers to the functioning elements of the lower shiftregister, and similarly shifts successively to the bottom of the lowest shiftregister, then vanishes. Thus, as soon as the 1 state vanishes, a new 1 is supplied from Q without delay, such as in the fundamental probability circuit. Therefore, when a trial pulse is applied to the terminal T, the probability that its pulse comes out of any one of m output terminals is proportional to the number of elements joined to the OR circuit. Ultimately, the probabilities in respective output terminals are as follows.

$$\left.\begin{aligned} P_1 &= \frac{w_1}{E}=\frac{1}{m}-V_1+\frac{1}{m}\sum_{i=1}^{m}V_i \\ P_2 &= \frac{w_2}{E}=\frac{1}{m}-V_2+\frac{1}{m}\sum_{i=1}^{m}V_i \\ &\vdots \\ P_m &= \frac{w_m}{E}=\frac{1}{m}-V_m+\frac{1}{m}\sum_{i=1}^{m}V_i \end{aligned}\right\} , \quad \cdots\cdots\cdots(2)$$

where,

$$V_i=\frac{v_i}{E}, \qquad (i=1, 2, \ldots, m) \qquad \cdots\cdots\cdots(3)$$

However, the relations of equation (2) hold only if all terminal voltages of the intensive circuit are $0 \leqq w_i \leqq E$. In particular, P_i becomes 0 for $w_i \leqq 0$, and the remaining P_i are divided in proportion to $w_i > 0$. For example, when $w_1 = -3$ (volt), $w_2 = 8$ (volt) and $w_3 = 5$ (volt), then $P_1 = 0$, $P_2 = 8/13$ and $P_3 = 5/13$.

Thus it is possible to freely vary the probability that a trial pulse applied to T comes out of any one of the output terminal $O_1, O_2, \ldots, O_m$ by suitably adjusting the regulation voltages $v_1, v_2, \ldots, v_m$.

4. Joint probability circuits and conditional probability circuits

If there are some classes of random variables $X=\{x_1, x_2, \ldots, x_l\}$, $Y=\{y_1, y_2, \ldots, y_m\}, \ldots, Z=\{z_1, z_2, \ldots, z_n\}$, the joint probability $P(x_i, y_j, \ldots, z_k)$ of these variables is considered as $l \times m \times \cdots \times n$ in all. Because of $\sum_i \sum_j \cdots \sum_k P(x_i, y_j, \ldots, z_k)=1$, a circuit which possesses the assigned joint probability distribution may be realized in the same form as in Fig. 1.

For a variable probability type, the circuit may be realized in the same form as in Fig. 2.

Next, in two classes of events $X=\{x_1, x_2, \ldots, x_l\}$ and $Y=\{y_1, y_2, \ldots, y_m\}$, the conditional probability of X, assuming Y, is written as

$$P(X|Y)=\begin{pmatrix} P(x_1|y_1) & P(x_2|y_1)\cdots P(x_l|y_1) \\ P(x_1|y_2) & P(x_2|y_2)\cdots P(x_l|y_2) \\ \vdots & \vdots \qquad\quad \vdots \\ P(x_1|y_m) & P(x_2|y_m)\cdots P(x_l|y_m) \end{pmatrix}.$$

The circuit which presents this probability may be realized by a little modification of the joint probability circuit, but may be realized also somewhat simply in the form of Fig. 3, for example.

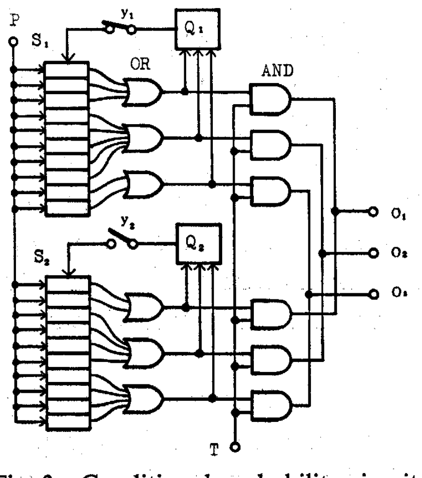

Fig. 3 Conditional probability circuit.

Fig. 3 shows a case of the conditional probability

$$P(X|Y)=\begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

which is obtained on the basis of $X=\{x_1, x_2, x_3\}$, $Y=\{y_1, y_2\}$ and the joint probability

$$P(X, Y)=\begin{bmatrix} P(x_1, y_1) & P(x_2, y_1) & P(x_3, y_1) \\ P(x_1, y_2) & P(x_2, y_2) & (Px_3, y_2) \end{bmatrix}=\begin{bmatrix} 0.18 & 0.30 & 0.12 \\ 0.08 & 0.16 & 0.16 \end{bmatrix},$$

where the figure shows the state with given y_1, but the switches y_1 and y_2 are closed or opened by the output of a probability circuit with probabilities of 0.6 and 0.4 respectively, as the marginal distribution of Y is $P(Y)=[P(y_1)\ P(y_2)]=[0.6\ 0.4]$.

Then, when a trial pulse is applied to the terminal T, its pulse appears from any one of the output terminals O_1, O_2 or O_3 with the probabilities 0.3, 0.5 and 0.2 respectively, since the switch y_1 is closed.

It is also possible to make the above as a variable probability type.

5. Concluding remarks

In order that the above mentioned various probability circuits act satisfactorily, attention must be paid mainly to the next two points during design and manufacture.

(1) Independence between trials — By increasing the number of elements of the shiftregister, the width of quantizing stair becomes smaller, therefore, the precision rises, but a large number of shift-pulses is necessary for the 1 state to make one round. Therefore, the average rate of the random shift-pulses must be so high that all trials may be mutually independent. Accordingly, the response speed of the elements of the shiftregisters must be made as high as possible.

(2) Error actions — If a trial pulse with a certain width of time is applied when the 1 state in a shiftregister is shifted by the shift-pulses, and is about to be transfered from a group of elements connected to one OR circuit to a group of the next OR circuit, then an error action may occur whose two output pulses appear simultaneously because a trial pulse may step over the two groups of OR circuits. Therefore, in order to minimize the rate of errors of this kind, the width of the trial pulses must be suitably narrow. Next, if a narrow trial pulse is applied during a transient response, then an error action where no output pulse appears may occur also. Therefore, the transient response of the elements of the shiftregisters etc. must be rapid enough.

Experimental equipment was manufactured with IC's etc., with attention to the above points, and some experiments were carried out. The results were relatively satisfactory.

Next, in regard to the field of application of variable probability circuits, an original application example which is characteristic for variable probability circuits may be considered. It is an example for the method of generation of stochastic interval processes, e.g., if the time varying signals are applied to the terminals of the regulation voltage, the probability varies with the time. Let us call this operation the probability modulation.

Thus, if the signals of any time function are applied as the regulation voltages and if a pulse sequence with known probability distribution is applied as the trial pulse, then an output pulse sequence with known probability distribution will be obtained. This method will be reported concretely in a separate manuscript.

Acknowledgement

We would like to thank Prof. Tsuneo Tamachi of Kyushu Univ., Prof. Itsuo Takanami, Prof. Yoshio Ebina, Mr. Kazutoshi Koga and Mr. Haruhiko Morichika of Yamaguchi Univ. for their kind discussions. And, we would like to thank Prof. Yasuyuki Iso of Kyushu Institute of Technology and Prof. Seigo Kanoh of Kyushu Univ. for their comments for this paper.

APPENDIX: Proof of probability in the shiftregister

Suppose that the n elements of the shiftregister S in Fig. 1 are $s_1, s_2, \ldots, s_n$, respectively, and s_1 is in the 1 state at $t=0$ as the initial condition. Further, suppose that the random shift-pulse sequence P arises according to the Poisson distribution with the mean value λ per unit time. Then the probability $P_k(t)$ that k pulses arise not later than t is

$$P_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}.$$

Thereupon, since the element s_i is in the 1 state at the time t for $k=nm+i$, $m=0$, 1, 2,..., the probability $P^i(t)$ that the element s_i is in the 1 state at t is

$$P^i(t) = \sum_{m=0}^{\infty} P_{nm+i}(t) = e^{-\lambda t} \sum_{m=0}^{\infty} (\lambda t)^{nm+i}/(nm+i)!.$$

Here, let $r=\lambda t$ apply and for $r \gg 1$ it may be considered that the above expression is approximately the same as in the case of $r=\infty$. Therefore,

$$P^i(t) \fallingdotseq \lim_{r\to\infty} \frac{1}{e^r} \left\{ \frac{r^i}{i!} + \frac{r^{n+i}}{(n+i)!} + \frac{r^{2n+i}}{(2n+i)!} + \cdots \right\}.$$

Differentiating numerators and denominators of the right hand side with respect to r,

$$= \lim_{r\to\infty} \frac{1}{e^r} \left\{ \frac{r^{i-1}}{(i-1)!} + \frac{r^{n+i-1}}{(n+i-1)!} + \cdots \right\}$$

$$\vdots$$

$$= \lim_{r\to\infty} \frac{1}{e^r} \left\{ \frac{r^1}{1!} + \frac{r^{n+1}}{(n+1)!} + \cdots \right\}$$

$$=\lim_{r\to\infty}\frac{1}{e^r}\left\{1+\frac{r^n}{n!}+\cdots\right\}$$

$$=\lim_{r\to\infty}\frac{1}{e^r}\left\{\frac{r^{n-1}}{(n-1)!}+\frac{r^{2n-1}}{(2n-1)!}+\cdots\right\}$$

$$\vdots$$

$$=\lim_{r\to\infty}\frac{1}{e^r}\left\{\frac{r^{i+1}}{(i+1)!}+\frac{r^{n+i+1}}{(n+i+1)!}+\cdots\right\}.$$

Summing up the above n identities,

$$n\cdot P^i(t)\fallingdotseq\lim_{r\to\infty}\frac{1}{e^r}\left\{1+r+\frac{r^2}{2!}+\cdots+\frac{r^i}{i!}+\frac{r^{i+1}}{(i+1)!}+\cdots\right\}$$

$$=1.$$

Therefore, if λt is sufficiently large,

$$P^i(t)\fallingdotseq 1/n. \qquad \text{(Q. E. D.)}$$

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