Electromagnetic Forces on an Infinite Plane Conductor by a Circular Exciting Coil in an Alternating Magnetic Field

By Nobuo TAKEHIRA*, Akio TANAKA** and Keiiti TODA***

(Received July 15, 1977)

Abstract

If a circular coil excited with AC is facing to an infinite plane conductor, an electromagnetic force is produced by eddy currents induced in the conductor. This force varies characteristically with the amplitude and frequency of the current in the coil, with the quality and thickness of the conductor and with the distance between the coil and the conductor.

So we make a precise analysis of these various characteristics by using Maxwell's equations. Furthermore, we make an experiment and obtain a very good agreement between the theoretical and the measured values of the force.

Introduction

When a conductor is placed close to an exciting coil, eddy currents are induced near the surface of it by the alternating magnetic field. The interaction between the eddy currents and the source magnetic field produces the electromagnetic force.

This force varies with the amplitude and frequency of the current carried in the coil, with various parameters (electrical conductivity, permeability and thickness) of the conductor and with the distance from the conductor to the coil. In this paper, we study the electromagnetic force between a circular coil and an infinite plane conductor. With respect to this problem, Basu and Srivastava[1], in the past, presented a theoretical analysis on the assumption that eddy currents are confined to the skin depth of the conductor. Furthermore, they solved the problem of a fast acting circuit breaker utilizing the repulsive force on the same assumption[2]. Lajotte and Drouet[3], however, pointed out that large discrepancies were, at lower frequencies, found between their measurements and the theoretical values according to Basu and Srivastava[2]. Then we make a precise analysis by Maxwell's equations and compare the theoretical values with the experimental ones.

Consequently the experimental results become in good agreement with the theoretical analysis.

Theory

The coordinate system of a circular exciting coil and an infinite plane conductor

* Department of Mechanical and Electrical Engineering, Tokuyama Technical College
** Department of Electrical Engineering, Ube Technical College
*** Department of Information and Electronics Engineering, Tokuyama Technical College

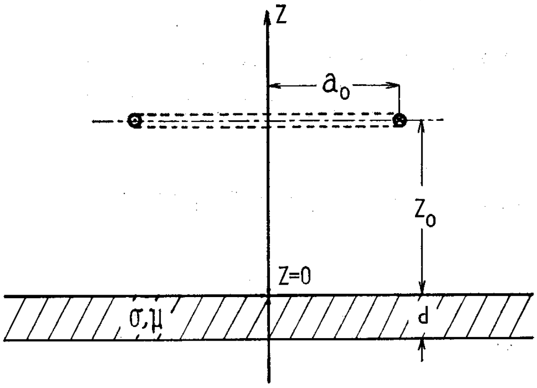

Fig. 1 A circular coil of a filament current loop facing to an infinite plane conductor and its coordinate system.

is shown on Fig. 1. The conductor is homogeneous and isotropic, and its electrical conductivity σ and permeability μ are respectively constant. The coil is assumed to be a filament loop, and the current I (rms) is flowing with angular frequency ω. On these assumptions, Maxwell's equations to be solved in the quasi-stationary state, in which displacement current can be negligible, are as follows:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \tag{1}$$

$$\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t \tag{2}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{3}$$

Inside of the conductor, the following equations are added moreover:

$$\boldsymbol{J} = \sigma \boldsymbol{E} \tag{4}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{5}$$

where a time variable exp $(j\omega t)$, which is included in all the electro-magnetic quantities, is omitted.

In order to solve these equations, since the phenomena are axially symmetric, we can use the vector potential $\boldsymbol{A}$ in the cylindrical coordinate system (r, θ, z). The vector potential is defined as follows:

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{6}$$

Outside of the conductor, from equation (1) we first obtain

$$\nabla \times \nabla \times \boldsymbol{A} = \mu_0 \boldsymbol{J} \tag{7}$$

Inside of the conductor, on the other hand, the following equation is given by equation (1), (2), (4) and (5).

$$\nabla \times \nabla \times \boldsymbol{A} = -j\omega\sigma\mu \boldsymbol{A} \tag{8}$$

where equation (3) can be satisfied by equation (6) necessarily.

Since both the exciting current and the vector potential have only azimuthal components in the cylindrical coordinate system, the following differential equation is obtained outside of the conductor:

$$\frac{\partial^2 A}{\partial r^2}+\frac{1}{r}\frac{\partial A}{\partial r}+\frac{\partial^2 A}{\partial z^2}-\frac{A}{r^2}=-\mu_0 I\delta(r-a_0)\delta(z-z_0) \tag{9}$$

where $\delta(\ )$ is the Dirac delta function.

Similarly we can reduce equation (8) to

$$\frac{\partial^2 A}{\partial r^2}+\frac{1}{r}\frac{\partial A}{\partial r}+\frac{\partial^2 A}{\partial z^2}-\frac{A}{r^2}=j\omega\sigma\mu A \tag{10}$$

Determining the solution by the use of the boundary conditions, the reflected vector potential A_{ref}, which is produced by eddy currents, becomes equation (11)[4),5)]:

$$A_{ref}=-\frac{\mu_0 a_0 I}{2}\int_0^\infty J_1(\zeta r)J_1(\zeta a_0)e^{-\zeta(z+z_0)}\lambda\, d\zeta \tag{11}$$

where $J_1(\ )$ is a first-order Bessel function. λ, which is called reflection coefficient, is a complex function expressing the influence of the conductor. It is defined by the following equation:

$$\lambda=\frac{\left(\gamma-\frac{\mu}{\mu_0}\zeta\right)\left(\gamma+\frac{\mu}{\mu_0}\zeta\right)-\left(\gamma-\frac{\mu}{\mu_0}\zeta\right)\left(\gamma+\frac{\mu}{\mu_0}\zeta\right)e^{-2\gamma d}}{\left(\gamma+\frac{\mu}{\mu_0}\zeta\right)\left(\gamma+\frac{\mu}{\mu_0}\zeta\right)-\left(\gamma-\frac{\mu}{\mu_0}\zeta\right)\left(\gamma-\frac{\mu}{\mu_0}\zeta\right)e^{-2\gamma d}} \tag{12}$$

where

$$\gamma=\sqrt{\zeta^2+j\omega\sigma\mu} \tag{13}$$

By using the vector potential A_{ref}, it can be easy to obtain the r component of the reflected magnetic flux density B_r contributing to the electromagnetic force in the axial direction. That is,

$$\begin{aligned}B_r&=-\partial A_{ref}/\partial z\\&=-\frac{\mu_0 a_0 I}{2}\int_0^\infty \zeta J_1(\zeta r)J_1(\zeta a_0)e^{-\zeta(z+z_0)}\lambda\, d\zeta\end{aligned} \tag{14}$$

Using equation (14), we can evaluate the force F as follows:

$$\begin{aligned}F&=\mathrm{Re}\left\{a_0\int_0^{2\pi} I^*\cdot Br\Big|_{z=z_0}^{r=a_0}\, d\theta\right\}\\&=-\mu_0\pi a_0^2 I^2\int_0^\infty \zeta J_1^2(\zeta a_0)e^{-2\zeta z_0}\,\mathrm{Re}\{\lambda\}\, d\zeta\end{aligned} \tag{15}$$

where * indicates complex conjugate and Re represents the real part.

These are the theoretical results of a filament current loop. We will now use these results to evaluate the electromagnetic force by a multi-layer circular coil of more practical interest as shown on Fig. 2. Applying the principle of linear superposition, the force of this coil is obtained by integration of the force of a number of filament

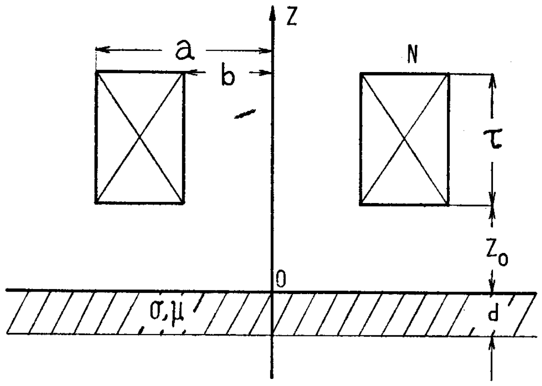

Fig. 2 A multi-layer circular coil and its coordinate system.

current loops over the cross section of the coil. After all, we may integrate equation (15) in the radial direction (from b to a) and in the axial direction (from z_0 to $z_0+\tau$). Then, the current density J in a N-turn multi-layer coil is given by

$$J=\frac{NI}{\tau(a-b)} \tag{16}$$

Therefore, the total electromagnetic force acting on the infinite plane conductor is as follows[5]:

$$\begin{aligned} F&=-\frac{\mu_0\pi N^2I^2}{\tau^2(a-b)^2}\int_0^\infty \zeta\left[\int_b^a rJ_1(\zeta r)dr\right]^2\cdot\left[\int_{z_0}^{z_0+\tau} e^{-\zeta z}dz\right]^2\cdot\mathrm{Re}\,\{\lambda\}\,d\zeta \\ &=-\frac{\mu_0\pi N^2I^2}{\tau^2(a-b)^2}\int_0^\infty \frac{S^2(\zeta)}{\zeta^5}e^{-2\zeta z_0}(1-e^{-\zeta\tau})^2\,\mathrm{Re}\,\{\lambda\}\,d\zeta \end{aligned} \tag{17}$$

where

$$S(\zeta)=\int_{b\zeta}^{a\zeta} xJ_1(x)dx=\zeta^2\int_b^a rJ_1(\zeta r)dr \tag{18}$$

Experiment

A schematic diagram of the experimental arrangement is shown on Fig. 3. A

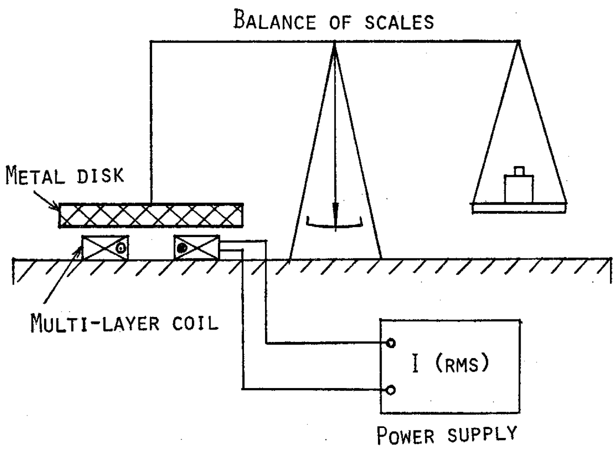

Fig. 3 Experimental arrangement.

nonmagnetic metal disk, which has a large diameter enough to be regarded as an infinite plane conductor, is coaxially mounted above the circular coil. The force acting on the disk is measured by a balance of scales. And the coil is energized in constant current mode using an oscillator and power amplifier.

The dimensions of the circular coil and the metal disks used in the experiments are shown on Table 1 and Table 2.

Table 1. Dimensions of multi-layer coil.

$a=26.98$ (mm),	$b=9.5$ (mm)
$\tau=12.0$ (mm),	$N=206$ (turns)
$I=5$ (A)	

Table 2. Dimensions of conductors.

Quality	Shape	Diameter (mm)	Thickness d (mm)	Conductivity σ ($\times 10^7$ S/m)	Permeability μ (H/m)
Copper (Cu)	Disk	130	2.0	4.19	μ_0 ($4\pi \times 10^{-7}$)
Aluminum (Al)	Disk	130	2.0	3.54	μ_0
	Disk	130	12.0	3.54	μ_0
Brass	Disk	130	2.0	1.60	μ_0

Experimental Results

Fig. 4 shows frequency characteristics of the materials which have the different conductivity and the same thickness. These characteristic curves are found to indicate the same tendency respectively. From 100 Hz to 1 kHz, both the variation with the frequency and the dependence on the conductivity of the force are large. Under 100 Hz or over 1 kHz, however, the dependence of the force on these factors is small. This is due to the relationship between skin depth and the thickness of the conductor. At lower frequencies the skin depth is much larger than the thickness, and then the former is smaller than the latter at higher frequencies. The frequencies where the skin depth is equal to the thickness (2 mm) are 1.5 kHz (copper), 1.8 kHz (aluminum) and 4.0 kHz (brass). Furthermore, it is evident that the force increases as either the frequency or the conductivity becomes high.

The frequency characteristics of the materials having the same conductivity and the different thickness is shown as Fig. 5. Although the force acting on the thick conductor is larger at low frequencies, it shows the same value as the force on the thin conductor at a certain frequency (where the thickness of the thin conductor is nearly equal to the skin depth). At the higher frequencies more than this frequency, the two curves of the force satulate gradually towards the same constant value, which is ob-

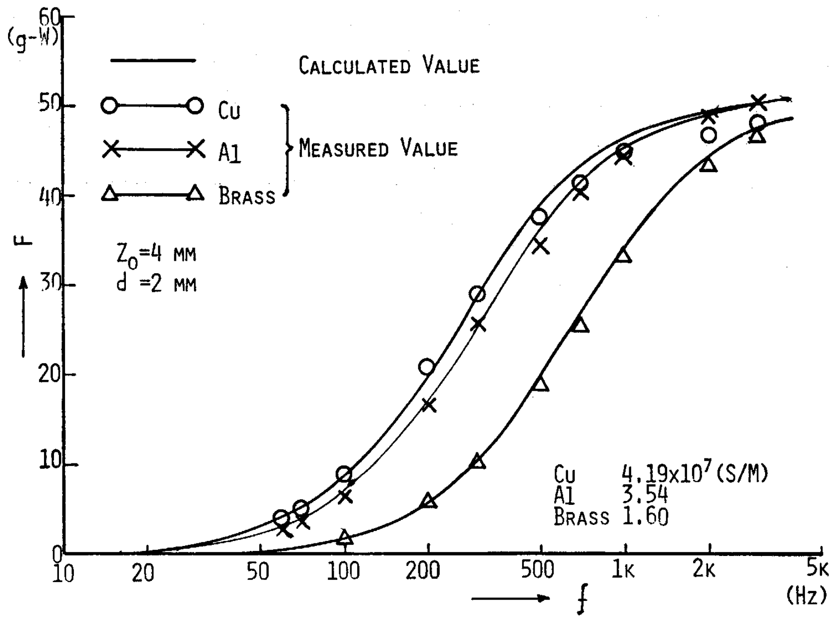

Fig. 4 Relationship between electromagnetic force and frequency.

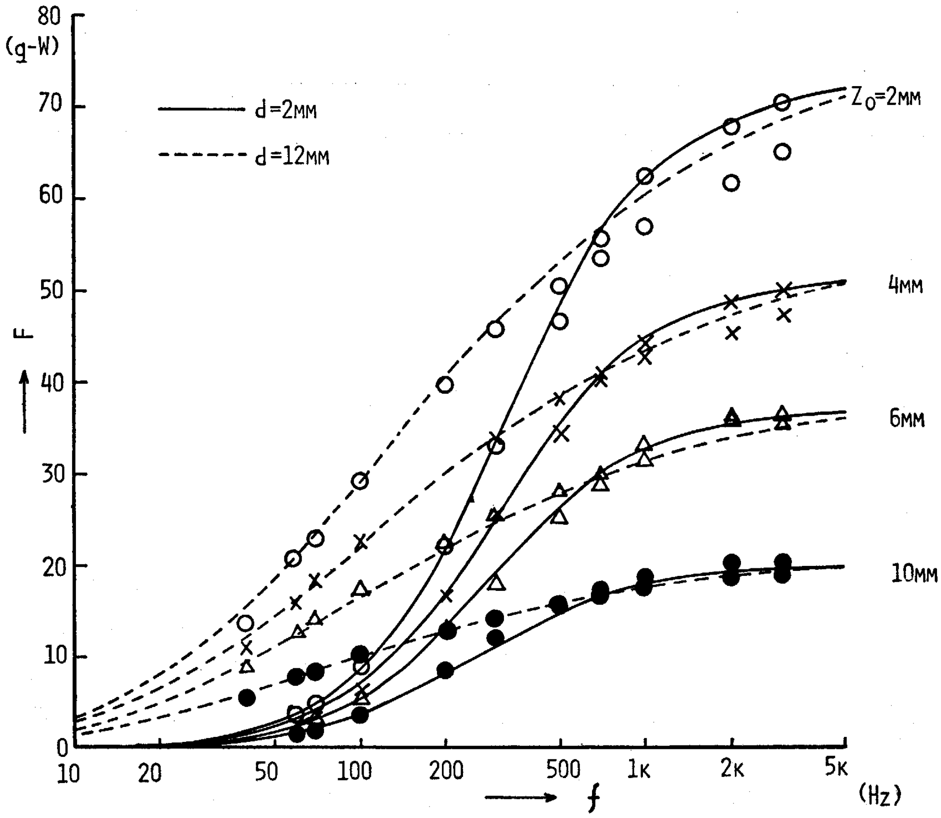

Fig. 5 Relationship between electromagnetic force and frequency (different thickness).

tained in the case of perfect conductor, with the thin conductor slightly exceeding the thick one in the force.

Fig. 6 shows the distance characteristics between the coil and the conductor. In

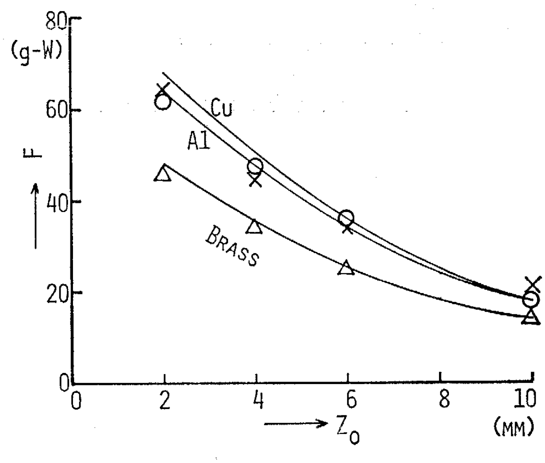

Fig. 6 Relationship between electromagnetic force and distance.

the case of any conductor, the force decreases exponentially as the distance increases.

Conclusion

In the present paper, the theoretical analysis of the electromagnetic force between the circular coil and the infinite plane conductor has been made by field theory. And the repulsive force acting on the nonmagnetic conductor has been measured. As a result, the theoretical predictions agree with the measurements very well.

The results are as follows:

1) There is a complex function called reflection coefficient which has a great influence on the electromagnetic force. Only in the reflection coefficient, various parameters (conductivity, permeability, thickness of the conductor and the exciting frequency) can exist. Therefore, this coefficient is seemed to be of great importance.

2) The force increases as the value of either the conductivity or the frequency increases. The increasing distance between the coil and the conductor makes the force decrease. Moreover, the skin depth, which is related with the frequency, will express the influence of the conductor thickness. At high frequencies, the force will approach the value of the perfect conductor.

Now that the various characteristics of the force at wide range of the frequency have been made evident and the theoretical analysis has been confirmed to be proper, we believe that this analysis can be applied to the optimization of various apparatus (magnetic levitation, magnetic metal forming and so on) utilizing the electromagnetic force.

References

1) Basu, S. and Srivastava, K. D.: "Electromagnetic Forces on a Metal Disk in an Alternating Magnetic Field", IEEE Trans. PAS, **88** [8], 1281–1285 (1969)

2) Basu, S. and Srivastava, K. D.: "Analysis of a Fast Acting Circuit Breaker Mechanism", IEEE Trans. PAS, **91**, 1197–1203 (1972)

3) Rajotte, R. J. and Drouet, M. G.: "Experimental Analysis of a Fast Acting Circuit Breaker

Mechanism — Electrical Aspects", IEEE Trans. PAS, **94**, 89–96 (1975)
4) Onoe, M.: "An Analysis of a Finite Solenoid Coil Near a Conductor", JIEEJ, **88**, 1894–1902 (1968) (in Japanese)
5) Toda, K., Takehira, N. and Tanaka, A.: 1976 National Convention Record I. E. E. J., No. 477, (1976) (in Japanese)