Transmission of the wave past a rigid vertical thin Barrier (3)

By Kazuo KANAYAMA

(Received June 24, 1977)

Abstract

The results of an experimental and theoretical investigation on the damping action of the submarged vertical thin barrier are presented.

The experimental data also are compared with published theories. Further more, in this paper, a new theory is presented for the transmission of wave passing through a submerged plane barrier. This theory is based on a consideration of the Wave Maker Theory by T. H. Havelock.[1]

1. Introduction

Theoretical and experimental results are presented for the transmission of water waves, passing over a rigid submarged vertical thin barrier located between regions of given depth and some distance above the sea bottom.

One of the problems in the design of such a structure is developed here together with the results of laboratory tests on a model structure under a variety of wave conditions.

The theory is based on consideration of wave power transmission, momentum transmission and wave maker theory.

Laboratory data are presented to show that these theory are useful from the engineering standpoint.

2. Theoretical Development

Energy Transmission Theory[2]

Author developed a theory for the partial transmission of gravity wave for a submarged vertical thin barrier (See Fig. 1).

We found that

$$K_t=\sqrt{\frac{\sinh 2k_0(h-d-d')+\sinh 2k_0h-\sinh 2k_0(h-d)+2k_0(h-d')}{\sinh 2k_0h+2k_0h}} \tag{1}$$

where k_0 denotes the wave number, h is a still water depth, d denotes a upper edge of the barrier and d' denotes a length of vertical barrier.

When $d\to 0$, $d'=d$. From Eq. (1), we immediately have next equation.

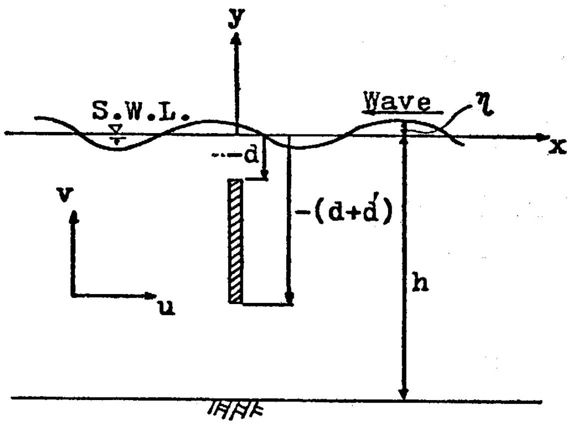

Fig. 1 Notation and Definition of Symbols.

$$K_t = \sqrt{\frac{\sinh 2k_0(h-d) + 2k_0(h-d)}{\sinh 2k_0 h + 2k_0 h}} \tag{2}$$

Eq. (2) is coincide with the Wiegel's theory[3] for the transmission of wave past a rigid vertical thin barrier extending from the water surface to some depth below the surface.

On the other hand, when $d' \to h-d$.

We immediately have next relation.

$$K_t = \sqrt{\frac{\sinh 2k_0 h - \sinh 2k_0(h-d) + 2k_0 d}{\sinh 2k_0 h + 2k_0 h}} \tag{3}$$

Eq. (3) is coincide with the Fuch's theory[4]) for the transmission of wave past a rigid vertical thin barrier extending from the sea bottom to some distance below the surface.

Momentum Transmission Theory[5]

In 1974 author developed a theory for the partial transmission of finite amplitude wave for this structure (See Fig. 2).

We found that

$$K_t = \sqrt[4]{\frac{\sinh 4k_0(h-d-d') - \sinh 4k_0(k-d) + \sinh 4k_0 h + 4k_0(h-d')}{\sinh 4k_0 h + 4k_0 h}} \tag{4}$$

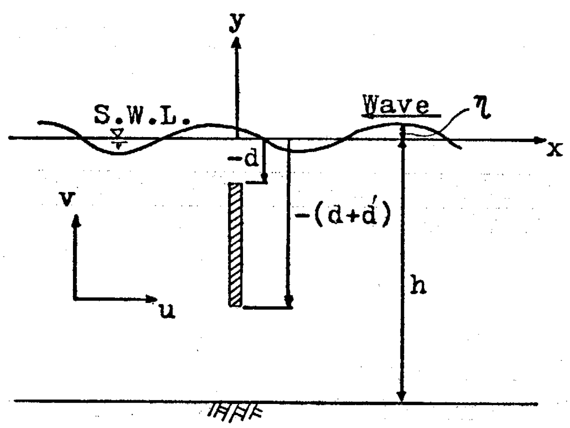

Fig. 2 Notation and Definition of Symbols.

In the Eq. (4), when $d \to 0$, $d' = d$. We immediately have next relation

$$K_t = \sqrt[4]{\frac{\sinh 4k_0(h-d) + 4k_0(h-d)}{\sinh 4k_0h + 4k_0h}} \tag{5}$$

Eq. (5) is coincide with the author's equation[6] for the rigid vertical thin barrier extending from the water surface to some depth below the surface.

On the other hand, when $d' = h - d$. We immediately have next relation

$$K_t = \sqrt[4]{\frac{\sinh 4k_0h - \sinh 4k_0(h-d) + 4k_0d}{\sinh 4k_0h + 4k_0h}} \tag{6}$$

It is a interesting point that Eq. (6) denotes a wave transmission coefficient for the rigid vertical thin barrier extending from the sea bottom to some distance below the water surface.

Wave Maker Theory

Concider the partial transmission of uniform small amplitude wave in finite water depth for a submarged vertical thin barrier that occupied a region between some distance above the sea bottom and a given depth.

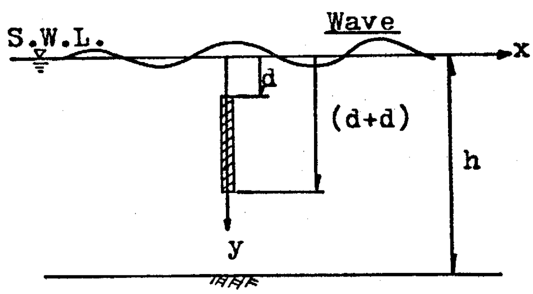

Fig. 3 Notation and Definition of Symbols.

We consider a two dimensional motion, in which a vertical plane occupies the line $x=0$, d, y, h, where the axis of y is taken vertically downwards and $y=0$ is the mean water surface. (See Fig. 3).

The wave motion, being such a could be produced from the rest by natural forces, is irrotational and simple harmonic.

The fluid is assumed to be incompressible and invisid so that a velocity potential $\phi(x, y, t)$ exists.

The velocity potential satisfies the next relation.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{7}$$

Neglecting the square of the fluid velocity at the free surface, and omitting the effect of capillarity, the condition at the surface is given as next relation.

$$\left(\frac{\partial^2 \phi}{\partial t^2} - g\frac{\partial \phi}{\partial y}\right)_{y=0} \Bigg| = 0 \tag{8}$$

Furthermore, the surface elevation is given by next relation.

$$\eta=\frac{1}{g}\left(\frac{\partial\phi}{\partial t}\right)\bigg|_{y=0} \tag{9}$$

For simple harmonic motion we assume a time factor $e^{i\sigma t}$, and equation (8) gives

$$\left(k_0\phi+\frac{\partial\phi}{\partial y}\right)\bigg|_{y=0}=0 \tag{10}$$

Furthermore, the boundary condition on the bottom $y=h$ is that no flow occures across this surface, ie;

$$\frac{\partial\phi}{\partial y}\bigg|_{y=h}=0 \tag{11}$$

The corresponding elementary solution of equation (7) are

$$\phi=e^{i(\sigma t-k_0x)}\cosh k_0(y-h) \tag{12}$$

where k_0 is the real positive root of

$$gk_0\tanh k_0h=\sigma^2 \tag{13}$$

and

$$\phi_j=e^{i\sigma t-kjx}\cos k_j(y-h) \tag{14}$$

where k_j is real positive root of next relation.

$$gk_j\tan k_jh+\sigma^2=0 \tag{15}$$

This equation has an infinite sequence of real roots, together with an imaginary root ik_0.

In the case of no barrier in the water, equation (12) and (14) give a solution of equation (7). From the linearity of equation (7), also next equation is satisfied equation (7).

$$\Phi=A'\phi+\Sigma B'\phi_j \tag{16}$$

where constant A' and B' are decided from boundary conditions.

In the case of existing of barrier in the water, we have the additional boundary condition.

$$\frac{\partial\phi}{\partial x}\bigg|_{x=0}=f(y)\sin\sigma t \tag{17}$$

where, we assume then the possibility of expanding a function $f(y)$ in the form

$$f(y)=A\cosh k_0(y-h)+\Sigma B_j\cos k_j(y-h) \tag{18}$$

where the summation extends over the real positive roots of the equation (15).

We find that the coefficient A and B_j are given by next equation.

$$A=\frac{4k_0}{2k_0h+\sinh 2k_0h}\int_0^h f(y)\cosh k_0(y-h)\,dy \tag{19}$$

$$B_j=\frac{4k_j}{2k_jh+\sin 2k_jh}\int_0^h f(y)\cos k_j(y-h)\,dy \tag{20}$$

From equation (17) and (18), we derived next relation.

$$\left.\frac{\partial \Phi}{\partial x}\right|_{x=0}=\{A\cosh k_0(y-h)+\Sigma B_j\cos k_j(y-h)\}\sin\sigma t \tag{21}$$

On the other hand, we obtain next relation from the equation (12), (14) and (16).

$$\left.\frac{\partial \Phi}{\partial x}\right|_{x=0}=-A'k_0e^{i\sigma t}\cosh k_0(y-h)-\Sigma B'k_je^{i\sigma t}\cos k_j(y-h) \tag{22}$$

From the equation (21) and (22), furthermore, we obtain next relation.

$$A'=Ak_0{}^{-1},\ B'=B_jk_j{}^{-1} \tag{23}$$

Substituting equation (23) into equation (16), we obtain after development

$$\begin{aligned}\Phi=&Ak_0^{-1}\cos(\sigma t-k_0x)\cosh k_0(y-h)\\&-\Sigma B_jk_j^{-1}e^{-k_jx}\sin\sigma t\cos k_j(y-h)\end{aligned} \tag{24}$$

Equation (24) gives a velocity potential in the case of existing barrier in the water. Considering the partial standing wave will arise on the barrier, we use a velocity distribution of partial standing wave for the boundary condition:

$$u\,|_{x=0}=\frac{gTa}{L}\left(1-\frac{a_r}{a}\right)\frac{\cosh k_0(y-h)}{\cosh k_0h}\sin\sigma t \tag{25}$$

where, a_r and a denote a reflected wave amplitude and incident wave amplitude respectively.

From equation (25) and (17), we obtain next relation for $f(y)$, $0<y<d$ and $d+d'<y<h$

$$f(y)=\frac{gTa}{L}(1-\sqrt{1-K_t^2})\frac{\cosh k_0(y-h)}{\cosh k_0h} \tag{26}$$

$$d<y<d+d'\quad f(y)=0 \tag{27}$$

where Kt is a wave transmission coefficient.

The value of coefficient A and B_j follow from equation (19) and (20), and from equation (24) we deduced the velocity potential in this case;

$$\begin{aligned}\phi=&\frac{gTa}{2u}(1-\sqrt{1-K_t^2})\cosh k_0(y-h)\sin(\sigma t-k_0x)\\&\times\frac{\sinh 2k_0(d-h)+\sinh 2k_0h-\sinh 2k_0(d+d'-h)+2k_0d-2k_0(d+d'-h)}{\cosh k_0h(\sinh 2k_0h+2k_0h)}\end{aligned} \tag{28}$$

where we neglect the flow disturbance neighboring the vertical plane barrier.

Substituting equation (28) into surface condition we lead to a next relation for surface elevation.

$$\eta=a(1-\sqrt{1-K_t^2})\cos(\sigma t-k_0x)$$
$$\frac{\sinh 2k_0(d-h)+\sinh 2k_0h-\sinh 2k_0(d+d'-h)+2k_0d-2k_0(d+d'-h)}{\sinh 2k_0h+2k_0h} \tag{29}$$

On the other hand, we postulate the transmitted wave form by next relation

$$\eta t=a_t\sin(\sigma t-k_0x+\varepsilon) \tag{30}$$

Considering the continuous of the wave form at $x=0$, we derive the following equation from equation (23) and (24),

$$a_t=a(1-\sqrt{1-K_t^2})$$
$$\frac{\sinh 2k_0h+2k_0(h-d)-\sinh 2k_0(h-d)+\sinh 2k_0(h-d-d')}{\sinh 2k_0h+2k_0h} \tag{31}$$

where we neglect the phase angle ε.

From the equation (31), we derive the next relation for the transmission coefficient Kt.

$$K_t=\frac{2W}{1+W^2},$$
$$W=\frac{\sinh 2k_0h+2k_0(h-d)-\sinh 2k_0(h-d)+\sinh 2k_0(h-d-d')}{\sinh 2k_0h+2k_0h} \tag{32}$$

In the equation (32), when $d\to 0$ $d'=d$, we immediately have next relation

$$K_t=\frac{2F}{1+F^2},\qquad F=\frac{\sinh 2k_0(h-d)+2k_0(h-d)}{\sinh 2k_0h+2k_0h} \tag{33}$$

In the equation (33), it is a interesting point that Function F is coincide with the Wiegel's theory[3] for the rigid vertical extending from the surface to some depth below the water surface. Furthermore, in the equation (32), when $d'\to h-d$, we immediately have next relation.

$$K_t=\frac{2F'}{1+F'^2},\qquad F'=\frac{\sinh 2k_0h-\sinh 2k_0(h-d)+2k_0d}{\sinh 2k_0h+2k_0h} \tag{34}$$

In the equation (34), it is a interesting point that function F' is coincide with the fuch's theory[4].

3. Test Apparatus and Testing Method

Fig. 4 shows the principal parts of the apparatus used for the testing. The experi-

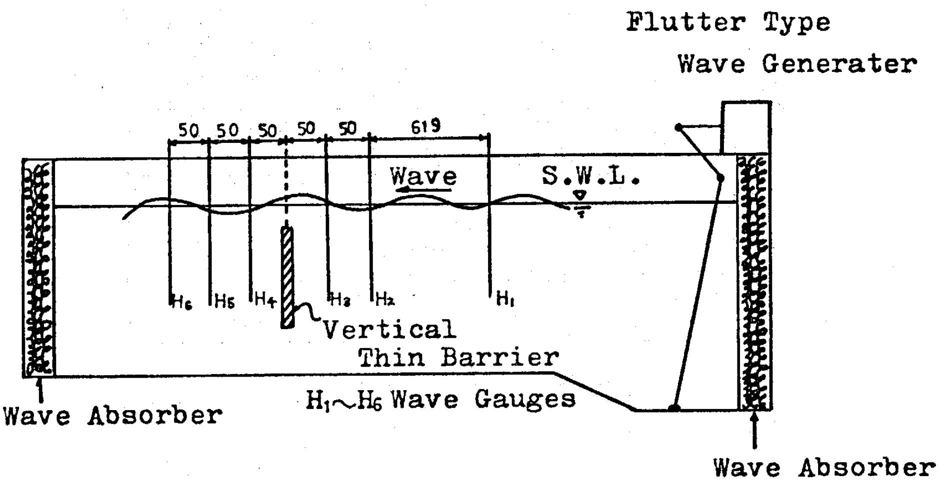

Fig. 4 Arrangement for the Test Channel.

ments were performed using a 0.7 meter wide, 0.9 meter depth and 20 meter long wave test channal. The rigid vertical thin barrier was fitted in to the channel about halfway between the wave generator and the opposite edge.

The wave generator was set to give a wave train of given height and period. The wave height were measured with the barrier instilled in the desired position. The wave generator was started and measurement were made on waves. Only the data obtained on the lee side wave mater used in computing the transmission coefficient *Kt*.

In analyzing the data, the first few waves were passed by as these not of constant period, therafter the analyzing was made for the next few waves. Considerable care was exercised in placing the barrier so that there would be no leakage between the barrier and the wall of the wave channel. Experimental condition were as following Table 1.

Table 1. Experimental Condition.

Wave steepness	0.0037 — 0.1056
Relative depth	0.11 — 0.43
Water depth	40 cm
Barrier length d′	8, 12, 16, 20, 24, 28, 32 cm
Barrier depth d	4 — 28 cm

Fig. 5 shows some examples wave record.

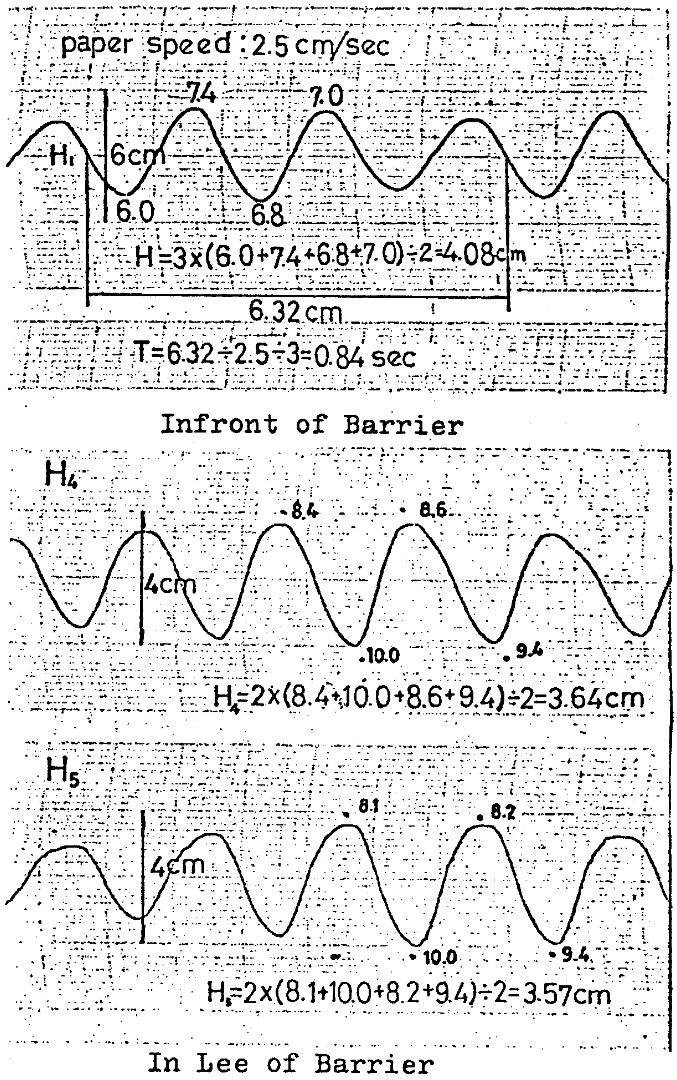

Fig. 5 Sample Wave Record.

4. Testing Results and Concideration

The experimental results are shown in Fig. 6, compared with the wave maker theory, momentum transmission theory and energy transmission theory.

In Fig. 6, it appears that these theories are useful to the engineering, but any improvement in the theory is needed.

The trend of increase in the value of transmission coefficient Kt with decreasing of value d'/h and increasing of relative depth h/L is consistent. This would be expected from the fact that, all the other condition being equal, the transported wave momentum and wave energy into the lee of barrier increase as the relative depth is increase and the value of d'/h is decrease.

Momentum transmission theory predict the transmission coefficient more closely than does the wave maker theory and energy transmission theory. Wave maker theory

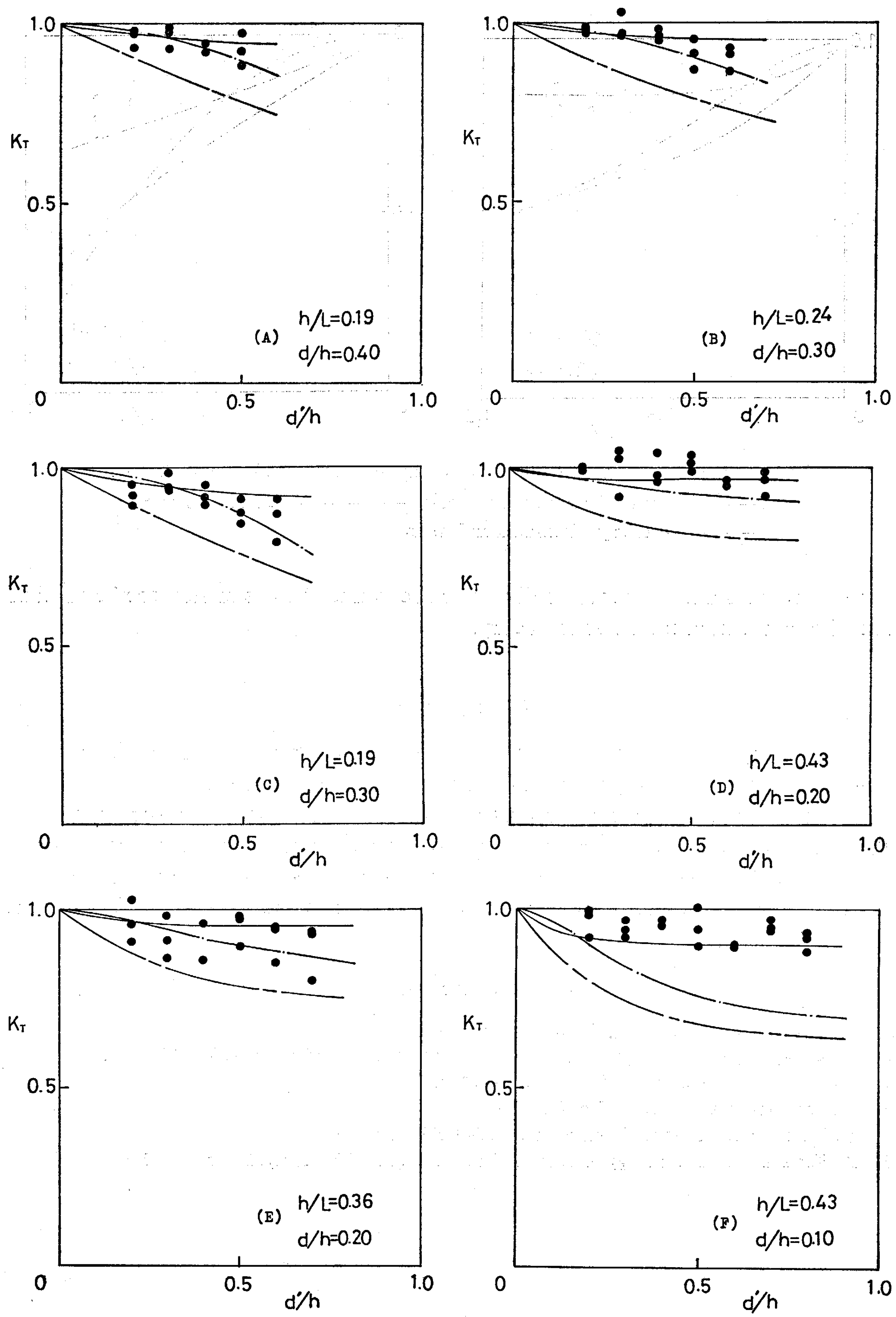
1.0
K_T
0.5
0
0.5
1.0
d'/h
(A)
h/L=0.19
d/h=0.40
(B)
h/L=0.24
d/h=0.30
(C)
h/L=0.19
d/h=0.30
(D)
h/L=0.43
d/h=0.20
(E)
h/L=0.36
d/h=0.20
(F)
h/L=0.43
d/h=0.10

Fig. 6 (A), (B), (C), (D), (E), (F).

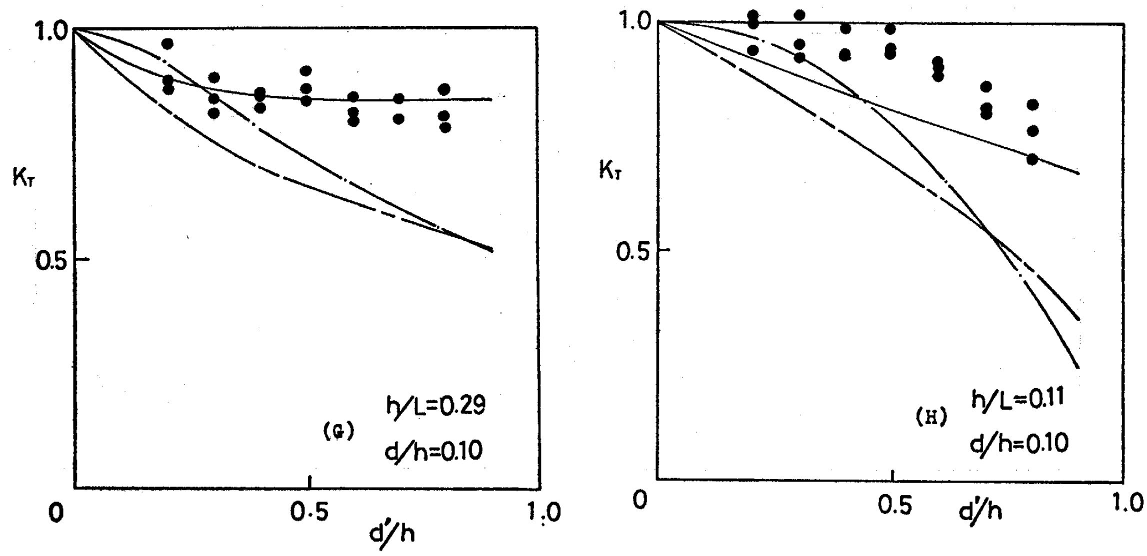

Fig. 6 Comparison of the Experimental Results with Calculated Results ((A)~(H)).
(—— Momentum Transmission Theory, ——·—— Wave Maker Theory, ——— - ——— Energy Transmission Theory.)

and energy transmission theory predicts a smaller value of transmission coefficient than does the momentum transmission theory.

5. Conclusion

The momentum transmission theory predicts the transmission coefficient adequately than does the energy transmission theory and wave maker theory.

A consistent trend of increasing transmission coefficient with decreasing a value of d'/h and with increasing of relative depth h/L is evident in the laboratory measurement.

The writer would like to express his appreciation to K. Tanaka for giving helpful sugestions, and to H. Ishida for performing the laboratorv experiments and calculation.

References

1) T. H. Havelock: Phil. Mag. S7 **8**, 569–576 (1929)
2) K. Kanayama: Technology Reports of The Yamaguchi University **1**, 571–578 (1976)
3) Wiegel. R. L A. S. C. E.: Vol. **WW1**, 1–12 (1960)
4) Fuchs: Trans A. G. U, **32**, 704–718 (1951)
5) K. Kanayama: Coastral Engineering, **21**, 393–399 (1974)
6) K. Kanayama: Technology Reports Of The Yamaguchi University, **1**, No. 4 (1976)