CONCEPTUAL EXTENSION OF STRESS INTENSITY TO AN ANGLED DEFECT II

—— A RHOMBIFORM CAVITY WITH ARBITRARY TIP ANGLES ——

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Singularity and distributions of the stresses at and around tips of an internal rhombiform cavity are examined by applying complex variable methods to the plane elastic problems of the defect-containing infinite sheet under symmetric and skew-symmetric loadings. The concept of the stress intensity in a crack problem is extended to the internal angled defect with an arbitrary tip angle, the general formulae for determining the stress singularity factors being given. It is shown that exact solutions for the general stress distributions as well as the local stresses can be derived by an effective use of the mapping function which composes the complex potentials.

Keywords: *Stress singularity factor, Strength of singularity, Rhombiform cavity, Conformal mapping, Schwartz-Christoffel transformation*

1. INTRODUCTION

The primary intention of the stress intensity factor in fracture mechanics is a perfect and unique description of the dominant and singular stress state at crack tip by means of a single parameter. In previous work[1] the concept of the stress intensity for a crack was extended to an externally cut V-shaped notch with an arbitrary included angle and a finite depth, which has been left to be solved notwithstanding its engineering importance. It was shown that the problem required the introduction of a mapping function with singularities of branch-point type and related complex potentials. It was further shown there that exact solutions for general distributions of the stresses as well as the stresses local to the notch tip could be derived by an effective use of the mapping function which composed the complex potentials.

In this work the analysis will be extended to an internally cut rhombiform cavity with an arbitrary tip angle under symmetric and skew-symmetric loadings, which bears also a great engineering importance. It will be shown that exact solutions for the general stress distributions as well as the stresses local to the tip of the internal cavity can be derived by a furhter effective use of the mapping function which composes the complex potentials.

2. INITIAL FORMULATION

The infinite sheet under symmetric and/or skew-symmetric loading weakened by an internal rhombiform cavity with an arbitrary tip angle 2β will be considered. The sheet lies in the complex z-plane, $z = x + iy$, with the tips and the other corners of the cavity described by $z = 0$, $-i2c$, $c\tan\beta - ic$ and $-c\tan\beta - ic$, and indicated by O, A, B and C, respectively, as illustrated in Figure 1, where $i = [-1]^{1/2}$.

For analyses we use a couple of complex potentials, $\phi(z)$ and $\chi(z)$, which are arbitrarily chosen analytic functions of the complex variable, z, but satisfy the required boundary conditions, and compose the well-known bi-harmonic Airy's stress function, $F(z) = Re[\bar{z}\,\phi(z) + \int^{z} dz\,\chi(z)]$.

For facilitation of the boundary condition consideration, an auxiliary complex plane, the ζ-plane, Figure 2, is introduced, and a function relationship $z = \omega(\zeta)$ is sought such that the unit circle, $\zeta = \sigma = e^{i\phi}$, and its exterior, D^+, conformally map into the boundary and the exterior region occupied by the sheet, Ω^+, Figure 1, respectively, with

$$\omega(i) = 0,\ \omega(-i) = -i2c,\ \omega(1) = c\tan\beta - ic$$

$$\text{and}\ \ \omega(-1) = -c\tan\beta - ic, \qquad (1)$$

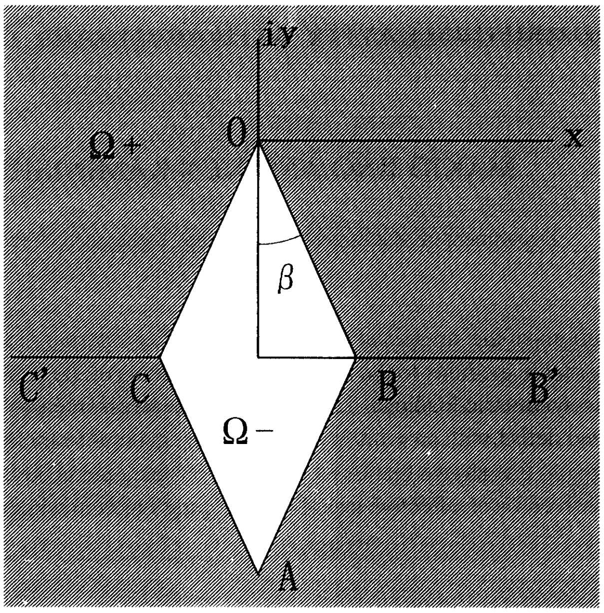

Fig. 1 Internal rhombiform cavity under symmetric and skew-symmetric loadings

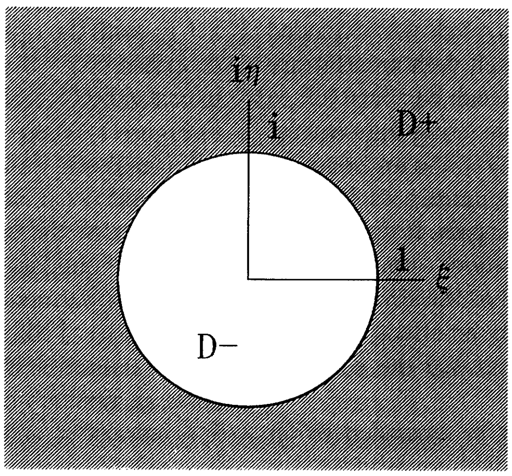

Fig. 2 Auxiliary complex plane, $\zeta = \xi + i\eta$

being defined. By application of the Schwartz-Christoffel transformation to this problem, it was found that the mapping function, $\omega(\zeta)$, can be given as a principal branch of

$$z = \omega(\zeta)$$
$$= C \int_i^\zeta d\zeta [\zeta^2+1]^{2n}[\zeta^2-1]^{1-2n}/2\zeta^2, \quad (2)$$

where n is related to 2β as

$$n = 1/2 - 2\beta/2\pi. \quad (3)$$

The constant C can be determined as shown in APPENDIX as

$$C = B(1/2, n)/\pi, \quad (4)$$

where B(p, q) is the betha function.

Thus obtained mapping function, $\omega(\zeta)$, is analytic for all finite points in D^+. The tips and the other corners of the cavity are described by the roots of $\omega'(\zeta) = 0$, which occur at $\zeta = \pm i$ and ± 1 on the boundary, the unit circle. The integral $\omega(\zeta)$, equation(2), can be expressed as

$$\omega(\zeta) = C \int_o^{(\zeta+1/\zeta)/2} dZ [Z^2/(Z^2-1)]^n, \quad (5)$$

which is characterized by the relationship propitious for the present analyses,

$$\omega(1/\zeta) = \omega(\zeta). \quad (6)$$

It should be noted that, if we assume integration starts at $\zeta = -i$ in equation(2), then the integral will be expressed, in place of equation(5), as

$$\omega(\zeta)$$
$$= \omega(-i) + C \int_o^{(\zeta+1/\zeta)/2} dZ [Z^2/(Z^2-1)]^n, \quad (5a)$$

where $\omega(-i) = -i2c$ from equation(1).

The complex potentials, $\phi(z)$ and $\chi(z)$, being functions of $z = \omega(\zeta)$, can also be considered as functions of ζ. Thus, we designate $\phi(z) = \phi[\omega(\zeta)]$ as $\phi(\zeta)$ and so on, which serves to minimize new notations and permits such definitions as $\phi'(z) = \phi'(\zeta)/\omega'(\zeta)$ and so forth. Primes are used to denote differentiation by the variable shown in the parentheses. Thus, the stresses, σ_ξ, σ_η and $\tau_{\xi\eta}$, and displacements, u_ξ and u_η, in curvilinear coordinates can be written as

$$\sigma_\xi + \sigma_\eta = 2\phi'(\zeta)/\omega'(\zeta) + \text{comp.conj.} \quad (7)$$

$$\sigma_\eta - \sigma_\xi + 2i\tau_{\xi\eta} = \{2/\overline{\omega'(\zeta)}\} \times$$
$$[\omega(\zeta) d\{\phi'(\zeta)/\omega'(\zeta)\}/d\zeta + \chi'(\zeta)], \quad (8)$$

$$2\mu(u_\xi + iu_\eta) = \kappa\phi(\zeta)$$
$$- \omega(\zeta)\overline{\phi'(\zeta)}/\overline{\omega'(\zeta)} - \overline{\chi(\zeta)}, \quad (9)$$

where μ and κ are elastic constants of the material, and bars denote complex conjugates.

In terms of the functions $\phi(\zeta)$ and $\chi(\zeta)$ the load-free boundary condition on ABOC of Figure 1 can be expressed as[2]

$$\phi(\sigma) + \omega(\sigma)\overline{\phi'(\sigma)}/\overline{\omega'(\sigma)} + \overline{\chi(\sigma)}$$
$$= \text{constant}, \quad (10)$$

and the solution requires the determination of the complex potentials $\phi(\zeta)$ and $\chi(\zeta)$ which are analytic in D^+ and satisfy the boundary condition(10).

When necessary, $\omega(\zeta)$, equation(5), can be developed in a power series in a domain of interest. Around the tip of the cavity, $|(\zeta+1/\zeta)/2| < 1$, $\omega(\zeta)$ can be expanded around $\zeta = i$ as

$$\omega(\zeta) = (iC/\nu) e^{+i(\pi/2)\nu} \sum_{k=o}^{\infty} a_k [(\zeta+1/\zeta)/2]^{\nu+2k},$$
$$\nu = 1 + 2n \quad (11)$$

by term-by-term integration after expansion around $\zeta = i$ of the integrand in a power series with respect to Z for $|Z| < 1$, where Z is given by

$$Z = (\zeta + 1/\zeta)/2. \quad (11a)$$

The coefficients, a_k, are defined by

$$a_o = 1, \text{ and}$$

$$a_k = \{\nu/(\nu+2k)\}(n+k-1)\cdots(n+1)n/k!$$
$$(k = 1, 2, 3 \cdots). \quad (11b)$$

For large $|\zeta|$, $|Z| > 1$, $\omega(\zeta)$ can be expanded around $\zeta = i$ as

$$\omega(\zeta) = C \sum_{k=0}^{\infty} b_k [2/(\zeta + 1/\zeta)]^{1-2k} - ic, \quad (12)$$

by term-by-term integration of the integrand expanded around $\zeta = i$ in a power series with respect to Z for $|Z| > 1$, where Z is given in equation(11a) again. The coefficients, b_k, are

$$b_o = 1, \text{ and}$$

$$b_k = -\{1/(2k-1)\}(n+k-1)\cdots(n+1)n/k!$$

$$(k = 1, 2, 3 \cdots). \quad (12a)$$

In terms of the formulations developed above the character of the stresses induced by the presence of the cavity will now be examined. To do this let $\phi(\zeta)$ first be defined in the interior of the unit circle, Figure 2, as

$$\phi(\zeta) = -\omega(\zeta)\bar{\phi}'(1/\zeta)/\bar{\omega}'(1/\zeta) - \bar{\chi}(1/\zeta),$$

$$\zeta \in D^-, \quad (13)$$

then the function $\phi(\zeta)$ is extended into D^-, following the extension concept of Muskhelishvili[3]. You will find the extended $\phi(\zeta)$ analytic in D^- excluding $\zeta = 0$. The function $\chi(\zeta)$ can now be expressed as

$$\chi(\zeta) = -\bar{\phi}(1/\zeta) - \bar{\omega}(1/\zeta)\phi'(\zeta)/\omega'(\zeta),$$

$$\zeta \in D^+, \quad (14)$$

which is clearly analytic for all the finite points in D^+. In equations(13) and (14) the bar notation is defined by

$$\bar{f}(1/\zeta) = \overline{f(1/\bar{\zeta})}. \quad (15)$$

3. DETERMINATION OF COMPLEX POTENTIAL $\phi(\zeta)$

Since we now know from the boundary condition consideration that $\chi(\zeta)$ can be expressed as equation(14), the problem reduces to the determination of $\phi(\zeta)$ which satisfies the loading conditions at infinity.

Examination of the function $\omega(\zeta)$, equation(11), suggests that $2\phi(\zeta)$ can be represented as

$$2\phi(\zeta) = iCA[\nu\,\omega(\zeta)/iC]^{\lambda}, \quad (16)$$

where A and λ are constants which depend on 2β; λ is assumed to be real and positive, $\lambda > 0$, in order for the displacements to be bounded at $\zeta = i$. The corresponding $2\chi(\zeta)$ is, from equation(14),

$$2\chi(\zeta) = iCB[\nu\,\omega(\zeta)/iC]^{\lambda}, \quad (17)$$

$$B(\zeta) = \bar{A}\,\bar{\varepsilon}^{\lambda}(\zeta) + A\lambda\,\bar{\varepsilon}(\zeta), \quad (17a)$$

where $\bar{\varepsilon}(\zeta)$ is defined by

$$\bar{\varepsilon}(\zeta) = -\bar{\omega}(1/\zeta)/\omega(\zeta), \quad (18)$$

which can be expressed, from the relationship(6), as

$$\bar{\varepsilon}(\zeta) = -\bar{\omega}(\zeta)/\omega(\zeta). \quad (18a)$$

On the load-free boundary, ABOC, $\varepsilon(\zeta) = e^{\mp i2\beta} = e^{\pm i2\gamma}$, where $2\gamma = 2\pi - 2\beta$. Then

$$B(\zeta) = \bar{A}\,e^{\pm i\lambda 2\gamma} + A\lambda\,e^{\pm i2\gamma} \; (\zeta = \sigma). (19)$$

Thus, $B(\sigma)$ turns out to be a pair of constants, which must be identical. Equating both the constants in equation(19) reduces to

$$Re A[\lambda \sin 2\gamma + \sin\lambda 2\gamma]$$

$$+ i Im A[\lambda \sin 2\gamma - \sin\lambda 2\gamma] = 0, \quad (20)$$

which implies that if A is real, i.e., for symmetric loading in the x-direction at infinity, λ must be the real part of solutions of the eigen equation,

$$\lambda \sin 2\gamma + \sin\lambda 2\gamma = 0. \quad (21)$$

If A is imaginary, i.e., for skew-symmetric loading in the y-direction at infinity, λ must be the real part of solutions of

$$\lambda \sin 2\gamma - \sin\lambda 2\gamma = 0. \quad (22)$$

In both the cases $B(\zeta)$ reduces to

$$B(\zeta) = A\lambda\cos 2\gamma + \bar{A}\cos\lambda 2\gamma \; (\zeta = \sigma). (22a)$$

In the area Ω^+, Figure 1, on the other hand, it will be found that $B(\zeta)$ varies with ζ, since $\bar{\varepsilon}(\zeta)$ varies there. For instance, along the line C'C and BB' $\bar{\varepsilon}(\zeta)$ varies as

$$\bar{\varepsilon}(\zeta) = -(x + ic)/(x - ic)$$

$$= (c^2 - x^2)/(c^2 + x^2) - i2cx/(c^2 + x^2)$$

$$= \exp[i\tan^{-1}\{-2cx/(c^2-x^2)\}],$$

which varies with x. If the counter-clockwise angle, in polar coordinates with pole at $z = 0$, with the positive y-axis is denoted by θ, then $x = c\tan(\pi + \theta)$, and

$$\tan^{-1}\{-2cx/(c^2-x^2)\} = \tan^{-1}[-\tan(2\pi + 2\theta)]$$

$$= -(2\pi + 2\theta).$$

It follows that

$$\bar{\varepsilon}(\xi) = e^{-i(2\pi+2\theta)} = e^{-i2\theta}, \quad (23)$$

and

$$B(\zeta) = A\lambda\,e^{-i2\theta} + \bar{A}\,e^{-i\lambda 2\theta}$$

$$(\text{ on C'C and BB'}), \quad (24)$$

which is smoothly continued at $\theta = \pm \gamma$ onto the value of $B(\zeta) = B(\sigma)$ given in equation(19).

In the complex ζ - plane, where the mapping function, $\omega(\zeta)$, is expressed as in equation(11) or (12) in accordance with the ζ area, it is understood that

$$B(\zeta) = A\lambda \cos 2\gamma + \bar{A} \cos \lambda 2\gamma$$

$$(\,|\,(\zeta + 1/\zeta)/2\,|\, \leqq 1\,), \quad (25)$$

and

$$B(\zeta) = A\lambda\,\overline{\varepsilon}(\zeta) + \bar{A}\,\overline{\varepsilon}^{\lambda}(\zeta)$$

$$(\,|\,(\zeta + 1/\zeta)/2\,|\, > 1\,). \quad (26)$$

Thus, the final forms of $2\phi(\zeta)$ and $2\chi(\zeta)$ will be represented in a power series as

$$2\phi(\zeta) = iC \sum_{k=1}^{\infty} A_k [\nu\,\omega(\zeta)/iC]^{\lambda k}, \quad (27)$$

$$2\chi(\zeta) = iC \sum_{k=1}^{\infty} B_k(\zeta) [\nu\,\omega(\zeta)/iC]^{\lambda k}, \quad (28)$$

$$B_k(\zeta) = A\lambda_k\,\overline{\varepsilon}(\zeta) + \bar{A}\,\overline{\varepsilon}^{\lambda k}(\zeta). \quad (28a)$$

General distributions of the stresses in a domain of interest may be expressed by substituting equation(27) and (28) into (7) and (8), with $\omega(\zeta)$ given in equation(11) or (12), as

$$\sigma_\xi = Re \sum_{k=1}^{\infty} (A_k \lambda_k \nu/2C)[\{2 - c_k(\zeta)\,\delta_1(\zeta)$$
$$- (1-\lambda_k)\,\delta_1(\zeta)\,\overline{\delta(\zeta)}\}(\nu\,\omega(\zeta)/iC)^{\lambda k-1}], \quad (29)$$

$$\sigma_\eta = Re \sum_{k=1}^{\infty} (A_k \lambda_k \nu/2C)[\{2 + c_k(\zeta)\,\delta_1(\zeta)$$
$$+ (1-\lambda_k)\,\delta_1(\zeta)\,\overline{\delta(\zeta)}\}(\nu\,\omega(\zeta)/iC)^{\lambda k-1}], \quad (30)$$

$$\tau_{\xi\eta} = Im \sum_{k=1}^{\infty} (A_k \lambda_k \nu/2C)[\{c_k(\zeta)\,\delta_1(\zeta)$$
$$+ (1-\lambda_k)\,\delta_1(\zeta)\,\overline{\delta(\zeta)}\}(\nu\,\omega(\zeta)/iC)^{\lambda k-1}], \quad (31)$$

where $\delta(\zeta)$, $\delta_1(\zeta)$ and $c_k(\zeta)$ signify $\delta(\zeta) = -\omega(\zeta)/\overline{\omega(\zeta)}$, $\delta_1(\zeta) = \omega'(\zeta)/\overline{\omega'(\zeta)}$ and $c_k(\zeta) = B_k(\zeta)/A_k$, respectively. Among the terms thus expanded attention will be focused on the terms of a dominant singularity in the following section.

4. DEFECT-TIP SINGULARITY AND θ - DEPENDENCE OF STRESSES

The amplitudes of stress singularities at a crack tip, being termed as stress intensity factors, are a widespread concept today. A general definition of them will be given by

$$K_I - iK_{II} = \lim_{\zeta \to \zeta_0} 2\phi'(\zeta)/\omega'(\zeta) \times$$
$$[e^{-i\delta} 2\pi \{\omega(\zeta) - \omega(\zeta_0)\}]^{1/2}, \quad (32)$$

where K_I and K_{II} are mode I and mode II stress-intensity factor respectively, $\omega(\zeta_0)$ the location of the crack tip, and δ the relative angle of the normal vector of the crack plane to the y-axis. The extention of the concept to a general angled defect would define stress singularity factors K_I and K_{II} for the defect, rewriting λ_1 as λ_I or λ_{II}, in accordance with the loading mode, as

$$K_I = Re \lim_{\zeta \to \zeta_0} 2\phi'(\zeta)/\omega'(\zeta) \times$$
$$[e^{-i\delta} \pi\nu \{\omega(\zeta) - \omega(\zeta_0)\}]^{1-\lambda I}, \quad (33)$$

and

$$K_{II} = -Im \lim_{\zeta \to \zeta_0} 2\phi'(\zeta)/\omega'(\zeta) \times$$
$$[e^{-i\delta} \pi\nu \{\omega(\zeta) - \omega(\zeta_0)\}]^{1-\lambda II}, \quad (33a)$$

where $1-\lambda_I$ and $1-\lambda_{II}$ denote strength of the mode I and mode II stress singularity, respectively. It is to be noted that a factor $\pi\nu$, which apperas in equations(33) and (33a), is defined as

$$\pi\nu = 2\pi - 2\beta, \quad (34)$$

and not 2π.

To examine defect tip singularities restrict attention to the domain $|\zeta - i| \ll 1$, where $\omega(\zeta)$ can be written as

$$\omega(\zeta) = (iC/\nu)\, e^{\mp i(\pi/2)\nu} [(\zeta + 1/\zeta)/2]^{\nu}. \quad (35)$$

If we describe the z-plane by polar coordinates with pole at the defect tip, $\zeta = i$, and θ the counter-clockwise angle with the y-axis, then

$$z = \omega(\zeta) = ir\, e^{i\theta} \quad (36)$$

The stresses in the immediate vicinity of the defect tip in polar coordinates can now be given as special cases of the general expressions(29) to (31), with $k = 1$, through the conversion formulae and writing λ_1 as λ, as follows:

For mode I loading, they are

$$\sigma_r = (ReA_1 \lambda\ \nu/2)[C/\nu r]^{1-\lambda} \times [(3-\lambda)\cos(1-\lambda)\theta$$
$$+ (\cos 2\lambda\ \gamma + \lambda \cos 2\gamma)\cos(1+\lambda)\theta], \quad (37)$$

$$\sigma_\theta = (ReA_1 \lambda\ \nu/2)[C/\nu r]^{1-\lambda} \times [(1+\lambda)\cos(1-\lambda)\theta$$
$$- (\cos 2\lambda\ \gamma + \lambda \cos 2\gamma)\cos(1+\lambda)\theta], \quad (38)$$

$$\tau_{r\theta} = (ReA_1 \lambda\ \nu/2)[C/\nu r]^{1-\lambda} \times [(1-\lambda)\sin(1-\lambda)\theta$$
$$- (\cos 2\lambda\ \gamma + \lambda \cos 2\gamma)\sin(1+\lambda)\theta], \quad (39)$$

and for mode II loading, they are

$$\sigma_r = (ImA_1 \lambda\ \nu/2)[C/\nu r]^{1-\lambda} \times [(3-\lambda)\sin(1-\lambda)\theta$$
$$+ (\cos 2\lambda\ \gamma - \lambda \cos 2\gamma)\sin(1+\lambda)\theta], \quad (37a)$$

$$\sigma_\theta = (ImA_1 \lambda\ \nu/2)[C/\nu r]^{1-\lambda} \times [(1+\lambda)\sin(1-\lambda)\theta$$
$$- (\cos 2\lambda\ \gamma - \lambda \cos 2\gamma)\sin(1+\lambda)\theta], \quad (38a)$$

$$\tau_{r\theta} = (ImA_1 \lambda\ \nu/2)[C/\nu r]^{1-\lambda} \times [(1-\lambda)\cos 1-\lambda)\theta$$
$$- (\cos 2\lambda\ \gamma - \lambda \cos 2\gamma)\cos(1+\lambda)\theta]. \quad (39a)$$

By applying the general definitions, equations(33) and(33a), to $2\phi(\zeta)$, equation(27), it is found that

$$K_I = ReA_1 \lambda\ \nu\,[\pi c]^{1-\lambda I}, \quad (40)$$

and

$$K_{II} = -ImA_1 \lambda\ \nu\,[\pi c]^{1-\lambda II}, \quad (40a)$$

where we write the real and imaginary part of A_1 as ReA_1 and ImA_1, respectively. In a limiting case of a crack, $\lambda_I = \lambda_{II} = 1/2$, equations(37) to (40a) lead to the conventional formulae.

5. CONCLUSIONS

Consecutively from previous work[1] the concept of the stress intensity in a crack problem was extended to an externally or internally cut sharp notch or defect with an arbitrary tip angle. It is shown that exact solutions for the general stress distributions and the stresses local to the tip of the internal defect can be derived by an effective use of the mapping function which composed the complex potentials.

APPENDIX Determination of C

By applying definition(1), $\omega(1) = c\tan\beta - ic$, to equation(5), we have

$$c\tan\beta - ic = C\int_0^1 dZ\,[Z^2/(Z^2-1)]^n$$

or equivalently

$$c\,e^{-i(\pi/2-\beta)}/\cos\beta = C\,e^{\mp i\pi n} I \quad (A1)$$

where the integral $I = \int_0^1 dZ\,[Z^2/(1-Z^2)]^n$ is found to be given by the betha function as

$$I = \int_0^1 dZ\ Z^{2n}(1-Z^2)^{-n}$$
$$= (1/2)B[(1+2n)/2,\ 1-n]. \quad (A2)$$

The last expression can be deformed in terms of the gamma function, $\Gamma(s)$, as

$$(1/2)B[(1+2n)/2,\ 1-n]$$
$$= (1/)\Gamma(1/2+n)\Gamma(1-n)/\Gamma(3/2)$$
$$= \{\Gamma(1/2+n)/\Gamma(1/2)\Gamma(n)\}\cdot\Gamma(n)\Gamma(1-n)$$
$$= \{1/B(1/2,n)\}(\pi/\sin\pi n)$$
$$= \{\pi/B(1/2,n)\}(1/\cos\beta) \quad (A3)$$

Remembering that $n = 1/2 - \beta/\pi$, it follows that C in equation(A1) is lead to

$$C = cB(1/2,n)/\pi, \quad (A4)$$

where it is assumed without loss of generality that C is real.

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任意先端角の菱形欠陥への応力強度（Stress intensity)概念の拡張

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モードIおよびモードII荷重の下で菱形欠陥の先端の周りに誘起される応力の特異性および一般的な応力分布をSchwartz-Christoffel変換の活用により解析的に導き、亀裂問題における応力強度(stress intensity)概念を前報に報告した表面V切欠きに引続き任意先端角の内部欠陥の解析に拡張した。応力分布の一般解は複素ポテンシャルを写像関数の固有値べき数展開することにより得られ、この場合欠陥先端近傍解が遠方の解にスムーズに接続できることを示すことができた。