An Approximation on the Discharge of Over-flow Spillways

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Abstract

In the preceding paper, the author proposed the method of determination of the control section in the over-flow from dam crest by applying to potential flow theory and estimation of the discharge coefficient in consideration of boundary layer development, and showed that, in case of that the over-flow depth are small, the decrease of discharge coefficient could be explained well. But, incomplete point still remain, for example, change of control section due to the development of boundary layer, the influence of gravity force on the development of boundary layer and the deviation of stream-line from coaxial circles on the discharge are not considered.

In this paper, the author intends to treat the case of broad over-flow spillway with vertical curvature which changed continuously, in considering the velocity component normal to botton surface and the influence of gravity force on the boundary layer development.

Introduction

The over-flow spillway is typical form of the discharge construction and widely used as ordinary or extraordinary discharge equipment on many dams. The determination of discharge coefficient of over-flow spillway is one of important items on hydraulic model experiment of individual dam.

Recently, the higher utilization of water resource is required, the higher accuracy of the discharge coefficient of over-flow spillway become a serious problem.

A large number of studies on the discharge coefficient have been done for a long time, and various formulas about it have been proposed. But, a large majority of those are experimental studies on individual dam model, and the theoretical studies in order to reduce the general expression of the discharge coefficient are limited.

The studies by Iwasaki[1], Ishii and Fujimoto[2,3], Fujimoto[4,5], Mura and Araki[6], Araki[7,8] are typical example of theoretical investigation. These studies are based on the potential flow theory fundamentally. The investigation by Araki in consideration of the velocity component perpendicular to the wall is worth noting. In these studies, there are still problems in relation to determination of the control section. And the investigation in consideration of the influence of the boundary layer development along the wall on the discharge coefficient has never been performed.

In preceding paper, it has been clarified that, in case of that over-flow depth are

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fairly small, the boundary layer development has an important role, because the boundary layer thickness are not negligible for over-flow depth relatively.

It may be expected that the influence of the velocity component perpendicular to the wall plays an important role on the discharge coefficient, as the discharge increase, the deviation of stream-line from coaxial circules are greater.

In this paper, the author intends to introduce the influence of the velocity component perpendicular to the wall into the determination of the location of control section as follow. At the first, we set up the general expression of fundamental equation in circular polar co-ordinate, and simplified by ordinary boundary layer approximation, as the velocity component perpendicular to the wall are relatively smaller than the velocity component parallel to the wall, and then, proposed to determine the location of control section by numerical method with trial and error.

Fundamental equation in cylindrical polar co-ordinate

At the time when we set up the general relation on the flow in open channel with vertical curvature which are continuously varied, it is convenient to use the cylindrical polar co-ordinates, and to denote the vector velocity of fluid at the point (r, θ, z), its components parallel to the co-ordinate axes being denoted by (v_r, v_θ, v_z).

The fluid being assumed incompressible. The equation of motion and the equation of continuity are written as follow.

$$\left.\begin{aligned}
&\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+v_z\frac{\partial v_r}{\partial Z}-\frac{v_\theta^2}{r}\\
&\qquad =X_r-\frac{1}{\rho}\frac{\partial P}{\partial r}+\nu\left\{\nabla^2 v_r-\frac{v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right\}\\
&\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+v_z\frac{\partial v_r}{\partial Z}+\frac{v_\theta v_r}{r}\\
&\qquad =X_\theta-\frac{1}{\rho}\frac{\partial P}{\partial r}+\nu\left\{\nabla^2 v_\theta+\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{v_\theta}{r^2}\right\}\\
&\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial Z}\\
&\qquad =X_z-\frac{1}{\rho}\frac{\partial P}{\partial Z}+\nu\{\nabla^2 v_z\}
\end{aligned}\right\} \qquad (1)$$

$$\operatorname{div}\boldsymbol{v}=\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial Z}=0 \qquad (2)$$

where,
$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial Z^2}.$$

The motion being two-dimensional, we may take Z as the co-ordinate at right angle to the plane of the motion. Then, v_z is zero and all quantities are independent of Z.

For the curves $\theta=$const. we take the normals to the wall, and for the curves $r=$const. we take the curves parallel to the wall, each of which intersects the normals at a constant distance from the wall. Then $R\theta$ is the distance measured along the wall from a fixed point, for flow past a cylinder, is taken as a forward stagnation point. While r is the normal distance from the wall. It will cause no confusion, and will allow us to treat plane and curved wall together. If we use x and y for $R\theta$ and r $(=R+y,\ R;$ curvature radius of the wall), so that, quite generally, x and y are distance along and perpendicular to the wall.

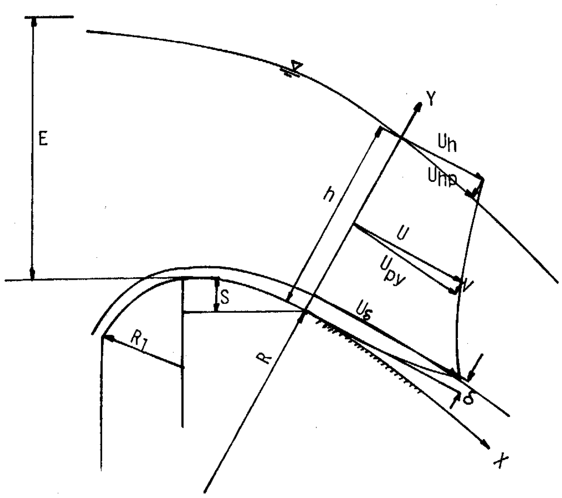

Fig. 1 Schematic appearance of flow and typical notation.

Rewrite u, v to replace v_θ, v_r as the velocity components parallel and perpendicular to the wall, respectively. If we now work out the component of the vector equation of motion, and write down the equation of continuity, we find

$$\left.\begin{aligned}
&\frac{\partial u}{\partial t}+\frac{R}{R+y}u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{R}{R+y}uv=g\sin\theta-\frac{1}{\rho}\frac{R}{R+y}\frac{\partial P}{\partial x}\\
&\qquad+\nu\left\{\frac{R^2}{(R+y)^2}\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{R\cdot y}{(R+y)^3}\frac{\partial R}{\partial x}\frac{\partial u}{\partial x}\right.\\
&\qquad\quad\left.+\frac{1}{R+y}\frac{\partial u}{\partial y}-\frac{u}{(R+y)^2}-\frac{R}{R(+y)^3}\frac{\partial R}{\partial x}v+\frac{2R}{(R+y)^2}\frac{\partial v}{\partial x}\right\}\\
&\frac{\partial v}{\partial t}+\frac{R}{R+y}u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{u^2}{R+y}=g\cos\theta-\frac{1}{\rho}\frac{\partial P}{\partial x}\\
&\qquad+\nu\left\{\frac{R^2}{(R+y)^2}\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{Ry}{(R+y)^3}\frac{\partial R}{\partial x}\frac{\partial v}{\partial x}\right.\\
&\qquad\quad\left.+\frac{1}{R+y}\frac{\partial v}{\partial y}-\frac{v}{(R+y)^2}+\frac{R}{(R+y)^3}\frac{\partial R}{\partial x}u-\frac{2R}{(R+y)^3}\frac{\partial u}{\partial x}\right\}
\end{aligned}\right\} \tag{3}$$

$$\frac{\partial u}{\partial x}+\frac{\partial}{\partial y}\left\{\frac{R+y}{R}v\right\}=0. \tag{4}$$

Flow over spillway

On the past studies on the flow over spillway, it is confirmed that the velocity distribution obtained from potential flow theory, except in vicinity of wall, agree with experimental value on a great part of flow cross a section. Based on this fact, in much the same way as boundary layer theory, it can be treat the flow over spillway with vertical curvature varied continuously, by dividing in two parts, potential flow occupied the major part of flow and boundary layer flow near the wall.

BOUNDARY LAYER; If we carry out a process of ordinary boundary layer approximation for Eq. (3). They reduce approximately to

$$\left.\begin{aligned} \frac{R}{R+y}u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{uv}{R+y}&=g\sin\theta-\frac{1}{\rho}\frac{R}{R+y}\frac{\partial P}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \\ -\frac{u^2}{R+y}&=-g\cos\theta-\frac{1}{\rho}\frac{\partial P}{\partial y}. \end{aligned}\right\} \tag{5}$$

Rearrange left hand of Eq. (5) by substituting Eq. (4),

$$\left.\begin{aligned} -u^2\frac{\partial(v/u)}{\partial y}&=g\sin\theta-\frac{1}{\rho}\frac{R}{R+y}\frac{\partial P}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \\ \frac{u^2}{R+y}&=g\cos\theta+\frac{1}{\rho}\frac{\partial P}{\partial y}. \end{aligned}\right\} \tag{6}$$

MAIN FLOW; In this region, the viscous terms may be neglected safely in accordance with the general custom. It can be considered as the velocity component perpendicular to the wall are relatively small compared with the velocity component parallel to the wall ($v/u \ll 1$). In other words, the boundary layer approximation hold in this flow regions. Then Eq. (3) reduce approximately to

$$\left.\begin{aligned} \frac{R}{R+y}u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{uv}{R+y}&=g\sin\theta-\frac{1}{\rho}\frac{R}{R+y}\frac{\partial P}{\partial y} \\ \frac{u^2}{R+y}&=g\cos\theta+\frac{1}{\rho}\frac{\partial P}{\partial y}. \end{aligned}\right\} \tag{7}$$

According to Bernoulli's equation, using symbols in Fig. 1.

$$E+S=\frac{P}{\rho g}+y\cos\theta+\frac{u_{py}^2}{2g}. \tag{8}$$

If we eliminate P from Eq. (7) and (8).

$$\frac{\partial u}{\partial y}=\frac{u}{R+y} \quad \text{or} \quad \frac{\partial}{\partial y}\{u(R+y)\}. \tag{9}$$

If we may take U_{pr} as the velocity at water surface ($y=h$), Eq. (9) reduce to following relation

$$\frac{U_{py}}{U_{ph}}=\frac{R+h}{R+y}. \tag{10}$$

Substitute the condition at water surface which $P/\rho=0$, at $y=h$ into Eq. (8), we fined that the velocity at water surface is as follow,

$$U_{ph}=\sqrt{2g(E+S-h\cos\theta)}. \tag{11}$$

Although there are some problems, in accordance with general custom, we assume that the velocity distribution in boundary layer was as follow.

$$\frac{U}{U_\delta}=\left(\frac{y}{\delta}\right)^{\frac{1}{n}}. \tag{12}$$

Where, U_δ is the velocity at outer edge of boundary layer, n is constant, for Blassius low $n=7$, and for manning-strickler resistance low $n=6$.

Let us carry on the relation between the discharge and specific energy of over-flow, considering the velocity component perpendicular to the wall and the boundary layer development.

If we work out the comparison of the velocity component, we find the relation between the velocity parallel and perpendicular to the wall.

$$U=U_{py}/\sqrt{1+(v/U_{py})^2}. \tag{13}$$

We set up the discharge per unit width by integrating Eq. (10)

$$\frac{q}{\sqrt{2gE}\cdot E}=\frac{R+h}{E}\left\{\int_0^\delta\left(\frac{y}{\delta}\right)^{\frac{1}{n}}\Big/\sqrt{1+\left(\frac{v}{u_{py}}\right)^2}dy\right.$$
$$\left.+\int_\delta^h\frac{1}{R+y}\Big/\sqrt{1+\left(\frac{u}{u_{py}}\right)^2}\right\}\sqrt{1+\frac{S}{E}-\frac{h}{E}\cos\theta}. \tag{14}$$

If we now neglect $(v/u_{py})^2$ for 1 to first approximation.

$$\frac{q_p}{\sqrt{2gE}\cdot E}=\frac{R+h}{E}\left\{\frac{n}{n+1}\frac{\delta}{R+\delta}+\ln\left(\frac{R+h}{R+\delta}\right)\right\}\sqrt{1+\frac{S}{E}-\frac{h}{E}\cos v}. \tag{15}$$

Furthermore, if we allow to neglect the boundary layer development, they reduce

$$\frac{q_n}{\sqrt{2gE}\cdot E}=\frac{R+h}{E}\ln\left(\frac{R+h}{R}\right)\sqrt{1+\frac{S}{E}-\frac{h}{E}\cos\theta}. \tag{16}$$

Let us set up the distribution of the velocity component perpendicular to the wall approximately. At the first approximation, it can be allow to rewrite Eq. (15) as the velocity parallel to the wall.

$$u=\frac{q}{(R+y)\left\{\frac{n}{n+1}\frac{\delta}{R+\delta}+\ln\left(\frac{R+h}{R+\delta}\right)\right\}}. \tag{17}$$

Differentiate above equation with respect to x, they reduce

$$\frac{\partial u}{\partial x}=\frac{-q}{(R+y)(R+h)\left\{\frac{n}{n+1}\frac{\delta}{R+\delta}+\ln\left(\frac{R+h}{R+\delta}\right)\right\}^2}\frac{dh}{dx}$$
$$+\frac{q\left\{\frac{n}{n+1}\frac{\delta}{R+\delta}-1\right\}}{(R+y)(R+\delta)\left\{\frac{n}{n+1}\frac{\delta}{R+\delta}+\ln\left(\frac{R+h}{R+\delta}\right)\right\}^2}\frac{d\delta}{dx}$$
$$+\frac{q\left\{\frac{n}{n+1}\delta(\delta-y)(R+h)-(R+y)(R+\delta)(h-\delta)+(R+\delta)^2(R+h)\ln\left(\frac{R+h}{R+\delta}\right)\right\}}{(R+y)^2(R+\delta)^2(R+h)\left\{\frac{n}{n+1}\frac{\delta}{R+\delta}+\ln\left(\frac{R+h}{R+\delta}\right)\right\}^2}\frac{dR}{dx}. \tag{18}$$

Integrate Eq. (4) by substituting Eq. (18) into

$$\frac{a}{u}=\frac{-\ln\left(\frac{R+y}{R\mp\delta}\right)}{\left(\frac{R+h}{R}\right)\left\{\frac{n}{n+1}\frac{\delta}{R+\delta}+\ln\left(\frac{R+h}{R+\delta}\right)\right\}}\frac{dh}{dx}$$
$$+\frac{\left\{\frac{n}{n+1}\frac{\delta}{R+\delta}-1\right\}\ln\left(\frac{R+y}{R}\right)}{\left(\frac{R+\delta}{R}\right)\left\{\frac{n}{n+1}\frac{\delta}{R+\delta}+\ln\left(\frac{R+h}{R+\delta}\right)\right\}}\frac{d\delta}{dx}$$
$$+\frac{\frac{n}{n+1}\frac{R\cdot\delta}{(R+\delta)^2}\left\{\frac{y}{R}\frac{R+\delta}{R+y}-\ln\left(\frac{R+y}{R}\right)\right\}-\frac{R(h-\delta)}{(R+h)(R+\delta)}\ln\left(\frac{R+y}{R}\right)-\frac{R}{R+y}\ln\left(\frac{R+h}{R+\delta}\right)}{\left\{\frac{n}{n+1}\frac{\delta}{R+\delta}+\ln\left(\frac{R+h}{R+\delta}\right)\right\}}\frac{dR}{dx}. \tag{19}$$

If we allow to neglect the boundary layer development, Eq. (17), (18) and (19) reduce respectively to

$$u=\frac{q}{(R+y)\ln\left(\frac{R+h}{R}\right)} \tag{20}$$

$$\frac{\partial u}{\partial x}=\frac{-q}{(R+y)(R+h)\ln^2\left(\frac{R+h}{R}\right)}\frac{dh}{dx}$$
$$+\frac{q\left\{h(R+y)-R(R+h)\ln\left(\frac{R+h}{R}\right)\right\}}{R(R+y)^2(R+h)\ln^2\left(\frac{R+h}{R}\right)}\frac{dR}{dx} \tag{21}$$

$$\frac{v}{u}=\frac{-\ln\left(\frac{R+y}{R}\right)}{\frac{R+h}{R}\ln\left(\frac{R+h}{R}\right)}\frac{dh}{dx}+\left\{\frac{\ln\left(\frac{R+y}{R}\right)}{\frac{R+h}{h}\ln\left(\frac{R+h}{R}\right)}+\frac{R}{R+y}\right\}\frac{dR}{dx}. \quad (22)$$

Differentiate Eq. (14), (15) and (16) with respect to x, under the condition flow discharge does not change with the location.

$$\left(\frac{dR}{dx}+\frac{dh}{dx}\right)\left\{\int_0^\delta\left(\frac{y}{\delta}\right)^{\frac{1}{n}}\Big/\sqrt{1+\left(\frac{v}{u_{py}}\right)^2}dy+\int_\delta^h\frac{1}{R+y}\Big/\sqrt{1+\left(\frac{v}{u_{py}}\right)_2}dy\right\}$$

$$+(R+h)\frac{d}{dx}\left\{\int_0^\delta\left(\frac{y}{\delta}\right)^{\frac{1}{n}}\Big/\sqrt{1+\left(\frac{v}{u_{py}}\right)^2}dy+\int_\delta^h\frac{1}{R+y}\Big/\sqrt{1+\left(\frac{v}{u_{py}}\right)_2}dy\right\}$$

$$+\frac{R+h}{2(E+s-h\cos\theta)}\left\{\int_0^\delta\left(\frac{y}{\delta}\right)^{\frac{1}{n}}\Big/\sqrt{1+\left(\frac{v}{u_{py}}\right)^2}dy+\int_\delta^h\frac{1}{R+y}\Big/\sqrt{1+\left(\frac{v}{u_{py}}\right)^2}dy\right\}$$

$$\left\{\frac{dS}{dx}-\cos\theta\frac{dh}{dx}-\frac{h}{R}\sin\theta\left(1-\theta\frac{dR}{dx}\right)\right\}=0 \quad (23)$$

$$\frac{dh}{dx}=\Bigg[\Bigg[\left(\frac{n}{n+1}\frac{\delta(R+h)}{(R+\delta)^2}-\frac{h-\delta}{R+\delta}\right.$$

$$+\left\{\frac{n}{n+1}\frac{h-\delta}{R+\delta}+\ln\left(\frac{R+h}{R+\delta}\right)\right\}\left\{1-\frac{h(R+h)\theta\sin\theta}{2R(E+S-h\cos\theta)}\right\}\Bigg]\frac{dR}{dx}$$

$$+\left\{\frac{n}{n+1}\frac{R(R+h)}{(R+\delta)^2}-\frac{R+h}{R+\delta}\right\}\frac{d\delta}{dx}$$

$$+\left\{\frac{n}{n+1}\frac{\delta}{R+\delta}+\ln\left(\frac{R+h}{R+\delta}\right)\right\}\frac{R+h}{2(E+S-h\cos\theta)}\left(\frac{dS}{dx}-\frac{h}{R}\sin\theta\right)\Bigg]$$

$$\Bigg/\left[1+\left\{\frac{n}{n+1}\frac{\delta}{R+\delta}+\ln\left(\frac{R+h}{R+\delta}\right)\right\}\left\{1-\frac{R+h}{2(E+S-h\cos\theta)}\right\}\right] \quad (24)$$

$$\frac{dh}{dx}=\Bigg[\left[\frac{h}{R}-\left\{1-\frac{h(R+h)\theta\sin\theta}{2R(E+S-h\cos\theta)}\right\}\ln\left(\frac{R+h}{R}\right)\right]\frac{dR}{dx}$$

$$+\frac{R+h}{2(E+S-h\cos\theta)}\ln\left(\frac{R+h}{R}\right)\left(\frac{h}{R}\sin\theta-\frac{dS}{dx}\right)\Bigg]$$

$$\Bigg/\left[1+\left\{1-\frac{(R+h)\cos\theta}{2(E+S-h\cos\theta)}\right\}\ln\left(\frac{R+h}{R}\right)\right]. \quad (25)$$

Those equation are fundamental equation for non-uniform flows in consideration with centrifugal force, the velocity component perpendicular to the wall being neglected in Eq. (24), and furthermore, the boundary layer development being neglected in eq. (25).

Boundary layer development

The fluid being assumed incompressible, using displacement thickness

$$\frac{\delta_*}{\delta}=\int_0^1\left(1-\frac{u}{u\delta}\right)d\eta, \quad \eta=y/\delta \tag{26}$$

and momentum thickness

$$\frac{\theta}{\delta}=\int_0^1\frac{u}{u_\delta}\left(1-\frac{u}{u_\delta}\right)d\eta$$

the momentum equation of boundary layer in consideration with gravity force being written as follow

$$\frac{\theta}{\delta}\frac{d\delta}{dx}=\left(\frac{u_*}{u_\delta}\right)^2+\frac{\delta/E}{u_\delta/\sqrt{2gE}}\left(1-\frac{\delta_*}{\delta}-2\frac{\theta}{\delta}\right)\frac{d(u_\delta/\sqrt{2gE})}{d(x/E)}$$
$$+\frac{1}{2}\frac{\delta/E}{u_\delta^2/2gE}\frac{d(P/\rho gE)}{d(x/E)}-\frac{1}{2}\frac{\delta/E}{u_\delta^2/2gE}\sin\theta. \tag{27}$$

Where, U_δ is the velocity at outer edge of boundary layer, U_* is shear velocity, for blasius low

$$\left(\frac{u_*}{u_\delta}\right)^2=0.0225\Big/\left(\frac{u_\delta\cdot\delta}{\nu}\right)^{\frac{1}{4}}, \quad \frac{\delta_*}{\delta}=\frac{1}{8}, \quad \frac{\theta}{\delta}=\frac{7}{72} \tag{28}$$

for manning-strickler resistance low

$$\left(\frac{u_*}{u\delta}\right)^2=0.0125\left(\frac{k_s}{\delta}\right)^{\frac{1}{3}}, \quad \frac{\delta_*}{\delta}=\frac{1}{7}, \quad \frac{\theta}{\delta}=\frac{3}{28}. \tag{29}$$

If we assume that the velocity at outer edge of boundary layer being increase linearly from zero in region αR_1 up-stream dam crest, in according to Pohlhausen approximation, the momentum thickness at dam crest being written as follow

$$\left(\frac{\theta}{E}\right)_{x=0}=0.0784\alpha\frac{R_1}{E}\Big/\left\{\frac{\sqrt{2gE}\cdot E}{\nu}\cdot\frac{u_{ph}}{\sqrt{2gE}}\frac{R_1+h}{R_1+\delta}\right\}. \tag{30}$$

A approximation to determine the control section

It is possible to determine the location of control section by applying Belange theorem to Eq. (14) substituted Eq. (19) into. But, this method involves great difficulties in solving the deduced equation by numerical integration. So that, the author propose a convenient approximation to determine the location of control section as following procedure.

At the first approximation, the boundary layer being neglected.

1) Obtain critical depth at each section by putting as numerator of Eq. (25) being zero.
2) Calculate dh/dx by Eq. (25) for several depth close to critical depth at each section, and then, obtain dh/dx for critical depth by interpolate with those values, because dh/dx being infinitive in case of $h=h_c$.
3) Estimate the values of v/u by substituting h_c and dh/dx which being required in procedure 1), 2) into Eq. (22).
4) Estimate the discharge at each section by integrating Eq. (14) substituted v/u of 3).
5) The control section can be obtained as the location at where the discharge being minimum. In following step, we take the boundary layer development into consideration.
6) As initial condition to calculate the development of boundary layer, using a suitable value of, the thickness of boundary layer at dam crest can be required from Eq. (30).
7) If we assume that the velocity at outer edge of boundary layer are same to the velocity calculated by Eq. (10), (11) and (13), using h for y. Then, the development of boundary layer can be calculated by integrating Eq. (27) numerically with try and error method.

The over-flow discharge can be required by repeating above procedure until the discharge of 4) step and the location of control section of 5) step converge.

Summary

In order to establish general expression on the discharge coefficient of over-flow spillways with vertical curvature varied continuously, we have proposed an approximation to determine the location of control section in consideration with the velocity component perpendicular to the wall. There are still problems that numerical and experimental investigation are not performed. The comparison of the experimental results in two-dimensional with calculated results by proposed approximation will reported at a later date.

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