Pattern Recognition by the Use of Karhunen-Lóeve Orthogonal System Applied to the Reformed Pattern

Toyoshi TORIOKA*

Abstract

A explanation is given to a method which forms the reformed pattern. And a method is also developed for pattern recognition by the use of Karhunen-Lóeve orthogonal system applied to the reformed pattern. Still more, computer-simulated experiments are carried into effects in order to the usefulness of the present method.

1. Introduction

A method is proposed for pattern recognition, in which 3×3 elements codes are used in order to form the reformed pattern and Karhunen-Lóeve system are applied to the reformed pattern. Karhunen-Lóeve system have been studied as a method for pattern recognition [1]~[3], and some results were reported in [4]~[6]. A extended method in reference [6] is proposed here, of which procedure are as follow. (1) Input patterns are lowered in dimension by a preprocessor to give the reformed patterns (3×3 codes). (2) Feature-coefficients are given by applying Karhunen-Lóeve method to these patterns, where the concept of auto-correlation is used. (3) When an unknown pattern is given, it is also lowered in dimension and then recognized by projecting it to the patterns of feature-coefficients thus obtained. It is essential for the present method to eliminate unnecessary part of information from a given pattern and to reformed it into an intrinsic pattern of a lower dimension. The degree of dependence is defined on the set of 3×3 codes and the relation between the recognition rate and the degree of dependence is also given.

2. Pre-Processing

When a given pattern is written in a 15×15 square for example, it becomes 225 dimensional vector so that its autocorrelation is represented 255×255 symmetric matrix. According to this cause, a great deal of memory-capacity and processing time is required to obtain the feature-coefficients from this matrix. A method of calculating the autocorrelation matrix in much lower dimension should be developed, even if the dimensionality of the original pattern is large.

* Department of Electrical Engineering, Technical Junior College of Yamaguchi University

Now, a pattern of 15×15 quantized mesh is scanned and its geometric parts (dege points, turning points) which coincide with prescribed 3×3 codes patterns are extracted, where original patterns and codes patterns take on the value 1 or 0 on the respective meshes[7]. The reader who wants to know the detail of the adopted code patterns can reder to the reference [6]. Now, let S be a set of 3×3 codes and C be a set of extracted features, denoted by

$$S=\{C^i \mid i=1\sim 22\} \qquad \cdots\cdots\cdots(1)$$

$$C=\{C^j \mid j=1\sim q\} \qquad \cdots\cdots\cdots(2)$$

where C^i is a 3×3 code matrix, $\mathbf{q}$ is the number of extracted code elements and C contains the inherent information of the original pattern. Next, let $\mathbf{F}_i$ be a map, expressed by

$$y_i=F_i(C^i)=(y_{i1},\ldots, y_{i16}) \qquad \cdots\cdots\cdots(3)$$

where y_i contains the same information C^i and some example of concrete maps, $\mathbf{F}_i(\mathbf{C}^i)$**'s** are showed **in Table 2.1.** In this Table 2.1, $\mathbf{C}_{mn}^i$ indicate the element of codes C^i. Still more, if a matrix Y is formed by these y_i's as follow.

$$Y=\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_i \\ \vdots \\ y_q \end{pmatrix}=\begin{pmatrix} F_1(C^1) \\ F_2(C^2) \\ \vdots \\ F_i(C^i) \\ \vdots \\ F_q(C^q) \end{pmatrix}=\begin{pmatrix} y_{11}, y_{12}, \cdots\cdots\cdots y_{116} \\ y_{21}, y_{22}, \cdots\cdots\cdots y_{216} \\ \vdots \quad \vdots \qquad\qquad \vdots \\ y_{i1}, y_{i2}, \cdots\cdots\cdots y_{i16} \\ \vdots \quad \vdots \qquad\qquad \vdots \\ y_{q1}, y_{q2} \cdots\cdots\cdots y_{q16} \end{pmatrix} \qquad \cdots\cdots\cdots(4)$$

then this matrix Y becomes an inherent matrix of original pattern whose elements takes on value 1 or 0. Furthermore, a simplified pattern should be reformed from matrix Y so as not to lose the characteristics or tendency of the original pattern as much as possible. If component x_i of the simplified pattern is defined by

$$x_i=\sum_{s=1}^{q} y_{si} \qquad \cdots\cdots\cdots(5)$$

then the pattern X is denoted by

$$X=[x_1, x_2, \cdots x_{16}]=[\sum_{s=1}^{q} y_{s1}, \sum_{s=1}^{q} y_{s2}, \cdots\cdots, \sum_{s=1}^{q} y_{s16}] \qquad \cdots\cdots\cdots(6)$$

This reformed vector X which is obtained by summing up the feature in matrix Y seems to preserve the characteristic configuration and tendency of the pattern. In other word, the vector represents a version of the original pattern which is lowered in dimension. The vector will contain sufficient information of Eq.(2). Finally, by a simple normalization, the normalized low-dimensional pattern vector X is obtained and this vector is called **the reformed pattern vector.**

Table 2.1 A set of maps, $Fi(C^i)'s$, Case(A)

Map Fi / Codes C^i	Meaning of the Maps Fi
C^1	$F_1=\{y_{1k}=C^1_{mn} \mid 1\leq m,\ n\leq 3 \text{ else } y_{1k}=0\}$
C^2	$F_2=\{y_{2k}=C^2_{mn} \mid 1\leq m,\ n\leq 3 \text{ else } y_{2k}=0\}$
C^3	$F_3=\{y_{3k}=C^3_{m'n} \mid m'=m+1\ 1<m,\ m\leq 3 \text{ else } y_{3k}=0\}$
C^4	$F_4=\{y_{4k}=C^4_{m'n} \mid m'=m+1\ 1\leq m,\ n\leq 3 \text{ else } y_{4k}=0\}$
C^5	$F_5=\{y_{5k}=C^5_{m'n'} \mid m'=m+1,\ n'=n+1\ 1\leq m,\ n\leq 3 \text{ else } y_{5k}=0\}$
C^6	$F_6=\{y_{6k}=C^6_{m'n'} \mid m'=m+1,\ n'=n+1\ 1\leq m,\ n\leq 3 \text{ else } y_{6k}=0\}$
C^7	$F_7=\{y_{7k}=C^7_{mn'} \mid n'=n+1\ 1\leq m,\ n\leq 3 \text{ else } y_{7k}=0\}$
C^8	$F_8=\{y_{8k}=C^8_{mn'} \mid n'=n+1\ 1\leq m,\ n\leq 3 \text{ else } y_{8k}=0\}$
C^9	$F_9=\{y_{9k}=C^9_{mn} \mid 1\leq m,\ n\leq 3 \text{ else } y_{9k}=0\}$
C^{10}	$F_{10}=\{y_{10k}=C^{10}_{m'n} \mid m'=m+1\ 1\leq m,\ n\leq 3 \text{ else } y_{10k}=0\}$
C^{11}	$F_{11}=\{y_{11k}=C^{11}_{mn} \mid 1\leq m,\ n\leq 3 \text{ else } y_{11k}=0\}$
C^{12}	$F_{12}=\{y_{12k}=C^{11}_{mn} \mid 1\leq m,\ n\leq 3 \text{ else } y_{12k}=0\}$
C^{13}	$F_{13}=\{y_{13k}=C^{13}_{m'n} \mid m'=m+1\ 1\leq m,\ n\leq 3 \text{ else } y_{13k}=0\}$
C^{14}	$F_{14}=\{y_{14k}=C^{14}_{m'n} \mid m'=m+1\ 1\leq m,\ n\leq 3 \text{ else } y_{14k}=0\}$
C^{15}	$F_{15}=\{y_{15k}=C^{15}_{mn} \mid m'm-1\ 2\leq m\leq 3,\ 1\leq n\leq 3 \text{ else } y_{15k}=0\}$
C^{16}	$F_{16}=\{y_{16k}=C^{16}_{m'n'} \mid m'=m+1\ 1\leq m,\ n\leq 3 \text{ else } y_{16k}=0\}$
C^{17}	$F_{17}=\{y_{17k}=C^{17}_{m'n} \mid m'=m+1\ 1\leq m,\ n\leq 3 \text{ else } y_{17k}=0\}$
C^{18}	$F_{18}=\{y_{18k}=C^{18}_{mn} \mid 1\leq m,\ n\leq 3 \text{ else } y_{18k}=0\}$
C^{19}	$F_{19}=\{y_{19k}=C^{19}_{m'n} \mid n'=n+1\ 1\leq m,\ n\leq 3 \text{ else } y_{19k}=0\}$
C^{20}	$F_{20}=\{y_{20k}=C^{20}_{m'n'} \mid m'=m+1\ n'=n+1\ 1\leq m,\ n\leq 3 \text{ else } y_{20k}=0\}$
C^{21}	$F_{21}=\{y_{21k}=C^{21}_{m'n'} \mid m'=m+1\ n'=n+1\ 1\leq m,\ n\leq 3 \text{ else } y_{21k}=0\}$
C^{22}	$F_{22}=\{y_{22k}=C^{22}_{m'n'} \mid m'=m+1\ n'=n>1\ 1\leq m,\ n\leq 3 \text{ else } y_{22k}=0\}$

3. The Karhunen-Lóeve Expansion

The basic idea of Karhunen-Lóeve system consits in expanding a pattern vector with a set of orthonormal vectors in order to extract the necessary information as much as possible. Let M and N be the numbers of the pattern classes and the typical pattern, respectively. Then, the set of typical input pattern in denoted by

$$X_N=\{X_m^{(n)} \mid m=1\sim M, n=1\sim N\}. \qquad \cdots\cdots\cdots(7)$$

Moreover, let B_m be a set of k orthonormal vectors

$$B_m=\{\beta_m{}^{(i)}, | i=1,2,\ldots\ldots, k\} \qquad \ldots\ldots\ldots(8)$$

$$(\beta_m^{(i)}, \beta_m^{(j)})=\delta_{m,}^{(j)} \quad \delta_m^{(i,j)}: \text{Kronecker's } \delta \qquad \ldots\ldots\ldots(9)$$

defined for each pattern class m, where k is the number of dimension of vectors. An input pattern is generally expanded by this B_m as follow.

$$X_m^{(n)}=a_{m1}^{(n)}\cdot\beta_m^{(1)}+a_{m2}^{(n)}\cdot\beta_m^{(2)}+\cdots\cdots+a_{mk}^{(n)}\cdot\beta_m^{(k)} \qquad \ldots\ldots\ldots(10)$$

where

$$a_{mi}^{(n)}=(\beta_m^{(i)}\cdot X_m^{(n)}) \qquad \ldots\ldots\ldots(11)$$

In the Karhunen-Lóeve expansion, the base B_m is chosen with reference to the stochastic properties of patterns belonging to class m. Let $P_m(X)$ be probability of occurrence of X in class m. Then the auto-correlation matrix is given by

$$G_m=\sum_{n=1}^{N} P_m(X_m^{(n)})\cdot X_m^{(n)t}\cdot X_m^{(n)} \qquad \ldots\ldots\ldots(12)$$

for the pattern of class m. The set B_m of the eigen vectors of Eq.(12) is called the Karhunen-Lóeve orthogonal system or shortly K-L system.

4. Method for Decision

In the expansion Eq.(10) of a pattern belong to class m by the K-L system $\{\beta_m^{(i)}\}$, the coefficient $a_{ms}^{(n)}$ are expected to vanish approximately,

$$a_{m,r+1}^{(n)}=a_{m,r+2}^{(n)}=\cdots\cdots=a_{m,k}^{(n)}=0 \qquad \ldots\ldots\ldots(13)$$

for s larger than r, where r is a certain integer $(r\geq 1)$. But this is not the case, if a pattern $X_m^{(n)}$ is expanded by the system $B'_m(m'\neq m)$. Therefore, the absolute value of an unknown pattern

$$T_m(X)=\sum_{i=1}^{k}(a_{mi})^2 \qquad a_{mi}=(\beta_m^i\cdot X) \qquad \ldots\ldots\ldots(14)$$

is nearly equal to

$$T_m(X)=\sum_{i=1}^{r}(a_{mi})^2 \qquad \ldots\ldots\ldots(15)$$

if X belongs to class m, but is much less than $T_m(X)$, if X does not belong to class m. Hence, $T_m(X)$ plays the role of a discriminant function for class m. Let

$$d(X)=\max_m\{T_m(X)|\ m=1\sim M\} \qquad \ldots\ldots\ldots(16)$$

An unknown pattern X is categorized into class m, if $T_m(X)$ has the largest value $d(X)$.

5. The Degree of Dependence

The vectors y_i, mapped 3×3 codes C_i, has only 16 components. For this reason, it is impossible that the vectors y_i's are formed independently to one another and the necessary features are extracted by these codes. But it is desirable condition for a system of pattern recognition that all feature are independent, since non-redundant codes carry more information than any other. The fact is obvious from information theory.

Now, the degree I_{ij} of dependence is defined. It is explained in the below whether or not the fact described abovs is truth really. Let us divide the set E of vectors y_i's into 3 subclasses as follow

$$E=\{E_1, E_2, E_3\} \quad \cdots\cdots\cdots(17)$$

where E_1, E_2, E_3 express the sets of codes vectors representing edge points, turning points of two arcs and turning points of three arcs, respectively. Then I_{ij} is defined by

$$I_{ij}=\sum_{s=1}^{16} y_{is}\cdot y_{js}/\sqrt{\sum_{s=1}^{16} y_{is}^2}\cdot\sqrt{\sum_{s=1}^{16} y_{js}^2} \quad \cdots\cdots\cdots(18)$$

Still more, if $I(E_\alpha, E_\beta)$ denotes the degree of dependence between subclass E_α and E_β, then $I(E_\alpha, E_\beta)$ is given from Eq.(18)

$$I(E_\alpha,E_\beta)=\begin{cases}\sum_{i=1}^{l_1}\sum_{j=1}^{l_2} I_{ij}/l_1\cdot l_2 & \text{:if } \alpha<\beta \\ \sum_{i=1}^{l_1}\sum_{j=1}^{l_1} I_{ij}/\{l_1\cdot(l_1-1)/2\} & \text{:if } \alpha=\beta\end{cases} \quad \cdots\cdots\cdots(19)$$

where l_1, l_2 are the number of elements in class E_α and E_β. From Eq. (18) and Eq.(19), I_{ij} and $I(E_\alpha, E_\beta)$ are calculated in three different sets of maps, $F_i(C^i)$'s (see Table 2.1). These different sets are named **case (A), case (B)** and **case (C)** respectively from now. The result of $I(E_\alpha, E_\beta)$ computed for the sets E_1, E_2, E_3 is given in **Table 5.1** and a example of the result I_{ij} is given in **Table 5.2.**

Table 5.1 The result of $I(E_\alpha, E_\beta)$

$I(E_\alpha,E_\beta)$ / Case	$I(E_1, E_1)$	$I(E_1, E_2)$	$I(E_1, E_3)$	$I(E_2, E_2)$	$I(E_2, E_3)$	$I(E_3, E_3)$
Case (A)	0.0714	0.2278	0.1083	0.4071	0.1885	0.4672
Case (B)	0.0714	0.2278	0.1365	0.4071	0.1384	0.5034
Case (C)	0.0714	0.2188	0.1492	0.4071	0.2555	0.3949

Table 5.2 The result of Iij between subclass E_1 and E_2, Case (A)

E_2 \ E_1	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
y_9	0.3162	0.6324	0.3162	0.3162	0.0000	0.0000	0.0000	0.3162
y_{10}	0.0000	0.3162	0.6324	0.3162	0.3162	0.0000	0.0000	0.0000
y_{11}	0.6324	0.6324	0.3162	0.3152	0.0000	0.0000	0.0000	0.0000
y_{12}	0.0000	0.3536	0.3536	0.3536	0.0000	0.0000	0.0000	0.3536
y_{13}	0.0000	0.3162	0.6324	0.6324	0.0000	0.3162	0.0000	0.0000
y_{14}	0.0000	0.0000	0.6324	0.6324	0.3162	0.6324	0.0000	0.0000
y_{15}	0.4082	0.8265	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

6. Experimental Results

To illustrate the above method for pattern recognition, computer-simulated experiments were carried out. Handwritten English characters A ~ H were given as pattern classes. The examples of the reformed patterns, obtained by Eq. (1)–(6) are shown **in Table 6.1**. Next, in order to calculate the K-L system, the following two probability distributions are assumed.

$$(1)\quad P_m(X)=1/10,\qquad N=10 \qquad \cdots\cdots\cdots(20)$$

$$(2)\quad P_m(X)=1/15,\qquad N=15 \qquad \cdots\cdots\cdots(21)$$

Table 6.1 Reformed patterns, Case (A)

Pattern \ Dimension	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$X{\cdot}X^t$
A	0	1	2	2	0	4	1	2	1	2	2	1	2	0	0	0	44
B	0	0	0	0	1	1	0	1	1	2	2	0	2	1	2	0	21
C	1	2	0	0	1	0	1	0	0	0	0	0	0	0	0	0	7
D	1	1	1	0	2	0	0	0	2	0	0	0	1	1	1	0	14
E	1	3	0	0	3	1	0	0	1	1	1	1	1	2	1	0	30
F	2	3	1	1	0	2	0	1	1	1	1	1	0	1	0	0	25
G	1	2	1	1	3	1	2	1	1	0	1	0	1	1	1	0	27
H	0	1	1	1	2	2	1	1	1	1	2	1	2	0	1	0	25

Table 6.1 Reformed patterns, Case (B)

Dimension / Pattern	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$X \cdot X^t$
A	0	1	3	2	0	1	3	4	1	3	1	0	0	0	0	1	52
B	0	2	0	0	0	2	3	2	0	3	1	0	0	0	0	1	32
C	1	2	0	0	1	0	1	0	0	0	0	0	0	0	0	0	7
D	1	1	1	0	2	0	0	0	2	0	0	0	1	1	1	0	14
E	2	3	0	0	2	2	1	1	2	0	2	0	2	2	2	0	43
F	2	4	1	1	0	2	1	2	1	1	0	0	0	0	0	0	33
G	1	2	0	1	2	0	1	2	1	1	1	1	1	1	1	1	23
H	0	1	1	1	1	0	1	3	1	2	1	2	1	0	1	1	27

Table 6.1 Reformed patterns, Case (C)

Dimension / Pattern	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$X \cdot X^t$
A	0	2	2	0	1	1	5	0	1	1	1	0	0	0	0	1	39
B	1	1	1	1	1	1	3	0	1	1	2	0	0	0	0	1	22
C	0	2	2	0	1	1	1	1	1	0	0	0	0	0	0	0	13
D	1	0	0	0	2	1	1	0	2	0	0	0	1	1	1	0	14
E	2	3	0	0	4	1	1	0	2	0	0	0	1	1	1	0	38
F	4	3	2	0	1	2	0	2	0	0	0	0	0	0	0	0	42
G	0	3	2	1	2	0	3	0	0	0	1	0	0	0	1	1	30
H	0	2	1	0	1	1	2	1	1	2	1	2	0	1	1	1	25

where N is the number of typical patterns. The K-L system obtained from $P_m(X)$ is given in **Table 6.2,** where the vectors corresponding to the largest eigenvalue only are shown ($r=1$). Next, two recognition experiments were carried out in the case of 72 unknown samples. Some experimental results are shown in **Table 6.3.** The maximum correct recognition rate, 92.2 percent was obtained from the experiments described above. The result is comparable with the correct recognition rate 98.0 percent by human being. Consequently, it is concluded that result is fairly good. The prcentage of correct recognition

Table 6.2 Eigen Vectors (KL-System), Case (A) and $P_m(X)=\frac{1}{15}$.

0.083587	0.086461	0.538681	0.309324
0.104289	0.067046	0.608192	0.261469
0.248154	0.182541	0.292387	0.244782
0.036493	0.004475	0.039627	0.033191
0.103623	0.250323	0.367754	0.548835
0.686938	0.454350	0.135155	0.027417
0.096423	0.056488	0.178072	0.095543
0.162326	0.043148	0.190667	0.033387
0.094783	0.145231	0.160201	0.528652
0.288435	0.431039	0.015515	0.027536
0.461602	0.555348	0.015514	0.114559
0.258038	0.290267	0.015513	0.027656
0.098057	0.057993	0.038957	0.280065
0.104872	0.275159	0.039416	0.166190
0.105478	0.035772	0.039928	0.207843
0.061225	0.004471	0.015517	0.033581
0.555838	0.557036	0.201591	0.094423
0.572115	0.570591	0.473119	0.103934
0.062412	0.137099	0.080385	0.008735
0.006389	0.202844	0.146526	0.277639
0.235799	0.156826	0.227188	0.121000
0.199524	0.234053	0.162398	0.608352
0.048355	0.057020	0.393664	0.033956
0.043107	0.238739	0.591845	0.240924
0.207608	0.137100	0.013445	0.201708
0.154614	0.190102	0.036980	0.052742
0.279647	0.223496	0.280328	0.488192
0.134563	0.172153	0.013843	0.178481
0.170420	0.042494	0.013450	0.348714
0.266749	0.159968	0.021439	0.082160
0.159512	0.063679	0.203246	0.099885
0.037875	0.008287	0.045127	0.069171

Table 6.3 The percentage of correct recognition

$P_m(X)$ / Case	Probability		the number of sample patterns
	$P_m(X)=\frac{1}{10}$	$P_m(X)=\frac{1}{15}$	
A	87.5	89.1	72
B	87.5	90.6	72
C	87.5	92.2	72

is also plotted for I_s in **Fig. 6.1,** where $I_s=\frac{1}{3!}\sum_{\alpha=1}^{3}\sum_{\substack{\beta=1\\ \alpha<\beta}}^{3} I(E_\alpha,E_\beta)$. From this Fig. 6.1, contradiction to the fact in section 5 become clear. But it is judged that this contradiction be brought by the result which lose the characteristic configuration and tendency of the original pattern by the preprosessing describ-ed above. The block diagram of pattern recognition system are shown **Fig. 6.2.**

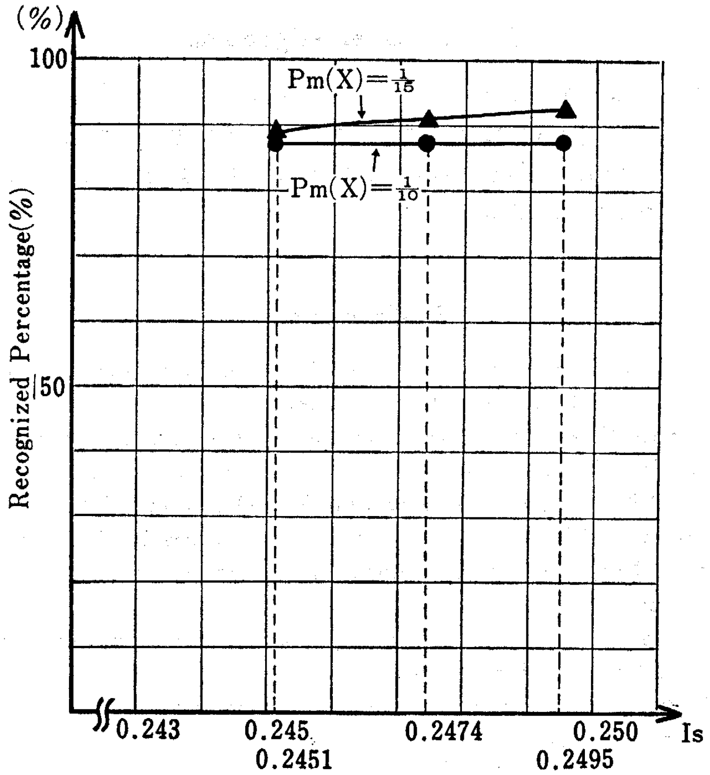

Fig. 6.1. The relation between the degree of dependence and recognized percentage.

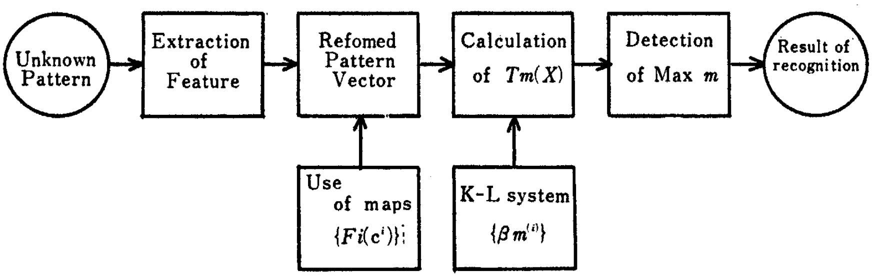

Fig. 6.2 Block diagram for pattern recognition system.

7. Conclusions

By the above results, the following three conclusions were obtained.

(1) A desirable percentage of correct recognition was obtained by the present method, which reduced given patterns in dimension by pre-processing.

(2) The processing-time and memory capacity were reduced sufficiently.

(3) Desirable result were obtained by choosing a apt set of maps, $\{F_i(C^i)\}$.

By conclusions (1) and (2), it is known that this method is more practical than the method studied earlier. A guide how to form a set of maps is given by conclusion (2). And **the extended method** in reference [6] were developed by introducing a set of **maps,** $\{\mathbf{F}_i(\mathbf{C}^i)\}$.

8. Acknowledgment

The author wishes to thank Prof. T. Hirata of the Yamaguchi University as well as Prof. S. Amari of the University of Tokyo for their kind discussion and valuable comments. He also thanks to miss Y. Nishimura for helping him in computer programming.

References

1) T. Iijima: "Theory of pattern recognition", Jour. Inst. Electoronics. Comm. Engrs. Japan, Vol. 46, No. 1, pp. 1582–1590 (1963) (in Japanese).
2) T. Iijima: "Basic Theory on feature extraction for visual pattern", Jour. Electoronics Comm. Engrs. Japan, Vol. 46, No. 11, pp. 1714–1721 (1963) (in Japanese).
3) S. Watanabe: "Knowing and Guessing", pp. 380–403, Wiley (1969)
4) S. Noguchi, et. al.: "Theory of feature extraction for pattern by Karhunen-Lóeve orthogonal system", Trans. Inst. Electoronics Comm. Engrs. Japan, Vol. 53–c, No. 12, pp. 897–903 (1970) (in Japanese).
5) Y. T. Chien and K. S. Fu: "Selection and observation in a pattern recognition system", Information and Control, 12, pp. 375–414 (1968)
6) T. Sera: "Pattern recognition by the codes of 3×3 elements with Karhunen-Lóeve orthogonal system", Information processing in Japan, Vol. 13, No. 4, pp. 210–217 (April 1972) (in Japanese)
7) T. Sera: "Consideration on the reformed pattern", Mem. Fac. Eng. Yamaguchi Univ., 22, pp. 235–241 (1972) (in Japanese)