Image Restoration for Small Computer Requirements

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Abstract

The problem solved in this paper is formulated as: given an observation of an noisy image against the background and given the statistics of the image and the noise, find an estimate of the image such that the computer-time and storage requirements of the estimator are modest.

Two-dimensional line scanning image model is applied to get the reasonable state space equation.

The resulting image restoration is performed by the scalar Kalman filter, smoother. Vector processing is also posslble to process simultaneously all rows of the image matrix.

1. Introduction

In recent years, there have been many attempts for estimating images from noisy data. In image processing, there are two techniques, non-recursive type and recursive type.

The non-recursive image processing techniques involve, for example, the inverse filter and the Wiener filter. These methods, however, have some difficulties and limitations as an image processing system. The computational burden can be quite high. In special cases, these problem may be somewhat alleviated with the use of FFT algorithms, but the required amount of calculation is still substantial.

The recursive image processing is desirable in general from the view point of computer burden. It has been shown that great computational savings have been attained over non-recursive methods[1]. The mild drawback of recursive method is the reqiurement of model constructions. But the potential gains in computational efficiency are a distinct advaneage.

A number of works have been made concerning these approaches. One-dimensional processing of the scan-ordered image is an approach to recursive processing of images[2].

All the recursive scan techniques are basically one dimensional. Recently, however, two dimensional recursive models for images have been considered[3),4].

In this paper, we develop a convinient algorithm for image restoration, in order to procese the noisy image data by use of small computer systems. An image model is the same one that was considered by Habibi[3] and Attasi[4].

The state space equation developed in this paper is so simple that it is very easy to implement the ordinary Kalman filter, smoother for image restoration.

Simulation work is presented to test the proposed algorithm.

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2. Dynamical Model of Image

We consider the monochromatic picture represented by a matrix intensities (gray levels) of dimension $M \times N$, with elements $x\ (i, j)\ (i=1, 2, \ldots, M; j=1, 2, \ldots, N)$. It is assumed that the image belongs to the class of images whose autocorrelation function can be separately represented by

$$\varepsilon\ [x(m,n)\ x\ (m+i, n+j)] = R_{ij} = \sigma^2 exp\ (-\gamma_1 \mid i \mid - \gamma_2 \mid j \mid). \quad (1)$$

ε represents the expectation operation. Given R_{00}, R_{10} and R_{01}, σ^2, γ_1 and γ_2 are calculated respectively as follows.

$$\sigma^2 = R_{00} \quad (2)$$

$$\gamma_1 = ln\ R_{00} - ln\ R_{10} \quad (3)$$

$$\gamma_2 = ln\ R_{00} - ln\ R_{01} \quad (4)$$

The image is assumed to be generated by the following model.

$$x\ (i,j) = a_1 x\ (i-1, j) + a_2 x\ (i, j-1) - a_1 a_2 x\ (i-1, j-1) + w\ (i,j) \quad (5)$$

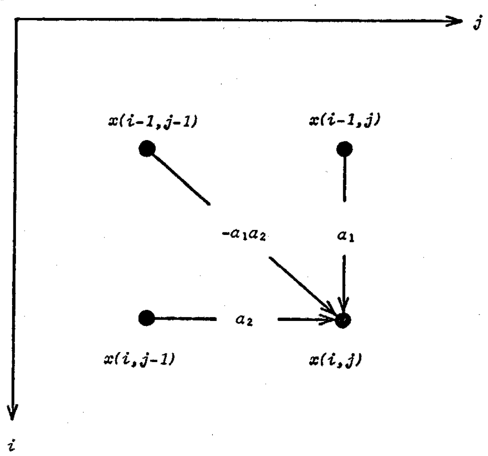

Fig. 1 Image model.

Fig. 1 represents this two-dimensional line-scanner model. Parameters a_1 and a_2 are the vertical and the horizontal correlation coefficients of the random field of the image, respectively and w is assumed to be white, gaussian noise with variance W. Then, they are represented as follows.

$$a_1 = exp\ (-\gamma_1) \quad (6)$$

$$a_2 = exp\ (-\gamma_2) \quad (7)$$

$$W = [1-(a_1)^2]\ [1-(a_2)^2] R_{00} \quad (8)$$

Noisy observation of an image $x(i, j)$ is represented by

$$y(i,j)=x(i,j)+v(i,j) \tag{9}$$

where v is white, gaussian noise whose variance is given as V.

The problem of image restoration is to obtain the estimate $x(i, j)$ of $x(i, j)$ from noisy observation image data Eq. (9).

3. Design of the Estimator

Consider the image model of Eq. (5). Given *a priori* informations $x(i, 1)$ for $i=1, 2, \ldots, M$, the following equation is obtained by neglecting the outside pixels of the image.

$$x(1, j+1)=a_2x(1, j)+w(1, j+1) \tag{10}$$

Added one to i in Eq. (5), the equation becomes to

$$x(2, j+1)=a_2x(2, j)+a_1w(1, j+1)+w(2, j+1) \tag{11}$$

where the relation of Eq. (10) is used. In general we have the following equations for $i=1, 2, \ldots, M; j=1, 2, \ldots, N-1$.

$$x(i,j+1)=a_2x(i,j)+\xi(i,j) \tag{12}$$

where

$$\xi(i,j)=\sum_{l=1}^{i}(a_1)^{l-1}w(i+1-l,j+1) \tag{13}$$

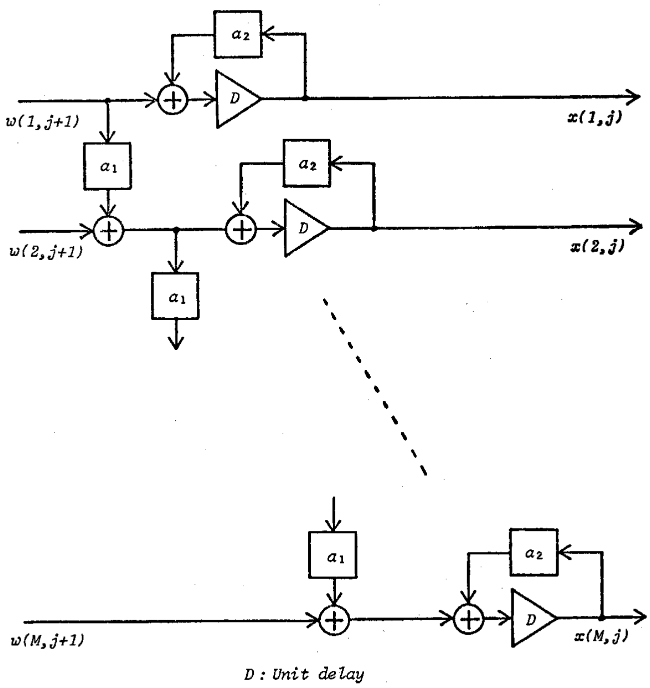

Fig. 2 Schematic illustration of the image model.

The schematic illustration of the developed state space model of an image is represented in Fig. 2.

The properties of noise ξ are shown as follows, for w is zero mean white, gaussian noise.

$$\varepsilon \{\xi(i,j)\}=0 \tag{14}$$

$$Var\{\xi(i,j)\}=W[1-(a_1)^{2i}]/(1-a_1) \tag{15}$$

Corresponding j to time t in Eq. (12), we can apply the Kalman filtering technique. Thus applying M Kalman filter, smother equations to Eq. (12), we can obtain image estimation algorithm represented in Table 1. The one-step smothing algorithm is adopted as the smoother due to its reasonability.

Table 1 Image Estimation Algorithm

Filtering:

$$\hat{x}(i,j+1/j+1)=\hat{x}(i,j+1/j)+K(i,j+1)[y(i,j+1)-\hat{x}(i,j+1/j)]$$
$$\hat{x}(i,j+1/j)=a_2\hat{x}(i,j/j)$$
$$K(i,j+1)=P(i,j+1/j)/[P(i,j+1/j)+V]$$
$$P(i,j+1/j)=(a_2)^2P(i,j/j)+W[1-(a_1)^{2i}]/(1-a_1)$$
$$P(i,j+1/j+1)=[1-K(i,j+1)]\ P(i,j+1/j)$$

Smoothing:

$$\hat{x}(i,j/j+1)=\hat{x}(i,j/j)+G(i,j)\ [\hat{x}(i,j+1/j+1)-\hat{x}(i,j+1/j)]$$
$$G(i,j)=a_2P(i,j/j)/P(i,j+1/j)$$
$$P(i,j/j+1)=P(i,j/j)+G^2(i,j)\ [P(i,j+1/j+1)-P(i,j+1/j)]$$

4. Experimental Results

Simulation work was performed on images of size 64×128 pixels with 32 gray levels to test the proposed algorithm. Fig. 3 shows the original image. Noisy images and restored images are represented in Fig. 4-Fig. 9, where the line printer is used as an image output equipment.

Signal to noise ratio (SNR) is defined as

SNR=peak to peak value/$(2\sqrt{V})$.

Improvement ρ (dB) with respect to restored image is also defined as follows.

$$\rho\,(dB)=10\ log_{10}\{\sum_{i=1}^{64}\sum_{j=1}^{128}[y(i,j)-x(i,j)]^2/\sum_{i=1}^{64}\sum_{j=1}^{128}[\hat{x}(i,j/j+1)-x(i,j)]^2\}$$

Table 2 Improvement ρ versus SNR.

SNR	5.0	3.0	2.5	2.1	1.5	1.25	1.0	0.75
ρ(dB)	3.90	5.19	5.69	6.13	7.31	8.00	8.36	10.25

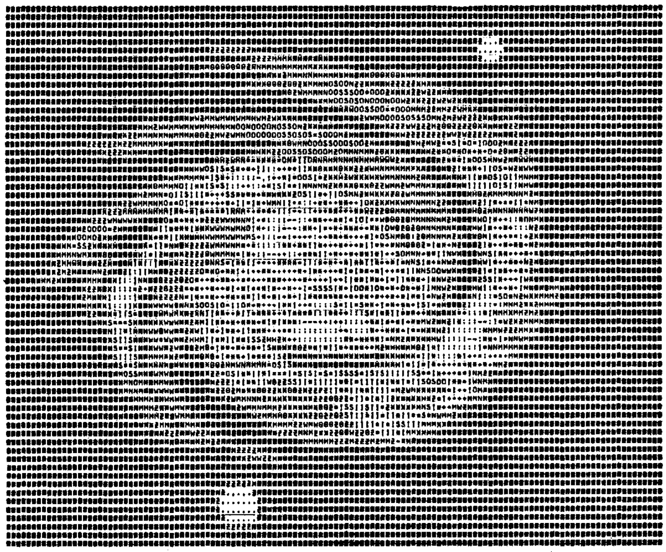

Fig. 3 Original image.

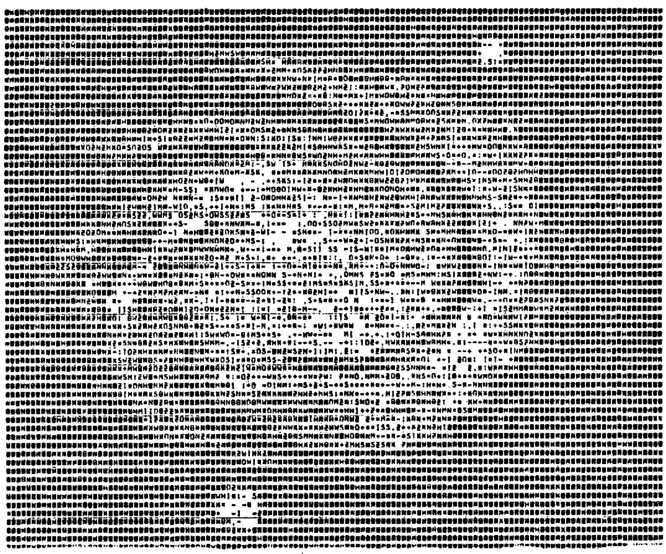

Fig. 4 Noisy image (SNR=3).

Table 2 represents the relation of ρ (dB) versus SNR.

As the value of SNR decrecses, the image improvement ρ represents good value. Intuitively, it seems that the efficiency of noise suppression becomes high under the large value of noise variance. It is observed that the restored image is blurred to the direction of scanning. This may be the general drawback when using the scanning image models. This phenomena are distinctly observed under low SNR.

We note also that the resolution is sacrificed for the aid of noise suppression effect because the Kalman filter is based on a minimum mean square estimation criterion. This problem is deeply concerned with the edge detection problem of an image.

It is apparent that the proposed image restoration technique has intuitively appealing

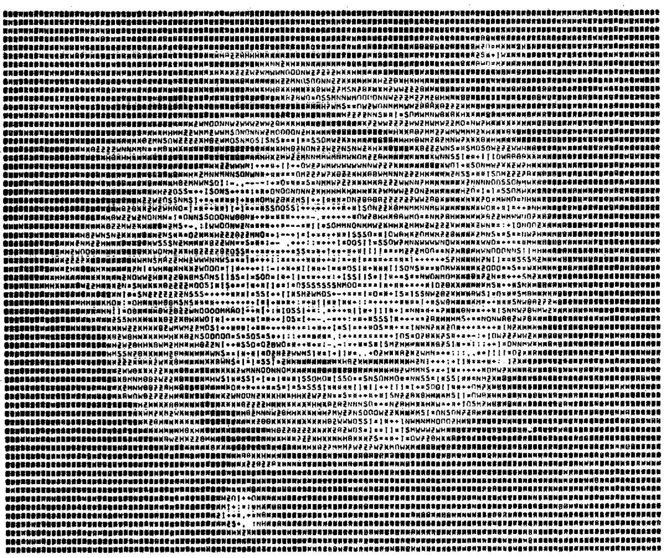

Fig. 5 Restored image (SNR=3).

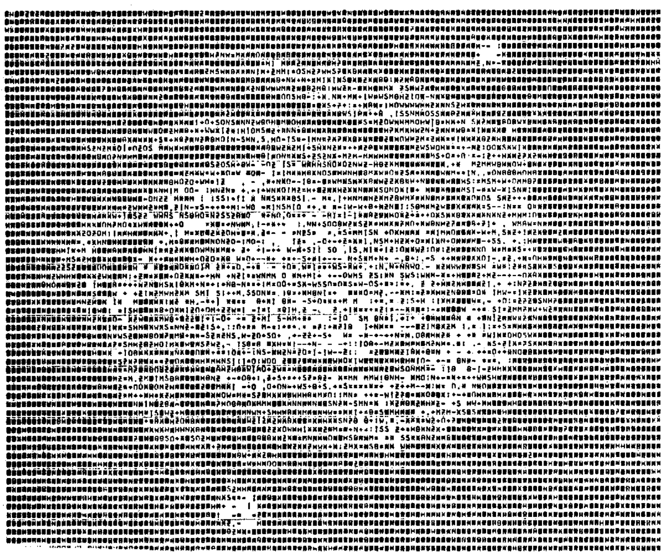

Fig. 6 Noisy image (SNR=2.5).

structure because it is possible to calculate simultaneously all rows of image matrix with a number of processing units such as micro-processors. Thus, we may expect to reduce extremely computation time in multi-processing systems.

Unfortunately the comparison with the proposed method and other methods had not been attempted for computational burden concerning other methods is so large that it is impossible to implement with the computer system by which the experimental results were obtained.

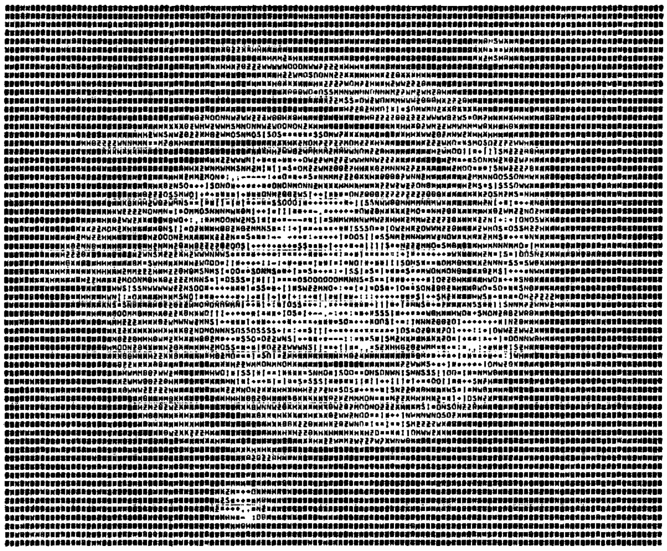

Fig. 7 Restored image (SNR=2.5).

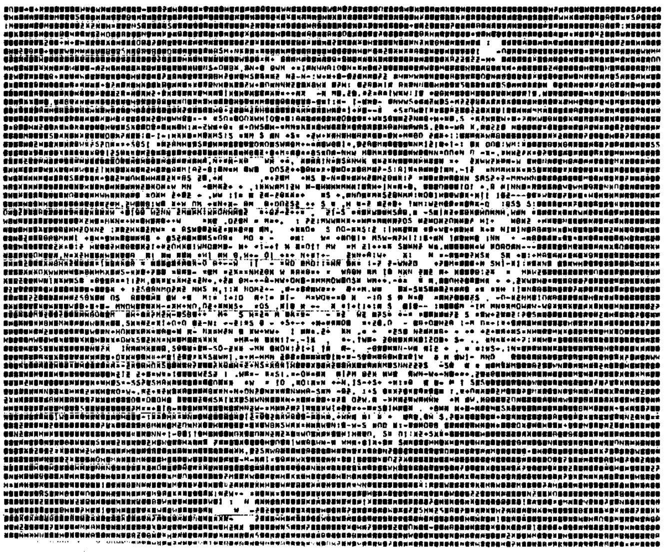

Fig. 8 Noisy image (SNR=1.5).

5. Conclusions

A very simple image restoration algorithm is developed by the use of the ordinary Kalman filter, smoother. The application of this estimator is illustrated by the experimentation.

The procedure is applicable to images characterized statistically by mean and correlation function. The limitation is the requirement of separability of the image covariance, but it allows one to obtaine detailed results. Large class of images in the real world satisfies these limitations.

In the context of noise suppression, an important problem is concerened with the

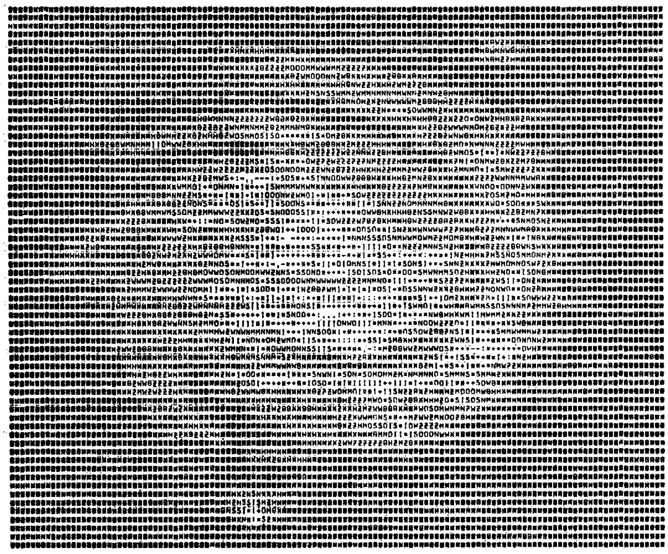

Fig. 9 Restored image (SNR=1.5).

detection of edges or boundaries between different regions in an image. The proposed method is not sufficient for this problem. This is the key questions for the future.

References

1) Willsky, A. S. "Digital Signal Processing and Control and Estimation Theory" (MIT Press, Mass) p. 138 (1979).
2) Nahi, N, E, and Franco, C. A. "Recursive Image Enhancement-Vector Processing," IEEE Trans. Comm. COM-21, 305-311 (1973).
3) Habibi, A. "Two-Dimensional Bayesian Estimate of Images," Proc. IEEE, 60' 878-883 (1972).
4) Attasi, S. "Modelling and Recursive Estimation for Double Indexed Sequences," in System Identification: Advances and Case Studies, edited by Mehra, R. K. and Lainiotis, D. G. (Academic Press, N. Y.) p. 289 (1976).